Electro-optics.

**Electro-optic Effect.**

Electro-optic material.

\[ \text{Electric field} \rightarrow \text{light} \]

The E-O effect is the change in the refractive index resulting from the applied Q or DC or low frequency electric field.

- A field applied to an anisotropic electro-optic material modifies its refractive index and thereby its effect on planar light.

- Refractive index changes that are proportional to \( E_{\text{applied}} \) - linear electro-optic effect or the Pockels effect.

- Refractive index changes \( \propto (E_{\text{applied}})^2 \) - quadratic electro-optic effect or the Kerr Effect.

Suppose \( \Delta n \sim 10^{-5} \) if we travel \( 10^5 \) wavelengths.

1 wavelength \( \sim 1 \mu m \) \( \Rightarrow \)

\[ k = \frac{2\pi n d}{\lambda} \rightarrow k(n+\Delta n)d. \]

\[ k \Delta n = \frac{2\pi n d}{\lambda} = 2\pi \Rightarrow \text{additional } 2\pi \text{ phase shift.} \]
Why interesting?

1. Lens made of a material whose \( n = n(E_{\text{applied}}) \) ⇒ controllable focal length
2. Prism whose beam bending is controllable can be used for a scanning device.
3. Optical phase modulator.
   \[
   \text{phase of light controllable.} \quad n \rightarrow n + \pi \alpha
   \]
4. Controllable wave retarders from anisotropic crystals.
5. Polarization rotation switch.

\[
\Gamma(E) = e^{i \alpha d E^2}
\]

Pockels Effect

\[n(E) = n + a_1 E + \frac{1}{2} a_2 E^2 + \ldots\]

\[n = n(E=0), \quad a_1 = (\frac{dn}{dE})|_{E=0}, \quad a_2 = (\frac{d^2n}{dE^2})|_{E=0}.
\]

We write

\[
\gamma = -\frac{2a_1}{n^3}, \quad \delta = -\frac{a_2}{n^3}.
\]

\[\Rightarrow n(E) = n - \frac{1}{2} \gamma n^3 E - \frac{1}{2} \delta n^3 E^2 + \ldots
\]

typically very small terms

Recall \( \eta = \frac{e_0}{\varepsilon} = \frac{1}{n^2} \)

\[
\Delta \eta = \left(\frac{dn}{dn}\right) \Delta n = \left(-\frac{2}{n^3}\right) \left(\frac{1}{2} \gamma n^3 E - \frac{1}{2} \delta n^3 E^2\right)
\]

\[= \gamma E + \delta E^2.
\]
\[ \eta(E) = \eta + rE + sE^2 : \text{impermeability}. \]

\[ \eta = \eta(0) \]

\(r\) explains definition of \(r\) & \(s\). \(\eta\) has simple form.

- **Pockels Effect**

\[ sE^2 \ll rE. \]

\[ \Rightarrow \eta(E) = \eta - \frac{1}{2} r n^3 E \]

\(r\) - Pockels coefficient or linear electro-optic coefficient.

\(\text{Pockels medium. (Pockels cell)}\)

\[ 10^{-12} < r < 10^{-10} \text{ m/V} \quad (1 \rightarrow 100 \text{ pm/V}). \]

Example: \(E = 10^6 \text{ V/m}\) \(10\text{kV across 1 cm}\)

\[ \frac{1}{2} r n^3 E \sim 10^{-6} \text{ to } 10^{-4}. \]

- **Pockels cells**: \(NH_4H_2PO_4 (ADP), \text{KH}_2PO_4 (KDP), \text{LiNbO}_3, \text{LiTaO}_3, \text{CdTe}.\)

- **Kerr Effect**

Centrosymmetric materials: liquids, gases, certain crystals

\(\eta(E)\) must be an even symmetric function

\[ \Rightarrow \eta(E) = \eta - \frac{1}{2} s n^3 E^2 \]

Kerr medium (or a Kerr cell). \(s\) - Kerr coefficient or quadratic electro-optic coefficient.

\[ 10^{-18} < s < 10^{-14} \text{ m}^2/\text{V}^2 \text{ in crystals}. \]

\[ 10^{-22} < s < 10^{-19} \text{ m}^2/\text{V}^2 \text{ in liquids}. \]

\[ E = 10^6 \text{ V/m} \quad \frac{1}{2} s n^3 E^2 \sim 10^{-6} \text{ to } 10^{-2} \text{ in crystals}. \]

\[ 10^{-10} \text{ to } 10^{-7} \text{ in liquids}. \]
**Electro-optic Modulators and Switches**

**Phase modulator**

\[ \phi = \eta(E) k_0 L \]
\[ = \frac{\pi \eta(E) L}{\lambda_0} \]

**Longitudinal modulator**

(Plane wave \( e^{i k_0 z} \) plane)

\( \lambda_0 \) - free space wavelength.

\[ \phi = \phi_0 - \pi \frac{n^3 E L}{\lambda_0} \]
\[ \phi_0 = \frac{2 \pi n L}{\lambda_0} \]

\( E = \frac{V}{d} \).

\[ \phi = \phi_0 - \pi \frac{V}{V_{\pi}} \]

\[ V_{\pi} = \frac{d \lambda_0}{L \eta n^3} \]

**Half-wave Voltage.**

\( V_{\pi} \) - half-wave voltage - phase changes by \( \pi \)

⇒ We can modulate the phase with application of the voltage.

\( V_{\pi} \) - important characteristic of the modulator.

- depends on material properties \( (\tau \& n) \).
- depends on \( \lambda_0 \).
- depends on aspect ratio \( \frac{d}{L} \).

If \( V \) applied along length ⇒ \( d = L \).

\( \tau \) - depends on direction of propagation.

- because crystal is anisotropic.

\[ V_{\pi} \sim 1 \rightarrow 10^3 \text{ V} \text{ for longitudinal modulation.} \]

100's V for transverse mods.
Modulation speeds: MHz $\rightarrow$ GHz, easily.

Dynamic Wave retarders.

- Anisotropic medium: Two normal modes $\frac{c_0}{n_1}, \frac{c_0}{n_2}$ see $n_1$ and $n_2$.
  Application of electric field $E$ modifies two refractive indices:

  $$n_1(E) = n_1 - \frac{1}{2} r_1 n_1^3 E$$

  $$n_2(E) = n_2 - \frac{1}{2} r_2 n_2^3 E$$

  $r_1, r_2$ = Pockels coefficients. After distance $L$
  phase retardation:

  $$\Gamma = k_0 (n_1(E) - n_2(E)) L$$

  $$= k_0 (n_1 - n_2) L - \frac{1}{2} k_0 (n_1 n_1^3 - n_2 n_2^3) E L.$$

- Applying voltage $V$ between two surfaces $B$ in the medium.

  $\Gamma = \Gamma_0 - \pi \frac{V}{V_{\pi}}$

  Phase retardation.

  $$\Gamma_0 = k_0 (n_1 - n_2) L \quad \text{phase retardation with } E=0.$$

  $$V_{\pi} = \frac{d}{\lambda_0} \frac{\lambda_0}{n_1 n_1^3 - n_2 n_2^3}.$$

  ⇒ electrical controllable dynamic wave retarder.

- Intensity Modulators: Use of a phase modulator in an Interferometer
Mach-Zehnder interferometer

\[ I_0 = \frac{1}{2}I_i + \frac{1}{2}I_i \cos \phi = I_i \cos^2 \frac{\phi}{2} \]

\( \phi = \phi_1 - \phi_2 \) - difference between phase shifts encountered by light

\[ T = \frac{I_0}{I_i} = \cos^2 \left( \frac{\phi}{2} \right) \]

Add modulator in branch 1 \( \Rightarrow \phi_1 = \phi_0 - \frac{\pi V}{V_{\text{II}}} \)

\[ \Rightarrow \phi = \phi_1 - \phi_2 = \phi_0 - \frac{\pi V}{V_{\text{II}}} \quad \phi_0 = \phi_0 - \phi_2 \]

\[ T(V) = \cos^2 \left( \frac{\phi_0}{2} - \frac{\pi V}{V_{\text{II}}} \right) \]

\( \phi_0 = \frac{\pi}{2}, T=0.5 \) or \( \phi_0 = n\pi \quad T(0) = 1 \quad T(V_{\text{II}}) = 0 \)

modulator switches the light on/off as \( V \quad 0/V_{\text{II}} \).

Intensity Modulators: Use of a retarder between crossed polarizer.
\[ T = \sin^2 \left( \frac{\Gamma}{2} \right) \]

If retarder is a Pockels cell, \( \Gamma \) linearly dependent on applied voltage \( \sqrt{V} \).

\[ \Rightarrow \quad \Gamma = \Gamma_0 - \frac{\pi V}{V_{\pi}} \quad \Rightarrow \quad T = \sin^2 \left( \frac{\Gamma_0}{2} - \frac{\pi V}{2 V_{\pi}} \right) \]

Linear operation: cell is biased to point B.

\[ \Gamma_0 = \frac{\pi}{2} \quad V \ll V_{\pi}. \]

\[ T(V) = \sin^2 \left( \frac{\pi}{4} - \frac{\pi V}{2 V_{\pi}} \right) = T(0) + \frac{dT}{dV} \bigg|_{V=0} V = \frac{1}{2} - \frac{\pi V}{2 V_{\pi}} \]

Taylor series expansion.

\[ \frac{\pi}{2 V_{\pi}} \] - sensitivity of modulator.

\( \Gamma_0 \) - adjusted optically (adding phase retarder, a compensator) or electrically by adding a constant bias voltage to \( V \).

i.e., \( V = V_0 + V_{\text{mod}} \).

Ratio between maximum and minimum transmittances is called the extinction ratio.

\[ \frac{T_{\text{max}}}{T_{\text{min}}} > 1000:1 \text{ are possible.} \]
Prism as Scanner.

\[ \theta_d = \theta - \alpha + \sin^{-1}\left(\frac{(n^2 - \sin^2 \theta)^{1/2} \sin \alpha}{\sin \theta \cos \alpha}\right), \]
determined by applying Snell's law twice.

if \( \alpha, \theta \) small. \( \Rightarrow \theta_d \approx (n-1) \alpha \).

\[ \theta_d = (n-1) \alpha \]

\[ \Delta \theta_d = \Delta \alpha n = -\frac{1}{2} \alpha \sin^3 \theta \frac{\sin^3 \theta}{d^2} \]

Incident light is scanned.

More convenient to use triangularly shaped electrodes on a cubic crystal.

Spatial Light Modulators. - modulate intensity of light at different positions by prescribed factors.

\[ \text{Out}(x, y) = I_i(x, y) T(x, y). \]

\( T(x, y) \) is controllable.
\[ \begin{align*}
V_0 &= V_1^+ - V_1^- \\
V_2 &= V_1^+ - V_2^- \\
V_3 &= V_2^+ - V_1^- \\
V_4 &= V_2^+ - V_2^- \\
\{ \text{can modify voltage across each device.} \}& \\
\Rightarrow & \text{controllable transmission}.
\end{align*} \]

\( \Rightarrow \) each e-modulator acts as a pixel as each gets smaller.

\[
T(x_i, y_i) \rightarrow T(x, y).
\]

\( i = 1, 2, 3 \ldots \)

(Used in Liquid-crystal spatial light modulator used for display).

Optically addressed E-O spatial light modulator

Photoconductive material: photoconductivity depends on light intensity \( G(x, y) \).

\( I_w(x, y) \) - image written in spatial intensity.

\[
G(x, y) \propto I_w(x, y) \quad E(x, y) \propto \frac{1}{G(x, y)} \times \frac{1}{I_w(x, y)}.
\]

\[
T(x, y) \propto E(x, y) \times \frac{1}{I_w(x, y)}.
\]
The Pockels Readout Optical Modulator

An ingenious implementation of this principle is the Pockels readout optical modulator (PROM). The device uses a crystal of bismuth silicon oxide, Bi₁₂SiO₂₀ (BSO), which has an unusual combination of optical and electrical properties: (1) it exhibits the electro-optic (Pockels) effect; (2) it is photoconductive for blue light, but not for red light; and (3) it is a good insulator in the dark. The PROM (Fig. 18.1-15) is made of a thin wafer of BSO sandwiched between two transparent electrodes. The light that is to be modulated (read light) is transmitted through a polarizer, enters the BSO layer, and is reflected by a dichroic reflector, whereupon it crosses a second polarizer. The reflector reflects red light but is transparent to blue light. The PROM is operated as