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Citation: Journal of Applied Physics 117, 154504 (2015); doi: 10.1063/1.4918313
View online: http://dx.doi.org/10.1063/1.4918313
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/117/15?ver=pdfcov
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Vertical electron transport in van der Waals heterostructures with graphene layers

V. Ryzhii,1,2,a) T. Otsuji,1 M. Ryzhii,3 V. Ya. Aleshkin,4 A. A. Dubinov,4 V. Mitin,1,5 and M. S. Shur6

1Research Institute for Electrical Communication, Tohoku University, Sendai 980-8577, Japan
2Center for Photonics and Infrared Engineering, Bauman Moscow State Technical University and Institute of Ultra High Frequency Semiconductor Electronics of RAS, Moscow 111005, Russia
3Department of Computer Science and Engineering, University of Aizu, Aizu-Wakamatsu 965-8580, Japan
4Institute for Physics of Microstructures of RAS and Lobachevsky State University of Nizhny Nogodor, Nizhny Novgorod 603950, Russia
5Department of Electrical Engineering, University at Buffalo, Buffalo, New York 1460-1920, USA
6Department of Electrical, Electronics, and Systems Engineering and Department of Physics, Applied Physics, and Astronomy, Rensselaer Polytechnic Institute, Troy, New York 12180, USA

(Received 4 February 2015; accepted 4 April 2015; published online 15 April 2015)

We propose and analyze an analytical model for the self-consistent description of the vertical electron transport in van der Waals graphene-layer (GL) heterostructures with the GLs separated by the barriers layers. The top and bottom GLs serve as the structure emitter and collector. The vertical electron transport in such structures is associated with the propagation of the electrons thermionically emitted from GLs above the inter-GL barriers. The model under consideration describes the processes of the electron thermionic emission from and the electron capture to GLs. It accounts for the nonuniformity of the self-consistent electric field governed by the Poisson equation which accounts for the variation of the electron population in GLs. The model takes also under consideration the cooling of electrons in the emitter layer due to the Peltier effect. We find the spatial distributions of the electric field and potential with the high-electric-field domain near the emitter GL in the GL heterostructures with different numbers of GLs. Using the obtained spatial distributions of the electric field, we calculate the current-voltage characteristics. We demonstrate that the Peltier cooling of the two-dimensional electron gas in the emitter GL can strongly affect the current-voltage characteristics resulting in their saturation. The obtained results can be important for the optimization of the hot-electron bolometric terahertz detectors and different devices based on GL heterostructures. © 2015 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4918313]

I. INTRODUCTION

Van der Waals heterostructures based on graphenelayers (GLs) separated by the barrier layers made of hBN, MoS2, WS2, and others have recently attracted a considerable interest (see, for example, the review paper1). These heterostructures can be used in novel electron and optoelectron devices. Several devices using these structures (among other GL-structures) have been proposed and realized.2–16 In particular, we proposed and evaluated the hot-electron bolometric GL-based detectors of terahertz (THz) radiation consisting of multiple-GL structures with the inter-GL barrier layers.1 In these devices (see Fig. 1), the extreme (top and bottom) GLs serve as the emitter and collector, respectively. In our previous work, considering the vertical electron transport in the multiple-GL heterostructures, we used a simplified model in which the spatial distribution of the electric field across the main portion of the GL detector structure was assumed virtually uniform. However, the features of the electron injection from the emitter GL and the electron capture into the inner GLs in the multiple-GL heterostructures, leading to a charging of the GLs adjacent to the emitter and deviation of the self-consistent electric-field distributions from uniform ones, can affect the characteristics of the vertical electron transport in such heterostructures. The cooling of the two-dimensional electron gas (2DEG) in the emitter GL due to the Peltier effect can add complexity to the characteristics. In this paper, we develop a model for the vertical electron transport in van der Waals GL-based heterostructures accounting for the thermionic electron emission from and the electron capture into GLs, self-consistent electric-
field distributions, and the electron cooling in the emitter GL. The developed model and obtained results might be useful for the optimization of the performance of the hot-electron bolometric THz detectors, particularly, their dark current characteristics and, hence, the detector detectionality, as well as the optimization of other devices based multiple-GL heterostructures.

II. GENERAL EQUATIONS OF THE MODEL

We consider multiple-GL heterostructures with the n-doped GLs and the sufficiently thick inter-GL barrier layers (tunneling non-transparent), made, for example, of WS$_2$. Figure 1 shows the GL-structure under consideration. The top and bottom GLs are connected to the pertinent electrodes. These GLs serve as the emitter and collector of electrons. The bias voltage $V$ is applied between the latter. The inner GLs are disconnected from any contacts, so that their potentials are floating. The bottom electrode serves as the collector of the electrons passed through all GLs (not captures into the inner GLs) and as the heat sink. The heterostructure can be supplied by the back electrode (not shown in Fig. 1), can provide the collection of the electrons passed through all GL, and plays the role of the extra heat sink. The terminal current is due to the electrons thermionically emitted from GLs and propagating above the tops of the barriers.

Since the thickness of the inter-GL barriers $d$ and the net thickness of the GL-heterostructure $dN$, where $N$ is the number of the barriers in the heterostructure, is relatively large [the number of GLs is equal to $(N + 1)$], the geometrical inter-GL capacitance is much smaller than the quantum capacitance, so that the effect of quantum capacitance can be disregarded.

The Poisson equation governing the electric potential distribution $\varphi = \varphi(z)$ in the direction perpendicular to the GL plane (the $z-$direction) and the equation governing the electron balance in each GL are presented in the following form:

$$\frac{d^2 \varphi}{dz^2} = \frac{4\pi e}{\kappa} \sum_{n=0}^{N} (\Sigma_n - \Sigma_D) \cdot \delta(z - nd),$$

$$\frac{\partial p_n}{\partial z} = \Theta_n, \quad \Theta_n = \Theta_D \exp\left(\frac{\mu_n - \Delta C}{T_n}\right).$$

Here, $e = |e|$ is the electron charge, $\Sigma_n$, $\Sigma_D$, and $\mu_n$ are the 2D densities of electrons and donors (assumed to be the same in all GLs) and the electron Fermi energy in the GL with the index $n = 1, 2, ..., N$, $\delta(z)$ is the Dirac delta function, $\kappa$ and $d$ are the barrier dielectric constant and thickness, $j$ is the density of the electron current across GLs, $p_n$ and $\Theta_n$ are the capture parameter (proportional to the probability of the electron capture into GLs), and the rate of the electron thermoemission from the $n$-th GL ($p_0 = 1, p_1, p_2, ..., p_N \leq 1$), respectively, $\Delta C$ is the band offset between the GL and the barrier material, and $T_n$ is the electron temperature (in the energy unit) in the $n$-th GL. In Eq. (2), we put

$$\Theta_D = \frac{2T_j^2}{\pi \hbar^2 v_w} \left(\frac{\Delta C}{T_j} + 1\right) \simeq \frac{\Sigma_D}{\tau_{exc}},$$

where $T_j$ is the lattice temperature, $\tau$ is the characteristic time of the momentum relaxation of electrons with the energy exceeding $\Delta C$ in GLs, $\hbar$ is the Plank constant, $v_w \simeq 10^8$ cm/s is the characteristic electron velocity in GLs, and $\tau_{exc}$ is the characteristic escape time from GLs. We disregard small variations of the electron density and the temperature in the pre-exponential factors.

Equation (2) is valid if the density of electrons above the barriers substantially exceeds its value in equilibrium.

The boundary conditions for the potential governed by Eq. (1) can be set as

$$\varphi|_{z=0} = 0, \quad \varphi|_{z=nd} = V.$$

As follows from Eq. (1), the electric field in the $n$-th barrier $(\partial \varphi/\partial z)|_n = E_n (n = 1, 2, 3, ..., N)$, i.e., at $(n - 1)d < z < nd$, is equal to:

$$E_{n+1} = E_n + \frac{4\pi e}{\kappa} (\Sigma_n - \Sigma_D).$$

According to Eq. (4), we obtain

$$d \sum_{n=1}^{n=N} E_n = V.$$

Assuming that the 2DEG in all GLs, except those fully depleted, is degenerate due to sufficiently high donor density ($\mu_n \gg k_BT$), one can use the following equation which relates the Fermi energy $\mu_n$ and the electron density $\Sigma_n$:

$$\mu_n = \hbar v_w \sqrt{\pi \Sigma_n}.$$

It is commonly believed that the probability of the electron capture to quantum wells (QWs) in the customary heterostructures is determined by the emission of optical phonons. The scattering of the electrons propagating over the barriers on the electrons located in GLs can also be rather effective mechanism of the capture of the former. Generally, the capture parameter $p_n$, which relates the current density $j$ and the rate of the capture and is proportional to the probability of the capture of electrons, depends on the electric-field distribution across the GL-structure. In the case of ballistic or near ballistic transport of electrons above the barriers, $p_n$ depends mainly on $E_n$. In the GL-structures with the collision-dominated electron transport, one can assume that $p_n$ is determined by the electric field around the $n$-th GL, in particular, by the electric fields $E_n$ and $E_{n+1}$. For the definiteness, we use the following approximation for the electric-field dependence (see the Appendix):

$$p_n = E_C/E_n.$$

Here, $E_C = \langle W_C \rangle v_T/b$ is the characteristic capture field, which is determined by the barrier layer and GL structural parameters: the average (averaged over the electron Maxwellian distribution in the barrier layer) probability of
the capture of electrons crossing a GL, \( W_c \), the electron mobility across the barrier, \( b = b(E) \), and the electron thermal velocity and the electron effective mass in the barrier layer, \( v_T = \sqrt{T_b/2m_b} \), respectively.

Using Eqs. (2) and (6), we obtain equations which relates \( \Sigma_0 \) and \( \Sigma_n \) and the current density \( j \)

\[
\exp\left( \frac{\mu_D \Sigma_0 / \Sigma_D - \Delta_c}{T_0} \right) = \frac{j}{j_D},
\]

for the emitter GL, and

\[
\exp\left( \frac{\mu_D \Sigma_n / \Sigma_D - \Delta_c}{T_n} \right) = \frac{jp_n}{j_D},
\]

for the inner GLs. Here, \( \mu_D = \hbar v_{FW} / \pi \Sigma_D \) and

\[
j_D \approx \frac{e \Sigma_D}{\tau_{exc}} \approx \text{const.}
\]

Generally, the electron temperature in the emitter GL \( T_0 \) can differ from the electron temperatures in the inner GLs \( T_n \) and the lattice temperature \( T_l \) in particular, due to the Peltier effect (see below). Neglecting for simplicity the electron heating in the barrier layers (see below), we get the electron temperatures in all inner GLs to be equal to the lattice temperature: \( T_n = T_l \).

Due to the smallness of the ratio \( E_C / E_D \), where \( E_D = 4 \pi e \Sigma_D / \kappa \), and the electric-field distribution can be partitioned into a smooth quasi-neutral (\( \Sigma_n = \Sigma_0 \)) portion with relatively small electric field in a wide range adjacent to the collector and a narrow high-field portion (high-electric-field domain) near the emitter GL. At \( \Sigma_D = 1.8 \times 10^{12} \text{ cm}^{-2} \) and \( \kappa = 4 \), one obtains \( E_D = 270 \text{ kV} \) and \( \mu_D = 150 \text{ meV} \). Due to relatively strong doping of GLs, the high-electric field is concentrated primarily in the emitter barrier layer (similar to that in the multiple-quantum well structures based on the standard semiconductors\[25-30]\). Considering this and using Eqs. (8) and (10), for the electric field in the quasi-neutral region \( E_{bulk} \) (in the barrier layers with \( n > M \), where \( M \) is the number of charged, i.e., fully and partially depleted GLs), we find

\[
E_{bulk} = E_C \frac{j}{j_D} \exp\left( \frac{\Delta_c - \mu_D}{T_l} \right).
\]

Taking into account that \( E_{Emitter} = E_1 = E_D (\Sigma_0 / \Sigma_D - 1) \), from Eq. (10), we obtain

\[
E_{Emitter} = E_D \left[ \left( \frac{\Delta_c + T_0 \ln \frac{j}{j_D}}{\mu_D} \right)^2 - 1 \right].
\]

Thus, the spatial distribution of the electric field is given by

\[
E_n = E_{Emitter} - (n - 1)E_D, \quad 1 \leq n \leq M,
\]

\[
E_n = E_{bulk}, \quad M < n \leq N.
\]

Using Eq. (6), we arrive at the following equation:

\[
ME_{Emitter} - (M - 1)E_D + (N - M)E_{bulk} = \frac{V}{d},
\]

with \( M \) is determined by the following inequalities:

\[
\left( \frac{E_{Emitter} - E_{bulk}}{E_D} \right) < M < \left( \frac{E_{Emitter} - E_{bulk}}{E_D} \right) + 1.
\]

The first two terms in the left-hand side of Eq. (16) correspond to the potential drop at the depleted region, whereas the third term is equal to the potential drop across the bulk of the heterostructure (across all other barrier layers). Equation (16) with Eqs. (12) and (13) yields the following current voltage characteristics (with the electron temperature in the emitter GL \( T_0 \) as a parameter):

\[
\begin{align*}
\Delta_c + T_0 \left( \frac{\mu_D - \Delta_c}{T_l} + \ln J \right)^2 & \leq \left( \frac{\Delta_c - \mu_D}{T_l} \right)^2 \frac{M - E_C J}{E_D} \frac{V}{V_D} + 2M - 1, \\
\frac{\Delta_c - \mu_D}{T_l} - \frac{E_C}{T_l} & \leq \frac{J}{j_D} \exp\left[ \frac{\Delta_c - \mu_D}{T_l} \right],
\end{align*}
\]

where \( J = \frac{d}{j_D} \exp\left[ \frac{\Delta_c - \mu_D}{T_l} \right] \).

III. THE PELTIER COOLING

The electrons leaving the emitter GL extract some energy from 2DEG in this GL. This can lead to the 2DEG cooling (the Peltier effect) and affect the electron thermionic emission from the emitter GL, the spatial distributions of the electric field and electron population in GLs, and the current density.

The density of the energy flux from the emitter GL is equal to \( Q = j (\Delta_c - 2\mu_D / 3) / e \). We assume that the main mechanism transferring the energy from the emitter GL lattice to 2DEG in this GL is associated with the GL’s optical phonons. In this case, the electron effective temperature \( T_0 \) in the emitter GL is governed by the following equation describing the balance between the energy taken away by the emitted electrons from the emitter GL and the energy which the 2DEG in this GL receives from the lattice:

\[
\frac{\hbar \omega_0}{\tau_0} \left[ \exp\left( - \frac{\hbar \omega_0}{T_0} \right) - \exp\left( - \frac{\hbar \omega_0}{T_l} \right) \right] \approx - \frac{\Delta_c - 2 \mu_D / 3}{T_l} \frac{j}{j_D}.
\]

Here, \( \hbar \omega_0 \) and \( \tau_0 \) are the optical phonon energy in GL and their characteristic spontaneous emission time, respectively. Equation (19) leads to

\[
1 - \frac{1}{T_0} = \frac{1}{T_l} - \frac{1}{\hbar \omega_0} \ln \left( 1 - \frac{J}{J_S} \right),
\]

with

\[
J_S = \frac{\tau_{exc}}{\tau_0} \left( \Delta_c - 2 \mu_D / 3 \right) \exp\left( \Delta_c - \mu_D - \hbar \omega_0 \right). \]

Figure 2 shows the dependences of the normalized electron temperature in the emitter GL \( T_0/T_l \) on the normalized...
current \( J \) calculated using Eq. (20). It is assumed that \( \tau_{\text{esc}} \sim \tau_0 \sim 10^{-12} \text{s} \), \( \Delta_c = 400 \text{ meV} \), \( \hbar \omega_0 = 200 \text{ meV} \), \( \mu_D = 150 \text{ meV} \) \( (\Sigma_D = 1.8 \times 10^{12} \text{ cm}^{-2}) \), and \( T_l = 25 \text{ meV} \) \( (T_l \approx 300 \text{ K}) \).

At the above parameters, \( J_S \approx 5 \times 10^{12} \text{ cm}^{-2} \text{s} \), \( J_D \exp[(\mu_D - \Delta_c)/T_l] \approx 1.45 \times 10^{14} \text{ A/cm}^2 \), and \( J_S = J_S \exp[(\mu_D - \Delta_c)/T_l] \approx 72.6 \times 10^{12} \text{ A/cm}^2 \).

As follows from Eq. (20) and seen from Fig. 2, the electron temperature in the emitter GL is lower than the lattice temperature due to the effect of thermoelectric (Peltier) cooling. When the current \( J \) approaches to \( J_S \) (the saturation current), \( T_0 \) drastically decreases.

### IV. ELECTRIC FIELDS VERSUS CURRENT AND CURRENT-VOLTAGE CHARACTERISTICS

Equation (12) yields the following expression for \( E_{\text{Bulk}} \) as a function of the normalized current \( J \):

\[
E_{\text{Bulk}} = E_C J.
\]

Using Eqs. (13) and (20), we find the following \( E_{\text{Emitter}} - J \) relation:

\[
E_{\text{Emitter}} = E_D \left[ \frac{\Delta_c + (\mu_D - \Delta_c + T_l \ln J)}{1 - (T_l/\hbar \omega_0) \ln (1 - J/J_S)} - 1 \right] \left[ \frac{\mu_D}{1 - (T_l/\hbar \omega_0) \ln (1 - J/J_S)} \right]^2.
\]

Apart from the \( T_0 - J \) relation, Fig. 2 shows also the dependence of \((E_{\text{Emitter}} - E_{\text{Bulk}})/E_D \) on the normalized current \( J \) correspond calculated using Eqs. (19), (22), and (23) for \( E_D = 270 \text{ kV/cm} \) and \( E_C = 1.5 \text{ kV/cm} \). As seen, the difference \( E_{\text{Emitter}} - E_{\text{Bulk}} \) steeply increases when \( J \) becomes close to the normalized saturation current \( J_S \). Using the value of the difference \((E_{\text{Emitter}} - E_{\text{Bulk}}) \) and inequalities (17), one can find the intervals of the normalized current corresponding to different numbers, \( M \), of the charged inner GLs.

Considering the above, we arrive at the following current-voltage characteristic:

\[
\frac{\Delta_c + (\mu_D - \Delta_c + T_l \ln J)}{1 - (T_l/\hbar \omega_0) \ln (1 - J/J_S)} - 1 \left[ \frac{\mu_D}{1 - (T_l/\hbar \omega_0) \ln (1 - J/J_S)} \right]^2 + \frac{(N - M) E_C}{M} \frac{V}{V_D + 2M - 1}.
\]

Figure 3 shows the current-voltage characteristics of the heterostructures with the numbers of the inter-GL barriers equal to \( N = 6 \) and \( N = 11 \) (the numbers of GLs are five and ten, respectively) for different values of \( E_C \). Other parameters are the same as for Fig. 2. The obtained characteristics clearly demonstrate the tendency to saturation with increasing voltage. At a relatively small \( E_C \), the current-voltage characteristics of the heterostructures with different \( N \) are rather close to each other. This is because at such values of \( E_C \) and \( N \), the potential drop across the quasi-neutral region is small in comparison with that across the depleted region, so that \( E_{\text{Emitter}} \approx V/d + (M - 1) E_D \) and, hence, the current is virtually independent on \( N \). However, when \( E_C \) is not so small, the potential drop across the quasi-neutral region becomes essential and dependent on \( N \). In this case, the current-voltage characteristics at different \( N \) can be markedly different (see curves for \( E_C = 7.5 \text{ kV/cm} \) in Fig. 3).

### V. POTENTIAL PROFILES

Considering the obtained current-voltage characteristics, one can find the voltage dependences of \( E_{\text{Emitter}} \) and \( E_{\text{Bulk}} \) corresponding to their current dependences shown in Fig. 2. Figure 4 shows these voltage dependences for the heterostructures with different \( E_C \). One can see that in both cases \( E_{\text{Emitter}} \gg E_{\text{Bulk}} \), the difference between these fields at \( E_C = 7.5 \text{ kV/cm} \) is markedly smaller.

Using Eqs. (14) and (15) and accounting for the calculated data for the \( E_{\text{Emitter}} - V \) and \( E_{\text{Bulk}} - V \) relations (see Fig. 4), one can find the spatial distributions of the electric potential (potential profiles) between the emitter and...
collector GLs. Figures 5 and 6 show examples of the potential profiles found for heterostructures with $N = 11$, $E_D = 270$ kV/cm, and $d = 5 \times 10^{-6}$ cm with $E_C = 1.5$ kV/cm and $E_C = 7.5$ kV/cm, respectively, at different voltages. One can see that the potential spatial distributions are substantially nonuniform: they steeply vary near the emitter GL (in the depleted, i.e., charged region) and are fairly smooth in the heterostructure bulk. It is also seen that an increase in the voltage leads to a change of the number of the charged GLs $M$ (from $M = 1$ to $M = 3$ at the parameters used for Fig. 5 and in the chosen voltage range). The transition from the voltage corresponding a certain $M$ to $M + 1$ results in the occurrence of jogs (actually rather weak) on the characteristics shown in Figs. 3 and 4. It is also seen that in a heterostructure with relatively large $E_C$ and moderate voltages, the potential distribution is rather smooth except the immediate vicinity of the emitter GLs, where the electric field is only slightly higher (see the line for $V = 1$ V in Fig. 6). This can justify the simplified model\textsuperscript{15} at not too small $p_n$.

VI. DISCUSSIONS

A. Limiting cases

In the voltages range not so close to the saturation, the Peltier cooling can be disregarded. In this range, the current-voltage characteristics can be expressed by simple equations. Taking into account that at moderate voltages only one GL is charged and the potential drops primarily across the barrier layer closest to the emitter GL (see Fig. 5), we obtain

$$E_{Emitter} \leq V/d, \quad (25)$$

$$j \simeq \exp \left[ \frac{\mu_D}{T_i} \left( \sqrt{1 + \frac{V}{V_D}} - 1 \right) \right] \simeq \exp \left( \frac{\mu_D}{2T_i V_D} V \right), \quad (26)$$

i.e.,

$$j \simeq j_0 e^D \exp \left( \frac{\mu_D - \Delta C}{T_i} \right) e^D \left( \sqrt{1 + \frac{V}{V_D}} - 1 \right),$$

$$\simeq j_0 e^D \exp \left( \frac{\mu_D - \Delta C}{T_i} \right) e^D \left( \frac{\mu_D V}{2 T_i V_D} \right),$$

$$= j_0 e^D \exp \left( \frac{\mu_D - \Delta C}{T_i} \right) \frac{1}{p_0^C}. \quad (27)$$

Here, $p_0^C = \exp (\mu_B V/2 T_i V_D)$. In the heterostructures with not too small $E_C$ and sufficiently large $N$, one has $E_{Emitter} < (N - 1) E_C$, i.e., the potential drops predominantly across the quasi-neutral region (bulk), and the following relation is true:

$$E_{Emitter} > E_{Bulk} \simeq \frac{V}{N d}. \quad (28)$$
In this limiting cases, the current density can be presented as

\[ J \approx \frac{V}{N_d E_C} \simeq 1. \]  

(29)

Hence,

\[ j \approx j_D \exp \left( \frac{\mu_D - \Delta C}{T_1} \right) \frac{V}{N_d E_C}, \]

\[ = j_D \exp \left( \frac{\mu_D - \Delta C}{T_1} \right) \frac{1}{p_C^{\text{eff}}}, \]  

(30)

where \( p_C^{\text{eff}} \) is the effective capture parameter, which, in this case, is determined by the electric field in the main portion of the heterostructure, namely, by \( E_{\text{Bulk}}; p_C^{\text{eff}} = E_C / E_{\text{Bulk}} \) [see Eq. (8)].

It is instructive that in the latter limit, the current density can be presented in the Ohmic form

\[ j \approx en_b \nu B / N_d, \]  

(31)

with the electron density in the barrier layers \( n_b \propto \sum_D / \langle W_C \rangle \exp[(\mu_D - \Delta C)/T_1]]. \)

Equations (27) and (29) coincide in form with those obtained previously using a simplified model. However, \( p_C^{\text{eff}} \) in these equations is determined by the electric-field spatial distribution and the number of GLs (instead of \( p_C = \text{const} \) or \( p_C \)) determined by the average electric field in Ref. 17. Moreover, \( p_C^{\text{eff}} \) can explicitly depend on the capture probability \( W_C \) and characteristic capture field \( E_C \) (see the Appendix) as in the case of Eq. (30), or be independent of these parameters as in the case of Eq. (27).

**B. Role of the \( p_n - E_n \) relation**

Above, we used the \( p_n - E_n \) relation given by Eq. (8). The deviation from this relation does not lead to qualitative change in the obtained result. To demonstrate this, let us assume now that (see, for example, Refs. 27 and 28)

\[ p_n = \exp(-E_n / E_C), \]  

(32)

where the characteristic field \( E_C \) generally differs from \( E_C \).

In this case, Eq. (12) should be replaced by the following equation:

\[ E_{\text{Bulk}} = E_C \left( \frac{\Delta C - \mu_D}{T_1} + \ln j / j_D \right). \]  

(33)

The second term in the left-hand side of Eq. (18) (and the only this term) should also be modified accordingly. Such a replacement does not affect the saturation of the current voltage characteristics and the value \( J_s \) (and \( j_s \)) and again leads to Eqs. (27) and (30). In the latter equation, there should be now \( p_C^{\text{eff}} \simeq \exp(-V/N_d E_C) \simeq 1. \)

The current across the GL-heterostructures under consideration essentially depends on the effective capture parameter \( p_C^{\text{eff}} \) [see, Eqs. (27) and (28)], which, in turn, depends on the electric-field distribution and the number of GLs in the heterostructure. Hence, the voltage dependence of the dark current, i.e., the current in the absence of irradiation in the hot-electron bolometric THz detectors on the base of these heterostructures proposed in Ref. 17, is more complex than that follows from the simplified model of such photodetectors. In particular, the dark current-voltage characteristics calculated using the present model explicitly depend on the number of GLs [see Fig. 3] in contrast to more simple model. The photocurrent caused by the THz radiation in the photodetectors in question and their responsivity also depend on \( p_C^{\text{eff}} \). Qualitative reasonings show that since the photodetector detectivity is determined by the responsivity, dark current, and the photoelectric gain \( g \propto 1/p_C^{\text{eff}} \), the effects of the electric-field nonuniformity (as well as the Peltier cooling effect) can pronouncedly influence the detectivity as a function of the voltage and number of GLs. Therefore, such effects should be taken into account considering the photodetector quantitative optimization. One of the parameter which should be optimized is the number of GLs in the photodetector. The quantitative study of the impact of the nonuniformity effects and the effect of the Peltier cooling on the photodetector responsivity and detectivity require a separate detailed treatment.

**C. Electron heating in the barrier layers**

In contrast to the cooling of the 2DEG in the emitter GL, the electron heating in the inner GLs, associated with the capture of relatively hot electrons heated in the barrier layers and cooled due the electron emission from these GLs, is relatively weak. This is because the energy brought to a GL by a captured electron and the energy swept away by an emitted electron are close to each other (these energies are close to \( \Delta C \) unless the electron temperature in the barrier layers \( T_b \ll \Delta C \). Due to this, \( T_n - T_1 \ll T_1 - T_0 \).

The effect of heating of the electrons propagating in the barrier layers is also should not be too marked. Indeed, assuming the relaxation of the electron energy obtained from the electric field (Joule heating power in the whole heterostructure is equal to \( jV \)) in the barrier layers is mainly due to the interaction of the electrons with the optical phonons in the barrier layers, for the estimate of the electron effective temperature, \( T_n \), in these barriers, one can use the following equation:

\[ E = 10 \text{ kV/cm} \]

[FIG. 7. Energy dependence of the net capture probability as well as the contributions of different capture mechanisms.]
\[ \frac{\hbar \omega_0^{(b)}}{\tau_0^{(b)}} \left[ \exp \left( -\frac{\hbar \omega_0^{(b)}}{k_B T_b} \right) - \exp \left( -\frac{\hbar \omega_0^{(b)}}{k_B T_t} \right) \right] = e b \left( \frac{V}{Nd} \right)^2. \]

Here, \( \hbar \omega_0^{(b)} \) and \( \tau_0^{(b)} \) are the optical phonon energy in the barrier layers and the characteristic time of their spontaneous emission. Equation (31) yields

\[ \frac{1}{T_b} = \frac{1}{T_t} - \frac{1}{\hbar \omega_0^{(b)}} \ln \left[ 1 + B_N \exp \left( \frac{\hbar \omega_0^{(b)}}{k_B T_t} \right) \right], \]

where

\[ B_N = \frac{\tau_0^{(b)} e b}{\hbar \omega_0^{(b)}} \left( \frac{V}{Nd} \right)^2. \]

Assuming \( \hbar \omega_0^{(b)} = 50 \text{ meV}, \ \tau_0^{(b)} = 10^{-2} \text{ s}, \ b = (10 - 15) \text{ cm}^2/\text{V} \cdot \text{s}, \ d = 10 - 50 \text{ nm}, \ T_t = 300 \text{ K}, \ N = 11, \) and \( V = 1 \text{ V}, \) we obtain \( B_N \exp(\hbar \omega_0^{(b)}/k_B T_t) \approx 0.4885 - 0.7328. \) As a result, from Eq. (35), we get the following estimate: \( T_b/T_t \approx 1.248 - 1.378 \) (or \( T_b \approx 374 - 413 \text{ K} \)). Since the average capture probability and the characteristic capture electric field slowly vary with the temperature (see Fig. 8), the pertinent effect should not essentially change the obtained characteristics. In principle, the effect of electron heating in the barrier layers can be accounted for by the inclusion of the capture probability. In this case, the quantum capacitance plays an insignificant role in comparison with the geometrical capacitance.

**E. Contact effects**

The voltage drop across the contacts, \( V_C, \) to the GL heterostructures under consideration can be fairly small. Indeed, setting \( j_D \exp(\mu_D - \Delta_C)/T_t \approx 14.5 \text{ A/cm}^2 \) (as above), considering the voltage range \( V = 1 - 2 \text{ V} \) in Fig. 3, for a device with the length (spacing between the side contacts) \( L = 100 \text{ µm}, \) for the current \( I = jL \approx 1 \text{ A/cm} \), we obtain \( I \approx 0.3 - 0.6 \text{ A/cm}. \) According to the experimental data \(^{31-34} \) for contacts to GLs, the contact resistance can be estimated as \( R_C = (100 - 500) \text{ Ωµm}. \) As a result, we find \( V_C \approx (1 - 15) \times 10^{-3} \text{ V}. \) These values are much smaller than the applied voltage (voltage drop between the emitter and collector GLs). Hence, the contact effects can be neglected. The necessity to apply voltages about few volts is due to a relatively high emitter-collector resistance, which is associated with current created by relatively small fraction of electrons overcoming the inter-GL barriers.

**VII. CONCLUSIONS**

Using an analytical model for the self-consistent description of the vertical electron transport in the van der Waals GL heterostructures with the GLs separated by the barrier layers, we calculated current-voltage characteristics and potential distribution in those heterostructures. The thermionic current over the barriers is controlled by the electron thermionic emission from the GLs and by the capture of electrons into GLs. At small applied voltages, the electric field is practically homogeneous through the structure with equal potential drop on each barrier. As the voltage increases, the probability of electrons capture into GLs decreases and the potential distributions (see Figs. 5 and 6) turn to be substantially inhomogeneous with well pronounced two regions: with high electric field near the emitter and low electric field region in the heterostructure bulk (high-field domain and quasi-neutral low-field quasi-neutral domains, respectively). An increase in the applied voltage gives rise to an increase of the number of charged, fully or partially depleted GLs and, therefore, to an increase in the width of the high-electric-field (charged) region. The electric field in the bulk is practically homogeneous. We demonstrated that the thermionic emission from the emitter GL can lead to a substantial cooling of the 2DEG in this GL (the Peltier effect). This effect results in the saturation of the current-voltage characteristics with increasing voltage. The high and low field domains were observed in conventional semiconductor heterostructures with quantum wells, but the current saturation in the van der Waals GL heterostructures under consideration is new phenomenon associated with the specifics of the emitter with 2DEG. The obtained results, which reveal the origin of the current-voltage characteristics nonlinearity in the van der Waals GL heterostructures, can be important for the optimization of the dark current.
characteristics of the THz hot-electron bolometers based on those heterostructures. An extended version of the developed model can also be applied for the consideration of the GL heterostructures response to incoming THz radiation and strict calculations of the responsivity and detectivity of such bolometric detectors.

ACKNOWLEDGMENTS

This work was supported by the Japan Society for promotion of Science (Grant-in-Aid for Specially Promoting Research 23000008), Japan and by the Russian Scientific Fund Foundation (Project #14 29 00277). The works at UB and RPI were supported by the U.S. Air Force Award No. FA9550-10-1-391 and the U.S. Army Research Laboratory Cooperative Research Agreement, respectively.

APPENDIX: CAPTURE PARAMETER

Let us introduce the probability \( W_C(\epsilon) \) of the capture of an electron crossing a GL. Considering the interaction of the electrons crossing the GL with the optical phonons in GL, optical phonons in the barrier layer material (WS\(_2\)), and with the electrons located in the GLs, one can obtain the dependence of \( W(\epsilon) \) on the electron energy \( \epsilon \). Figure 7 shows an example of the energy dependence of the capture probability \( w_C(\epsilon, E) \) for the electron density in GLs equal to \( 10^{12} \) cm\(^{-2} \) and the electric field of \( 10^4 \) V/cm.

At not too low voltages when the electron system in the barrier layers is sufficiently far from equilibrium, the rate of the capture \( C_{GL} \) into a GL of electrons incident from left and right is

\[
C_{GL} \propto \int_0^\infty dp_z v_z W_C(\epsilon_z) \times \left\{ \exp\left[ -\frac{(p_z - m_b u_b)^2}{2m_b T_b} \right] + \exp\left[ -\frac{(p_z + m_b u_b)^2}{2m_b T_b} \right] \right\}
\]

\[
\simeq 2 \int_0^\infty dv_z W_C(\epsilon, E) \exp\left( -\frac{\epsilon}{T_b} \right), \quad (A1)
\]

The current through the GL (assuming that the capture probability \( W_C(\epsilon) \) is small) is given

\[
j \propto \int_0^\infty dp_z v_z \times \left\{ \exp\left[ -\frac{(p_z - m_b u_b)^2}{2m_b T_b} \right] - \exp\left[ -\frac{(p_z + m_b u_b)^2}{2m_b T_b} \right] \right\}
\]

\[
\simeq 2 \int_0^\infty dv_z \exp\left( -\frac{\epsilon}{T_b} \right) \left( \frac{\sqrt{2m_b u_b}}{T_b} \right). \quad (A2)
\]

Here, \( T_b, m_b, \) and \( b(E) \) are the effective electron temperature, the effective mass, and mobility in the material of the barrier layers, respectively, and \( u_b = b(E)E \) is the drift velocity across the barrier.

Thus introducing the capture parameter \( p_C \) as

\[
C = p_C j/e,
\]

we obtain

\[
p_C = \frac{\int_0^\infty dv_C(\epsilon) \exp\left( -\frac{\epsilon}{T_b} \right)}{\sqrt{2\pi} u_b \int_0^\infty dv_C(\epsilon) \exp\left( -\frac{\epsilon}{T_b} \right)},
\]

where \( W_C(\epsilon) \) is the average capture probability, \( v_T = \sqrt{T_b/2m_b} \) is the thermal velocity (in the barrier layer), and \( b(E) \) is the electron mobility in the barrier layers. Figure 8 shows the average capture probability as function of the electron temperature in the barrier \( T_b \). As seen, the average capture probability is weakly dependent on the electric field (affecting the electron wave functions in the barrier layers) in a wide range of the latter (at least up to \( 10^5 \) V/cm).

If the electric field \( E \) is smaller than that corresponding to the electron velocity saturation in the material of the barriers \( E_0 \), Eq. (A4) yields \( p_C = E_C/E_0 \), i.e., Eq. (8), where \( E_C \approx (W_C)v_T/b \). Setting the low-field mobility perpendicular to the layers equal to \( b = 10 - 15 \) cm\(^2\)/V\( \cdot \)s, for \( T_b = 300 - 400 \) K, we obtain \( W_C(\epsilon) \sim 1.5 \times 10^{-3} \) (see Fig. 8). This leads to the following estimate: \( E_C \approx (1.0-1.5) \) kV/cm. Lower transverse mobility, higher doping of GLs, and the inclusion of other mechanisms (say, the capture associated with the plasma emission) can result in some increase in \( W_C(\epsilon) \) and \( E_C \). In the main text, we assume that \( E_C = 1.5 \) and 7.5 kV/cm. Note that Eq. (8) is valid at \( E_0 \approx E_C \).


As it should follow from Eq. (A3):

\[
p_C \approx \left( \frac{W_C v_T}{b(E)E} \right), \quad (A4)
\]

where \( W_C(\epsilon) \) is the average capture probability, \( v_T = \sqrt{T_b/2m_b} \) is the thermal velocity (in the barrier layer), and \( b(E) \) is the electron mobility in the barrier layers. Figure 8 shows the average capture probability as function of the electron temperature in the barrier \( T_b \). As seen, the average capture probability is weakly dependent on the electric field (affecting the electron wave functions in the barrier layers) in a wide range of the latter (at least up to \( 10^5 \) V/cm).

If the electric field \( E \) is smaller than that corresponding to the electron velocity saturation in the material of the barriers \( E_0 \), Eq. (A4) yields \( p_C = E_C/E_0 \), i.e., Eq. (8), where \( E_C \approx (W_C)v_T/b \). Setting the low-field mobility perpendicular to the layers equal to \( b = 10 - 15 \) cm\(^2\)/V\( \cdot \)s, for \( T_b = 300 - 400 \) K, we obtain \( W_C(\epsilon) \sim 1.5 \times 10^{-3} \) (see Fig. 8). This leads to the following estimate: \( E_C \approx (1.0-1.5) \) kV/cm. Lower transverse mobility, higher doping of GLs, and the inclusion of other mechanisms (say, the capture associated with the plasma emission) can result in some increase in \( W_C(\epsilon) \) and \( E_C \). In the main text, we assume that \( E_C = 1.5 \) and 7.5 kV/cm. Note that Eq. (8) is valid at \( E_0 \approx E_C \).