

# Damping of plasma waves in two-dimensional electron systems due to contacts

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We report on the numerical study of plasma waves in two-dimensional electron systems with contacts using the kinetic electron transport model. We simulate the time- and position-dependence of plasma waves in the system by initially adding small perturbation of the electron concentration from the steady

state. We demonstrate that in such a system the damping of plasma waves caused by “out of phase” electrons exiting the channel through the contacts occur. We find that the damping rate is proportional to the electron average velocity and inversely proportional to the channel length.

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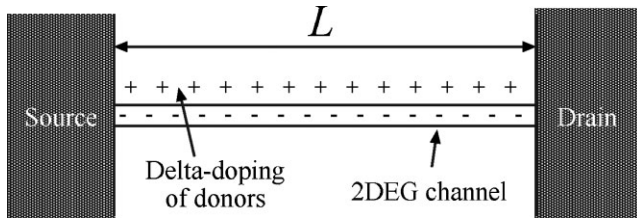
**1 Introduction** Generation and detection of THz waves by semiconductor devices have been one of the most challenging issues for decades. For this purpose, utilization of plasma waves in heterostructure two-dimensional electron gases (2DEGs) is one possible way [1]. Such 2DEGs can serve as resonant cavities of plasma waves, and their frequencies can be up to several THz if the electron concentration in the 2DEGs is  $\sim 10^{12} \text{ cm}^{-2}$  and the channel length is below  $1 \mu\text{m}$ . A number of both theoretical and experimental works have been conducted to realize such “plasma-wave devices” for THz generation and detection (see, for example, Ref. [2] and references therein).

The most important characteristics of the devices are the plasma frequency  $\omega$  and the damping rate  $\gamma$ . The latter determines the linewidth of resonant peaks for plasma-wave THz detectors [1], whereas it limits the condition of Dyakonov–Shur instability [3] and other instability mechanisms [4, 5] for the generators. In both cases, the smaller damping rate is preferred for the better performance of the devices. In the orthodox theory of plasma-wave devices, the damping rate is simply proportional to the electron collision frequency  $\nu$ :  $\gamma = \nu/2$ . Since electron mobilities in heterostructure 2DEGs are extremely high at low temperatures, very

high performance of plasma-wave devices has been expected.

Experimental reports on the detectors [6, 7], however, showed very wide broadening of the resonant peaks, *i.e.*, very low  $\omega/\gamma$  ratio, even at 4.2 K. To explain this discrepancy between theory and experiments, further theoretical works have been conducted. Mechanisms such as the decrease in the plasma frequency by cap regions [8], the radiative damping [9], and the damping related to the Ohmic loss in non-ideally conducting contacts [10] were investigated. Recently, it has been suggested that the plasma-wave mode in the lateral direction of the 2DEG is responsible for the discrepancy [11, 12].

In this paper, we demonstrate a mechanism of plasma-wave damping in 2DEGs with contacts which is related to the electron average velocity and the channel length. To our best knowledge, this mechanism is not reported so far in the literature. We show by the use of electron kinetic transport model that in the 2DEG with short length, even though the electron transport can be considered to be ballistic, the damping of plasma waves occurs. We calculate the characteristic damping rate extracted from the simulation of plasma waves in the 2DEG, and find that the damping rate is proportional to the electron thermal velocity and inversely proportional to the channel length.



**Figure 1** Geometry of the 2DEG system under consideration.

**2 Model of simulation** We consider a system comprising a 2DEG with length  $L$  and contacts with sufficiently large thickness (the geometry of the system is depicted in Fig. 1). We set the  $x$ - and  $z$ -axes corresponding to the parallel and perpendicular directions to the 2DEG, respectively, and the origin at its center. As standard heterostructure 2DEGs, we assume the delta-doping of donors slightly above the 2DEG with the concentration  $\Sigma_d$ . The potential at the contacts is assumed to be fixed to zero:  $\varphi|_{x=\pm L/2} = 0$ . This implicitly means that the Ohmic loss of electrons in the contacts, which is an additional damping mechanism of the plasma waves in the 2DEG discussed in Ref. [10], is neglected in this study.

We use the quasi-classical kinetic electron transport model to describe the system under consideration. In case of the ballistic electron transport, the electron distribution function  $f=f(t, x, p_x, p_y)$  obeys the Vlasov equation

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + \frac{\partial \varphi}{\partial x} \bigg|_{z=0} \frac{\partial f}{\partial p_x} = 0. \quad (1)$$

The potential  $\varphi$  in Eq. (1) is found from the 2D Poisson equation to be solved self-consistently with Eq. (1) through the electron concentration:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = \frac{4\pi e(\Sigma - \Sigma_d)}{\varepsilon} \delta(z), \quad (2)$$

where  $e=|e|$  is the electron charge,  $\varepsilon$  is the dielectric constant, and

$$\Sigma = \frac{2}{(2\pi\hbar)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f dp_x dp_y \quad (3)$$

is the electron concentration in the 2DEG. For the sake of simplicity, we in Eq. (2) assume that the distance between the delta-doping of donors and the 2DEG can be negligibly small.

In such a system the hydrodynamic model, which replaces Eq. (1) by Euler and continuity equations, predicts undamped plasma modes with the perturbed potential proportional to  $\cos(\omega_n t) \sin[n\pi(x/L - 1/2)]$ , where  $\omega_n = \sqrt{n}\omega$  ( $n = 1, 2, 3, \dots$ ) and  $\omega$  is the plasma frequency given by  $\omega = \sqrt{2\pi^2 e^2 \Sigma_d / m \varepsilon L}$ , where  $m$  is the electron effective mass.

**3 Results and discussion** We conducted simulation of standing plasma waves for the system under consideration by adding a nonuniform perturbation (whose position-dependence is expressed as  $\cos(kx)$ , where  $k=\pi/L$ , corresponding to the first mode of plasma waves in the system) to a steady-state distribution function of electrons. We set the perturbation sufficiently small compared with the steady state so that the plasma wave is linear. The boundary conditions for the distribution function are given by  $f|_{x=\pm L/2} = \{1 + \exp[(E_p - E_f)/k_B T]\}^{-1}$  and similar at  $x = -L/2$ , where  $E_p = (p_x^2 + p_y^2)/2m$ ,  $E_f$  is the Fermi energy in the contacts,  $k_B$  is the Boltzmann constant, and  $T$  is the temperature. Here, the contacts are assumed to be made of heavily doped semiconductor, so that the Fermi energy  $E_f$  is measured from the bottom of the conduction band in the contacts. We adapted the so-called splitting Scheme [13] to solve Eq. (1). Besides, in solving Eq. (2) numerically, the delta function in Eq. (2) was replaced by  $\theta(d/2 - |z|)$ , where  $\theta$  is the step function.

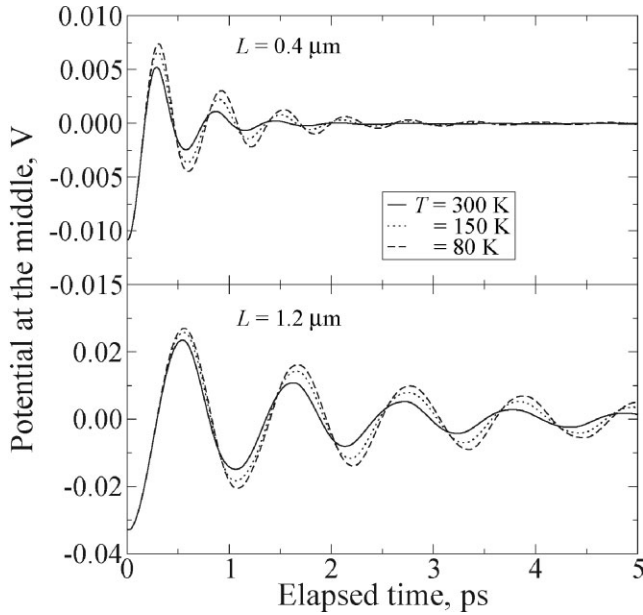
Parameters of the system were chosen for GaAs-based heterostructure ( $\varepsilon = 12$  and  $m = 6.1 \times 10^{-29}$  g) with  $L = 0.4\text{--}1.2 \mu\text{m}$ ,  $\Sigma_d = 0.5 \times 10^{12} \text{ cm}^{-2}$ , and  $T = 5\text{--}300$  K. The Fermi energy  $E_f$  was chosen so that the contacts provide the effective electron concentration  $\Sigma_c$  at edges of the 2DEG, i.e.,  $\Sigma|_{x=\pm L/2} = \Sigma_c$ , for the steady-state distribution (therefore, it is given by the expression  $E_f = k_B T \log[\exp(\pi\hbar^2 \Sigma_c / m) + 1]$ ). To avoid complication by the effect of built-in electric field at the edges, we assume  $\Sigma_c = \Sigma_d$ . This implies that the steady-state potential and electron concentration are uniform along the channel. (In the real situation,  $\Sigma_c > \Sigma_d$  and the built-in electric field that accelerates electrons toward the contacts appears. Our simulation showed that this does not affect significantly our results discussed below).

The time-dependence of the perturbed potential at a fixed point  $x=0$  in the 2DEG is shown in Fig. 2. It evidently illustrates that, contrary to the result predicted by the hydrodynamic model, the oscillation of the potential (manifesting the plasma wave) is damped out very rapidly. We stress that our model does not account for the collisional damping nor the radiative damping. It should also be mentioned that the time-dependence of the potential slightly deviates from  $\propto e^{-\gamma t} \cos(\omega t)$ . Moreover, the position-dependence is apparently not proportional to  $\cos(kx)$  as shown in Fig. 3. These facts show the significant difference of plasma waves predicted by the kinetic model and by the hydrodynamic model.

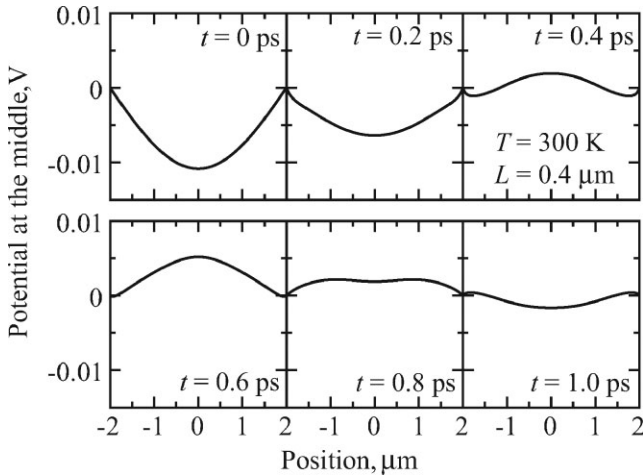
To study the damping quantitatively, we evaluate the characteristic damping rate as follows:

$$\gamma = \frac{1}{N} \sum_{n=1}^N \frac{\log(|\delta\varphi|_{x=0, t=t_{n-1}} / |\delta\varphi|_{x=0, t=t_n})}{t_n - t_{n-1}}, \quad (4)$$

where the absolute value of the potential at  $x=0$  has maxima at  $t = t_n$ ,  $n = 0, 1, 2, \dots$ , and  $N$  is the number of the maxima.



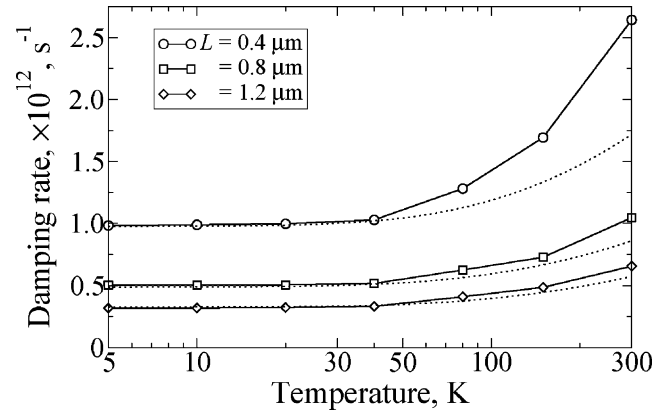
**Figure 2** The time-dependence of the potential at  $x=0$  with different temperatures and different channel lengths.



**Figure 3** The position-dependence of the potential distribution in the 2DEG at  $t=0$ –1 ps.

Equation (4) would give the exact damping rate if the potential would be expressed as  $\delta\varphi \propto e^{-\gamma t} \cos(\omega t) \cos(kx)$ .

Figure 4 shows the characteristic damping rate of plasma waves as a function of temperature. The damping rate becomes smaller at lower temperature but becomes almost constant below some temperature. As can be seen, the curves in Fig. 4 very much look like the temperature dependences of thermal velocities. This suggests that the damping demonstrated in the system might be related to the spread of electron distribution in velocity (momentum) space due to the degeneracy and finite temperature. In fact, those curves match the following empirical formula



**Figure 4** Characteristic damping rates *versus* temperature with different channel lengths.

of the damping rates (excellent matching especially at low temperature):

$$\gamma_{\text{th}} = a \frac{v_{\text{th}}}{L}, \quad (5)$$

where the constant  $a$  is approximately equal to  $a=3$ . The damping rates calculated using Eq. (5) are depicted by dotted curves in Fig. 4. Here,  $v_{\text{th}} = \langle |v_x| \rangle$  is the averaged absolute value of  $v_x$  for the steady-state distribution function with given parameters:

$$v_{\text{th}} = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |v_x| f_0 dp_x dp_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_0 dp_x dp_y}. \quad (6)$$

This matching clearly proves that the damping is related to the thermal distribution of the electron velocity in the 2DEG. Moreover, it is also associated with the presence of the contacts because such a damping does not occur when the edges of the 2DEG are reflective, *i.e.*, the 2DEG is bounded by potential barriers without the contacts (simulation of such a system using the same model showed plasma waves with no damping). We shall explain below how these factors lead to the damping.

First, let us consider how the hydrodynamic model describes plasma wave in our system. Mathematically, when Fourier-transforming the linearized hydrodynamic equations with the Poisson equation, they are reduced to a single eigenvalue equation (e.g., see Eq. (5) in Ref. [10]). Plasma modes for the system are determined by solving that equation, and for any initial condition of perturbed electron concentration the solution can be expressed as a linear combination of the plasma modes. Since the plasma modes do not have any damping factors if we neglect the collisional damping, so is their linear combinations. It implies that the plasma wave conserves its energy in the whole system including the contacts, even though the electrons can exit/enter the 2DEG and the number of electrons in the 2DEG oscillates with time (the latter is obvious if one considers the

electron concentration profile imagined from its relation to the potential). Thus, the plasma wave is maintained without damping.

In the hydrodynamic model, electron velocity is represented by a single position-dependent value and the spread of the electron distribution in velocity space is characterized by the pressure term in the hydrodynamic equations, which contributes to the relatively small change in the plasma frequency in the system under consideration [14]. This velocity is related to the time-dependent parts of the potential and the electron concentration by the hydrodynamic equations. In fact, the electron concentration and velocity “cooperate” so that the restoring force exerted on the electrons makes the oscillation of these quantities have no damping, in spite of the fact that the electrons can exit/enter the 2DEG. In this sense, we can say that the velocity of electrons is “in phase” with the plasma wave.

This is not the case when considering, with the use of the kinetic model, the thermal spread of the electron velocity, *i.e.*, when electrons at the same position have different velocities. Such electrons are “out of phase” with the plasma wave. Since some electrons move away from the 2DEG faster than in-phase velocity, the restoring force becomes different from that “prescribed” by the hydrodynamic model, and so is the electron concentration. In addition, the faster electrons exiting the 2DEG are absorbed by the contacts and do not contribute to the plasma wave any more. This results in the energy loss, *i.e.*, the damping of the plasma wave. The degree to which electrons are out-of-phase is characterized by the mean speed of electrons  $v_{th} = \langle |v_x| \rangle$ , and the damping rate is characterized by  $v_{th}/L$ , the inverse of the time that electrons with that speed spend in the 2DEG. Thus, the damping rate is described by Eq. (4). Since  $v_{th} \rightarrow 0$  at  $T \rightarrow 0$  if the steady-state distribution would be of Maxwell, electrons would become in-phase in this limit (our simulation for this case showed the damping rate given by Eq. (4), where  $a$  is replaced by 1.82). This limit can be considered as the hydrodynamic limit of the kinetic model. Note, however, that in reality any systems have Fermi steady-state distribution at sufficiently low temperatures, so electrons can never be in-phase.

This damping can also be considered as a ballistic effect of short-channel transistors, where the effective “ballistic” mobility severely reduces the overall mobility [15]. The damping rate (4) is inversely proportional to the “ballistic” mobility [15] as the collisional damping is inversely proportional to the collisional mobility. By simply adding the “ballistic” damping rate to the collisional one, we obtain the total damping rate  $\gamma = \nu/2 + \gamma_{th}$ . It is worth mentioning that the condition for the electron ballistic transport,  $\nu < v_{th}/L$ , is

equivalent to the condition of the validity of the kinetic model used here.

The electron–electron scattering is essential when the electron concentration is high. Intuitively, disregarding relatively weak effects of electron viscosity [3], the electron–electron scattering might not influence significantly on the damping since it just brings the local equilibrium to the 2DEG but the fact that electrons are out-of-phase remains.

**4 Conclusions** In summary, we conducted the simulation of plasma waves in a system consisting of a 2DEG and source/drain contacts based on the quasi-classical kinetic transport model of electrons. We demonstrated that in such a system the damping of the plasma waves occurs even when neglecting the collisional damping. The mechanism of the damping is related to the electrons exiting the 2DEG into the contacts and the characteristic damping rate is proportional to the average electron velocity as well as inverse of the 2DEG length.

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