

Deformation Electron-Phonon Coupling in Disordered Semiconductors and Nanostructures

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(Received 13 January 2005; published 8 April 2005)

We study the electron-phonon relaxation (dephasing) rate in disordered semiconductors and low-dimensional structures. The relaxation is determined by the interference of electron scattering via the deformation potential and elastic electron scattering from impurities and defects. We have found that in contrast with the destructive interference in metals, which results in the Pippard ineffectiveness condition for the electron-phonon interaction, the interference in semiconducting structures substantially enhances the effective electron-phonon coupling. The obtained results provide an explanation to energy relaxation in silicon structures.

DOI: 10.1103/PhysRevLett.94.136602

PACS numbers: 72.10.Di

Elastic electron scattering from impurities and defects drastically changes the electron-electron and electron-phonon (*e-ph*) interaction and modifies the temperature dependencies of the relaxation or dephasing rate. As a result of the diffusive motion of electrons, the electron-electron interaction is significantly enhanced in bulk and low-dimensional conductors [1]. Recent theoretical [2,3] and experimental [2,4] studies have shown that the piezoelectric *e-ph* coupling is also enhanced in semiconductors with a short electron mean free path. The effects of disorder on the deformation *e-ph* coupling are more complicated. In impure metals, the deformation coupling originates from “pure” electron-phonon scattering, electron scattering from vibrating impurities, and various interference processes. If electron scatterers vibrate in the same way as host atoms, the destructive interference of scattering mechanisms [5,6] results in the Pippard ineffectiveness condition [7], which means suppression of the *e-ph* relaxation. In this case, at low temperatures the relaxation rate modifies from T^3 dependence in the pure materials to T^4 dependence in the impure metals. However, even a small amount of static scatterers (e.g., tough boundaries) or an incomplete drag of impurities and defects increases the *e-ph* relaxation [8]. Disorder-suppressed relaxation is observed in disordered metallic films [9,10], while alloys commonly demonstrate the disorder-enhanced relaxation with T^2 dependence of the relaxation rate [11].

Recently, there has been significant interest in the electron relaxation in disordered semiconductors and structures, where the electron relaxation is determined by electron-phonon scattering via the deformation potential (DP). The relaxation rate has been measured in Si crystals containing a δ (Sb) layer [12,13] and in Si films [14]. Experimental results, including T^4 dependence of the electron relaxation rate, were associated with the Pippard ineffectiveness condition, obtained for metals. However, in the temperature ranges investigated in Refs. [12–14], DP is strongly screened and the relaxation rate in pure 2D and

quasi-2D structures follows to T^5 dependence [15,16]. Therefore, the ineffectiveness would result in the T^6 rather than the T^4 dependence.

It is not surprising that the theory developed for metals [5–7] fails to describe semiconductors. Indeed, DP in metals and semiconductors has a different nature [17]. In metals, DP is associated with electron gas compressibility, while in semiconductors this contribution is negligible due to small carrier concentrations. DP in semiconductors results mainly from a shift of the conduction band edge under the deformation, while in metals such contribution is small because of strong screening. It is important that DP has different tensor structures in metals and semiconductors [17,18], and this difference clearly manifests itself even in weakly disordered conductors [19]. Here we show that the tensor structure of DP plays a crucial role in kinetics of strongly disordered conductors: in contrast to the destructive interference in metals, the electron-phonon-impurity interference in semiconducting structures substantially enhances *e-ph* coupling.

Here we report results on the *e-ph* relaxation in disordered bulk semiconductors, two-dimensional electron structures, and multichannel one-dimensional conductors interacting with 3D phonons. Effects of disorder are described by the dimensionless parameter ql , where q is the characteristic momentum transferred to the electron due to *e-ph* scattering, $l = v_F \tau$ is the electron mean free path due to scattering from impurities, and v_F is the Fermi velocity. In the impure limit, $ql \ll 1$, a phonon interacts with an electron that diffuses in the interaction region, $L \sim 1/q \gg l$. In bulk conductors, q is the wave vector of a thermal phonon, $q_T = T/u$ (u is the sound velocity), and the crossover to the impure limit occurs at $T \sim u/l$.

In low-dimensional conductors, the characteristic momentum q is determined by the phonon wave vector component q_{\parallel} , which is parallel to the conductor. In two-dimensional systems, q_{\parallel} is of the order of q_T and, as well as in bulk semiconductors, the crossover is described

by the parameter $q_T l = Tl/u$. In 1D channels the transferred momentum q_{\parallel} is $\sim(u/v_F)q_T$ and effects of disorder become important at significantly higher temperatures, $T\tau \sim q_{\parallel}l < 1$. The same parameter $T\tau$ describes modification of the electron-electron interaction [1].

Investigating the electron energy relaxation, we focus our attention on the time scale much longer than the electron momentum relaxation time. In this time domain, electron-phonon kinetics is described by the angle-averaged electron and phonon distribution functions, n_e and N_{ω} . We consider interference processes, which are characterized by the momentum transfer much smaller than the Fermi momentum. In this case, the interference of electron-phonon and electron-impurity scattering is taken into account by the electron self-energy diagram shown in Fig. 1. The corresponding electron-phonon collision integral is [6,8]

$$I_{\epsilon} = -4\tau \int \frac{d\mathbf{q}d\omega}{(2\pi)^4} \frac{\gamma^2}{|\epsilon_n^R|^2} \Im D^R(q, \omega) \Re \frac{\zeta_n}{1 - \zeta_n} \times N_{\omega} n_{\epsilon}(1 - n_{\epsilon+\omega}) - (1 + N_{\omega})(1 - n_{\epsilon})n_{\epsilon+\omega}, \quad (1)$$

where $D^R(q, \omega)$ is the phonon Green function

$$D^R(\mathbf{q}, \omega) = (\omega - \omega_{\mathbf{q}} + i0)^{-1} + (\omega + \omega_{\mathbf{q}} + i0)^{-1}, \quad (2)$$

$\gamma = D_0(\mathbf{q} \cdot \mathbf{e})/(2\rho\omega_{\mathbf{q}})^{1/2}$ is the vertex of the electron-phonon interaction, D_0 is the constant of DP, and \mathbf{e} is the phonon polarization vector.

In the collision integral I_{ϵ} , ζ_n is an integral over the impurity-averaged electron Green functions [6],

$$\zeta_n = \frac{1}{\pi\nu_n\tau} \int \frac{d\mathbf{p}}{(2\pi)^n} G^A(\mathbf{p}, \epsilon) G^R(\mathbf{p} + \mathbf{q}, \epsilon + \omega) = \begin{cases} \frac{\arctan(ql)}{(ql)} & n = 3, \\ \frac{1}{\sqrt{1+(ql)^2}} & n = 2, \\ \frac{1-i\omega\tau}{(1-i\omega\tau)^2+(ql)^2} & n = 1, \end{cases} \quad (3)$$

where ν_n is the two-spin electron density of states in n -dimensional electron system. We limited our consideration by the condition $ql > u/v_F \sim 10^{-2}$, which allows us to put $\omega = 0$ in ζ_3 and ζ_2 .

The screening of DP is described by the dielectric function $\epsilon_n^R(q, \omega)$. Further calculations show that in the 3D and 2D electron systems the characteristic frequencies $\omega \sim T$

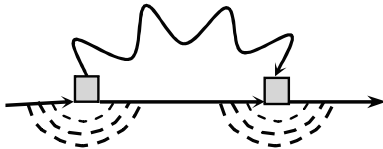


FIG. 1. Electron self-energy diagram. A wavy line stands for e -ph scattering, a dotted line stands for elastic electron scattering from random potential, and a straight line stands for the electron Green function.

are small compared with Dq^2 (D is the diffusion coefficient). In this limit the dielectric function is

$$\epsilon_n^R(q) = \begin{cases} 1 + (\kappa_3/q)^2, & \kappa_3^2 = 4\pi e^2 \nu_3 \quad 3D; \\ 1 + (\kappa_2/q_{\parallel}), & \kappa_2 = 2\pi e^2 \nu_2 \quad 2D. \end{cases} \quad (4)$$

For 1D conductors we should take into account the dynamical character of electron screening. If $q_{\parallel}r \gg 1$ (r is the conductor radius), the dielectric function is

$$\epsilon^R = 1 + e^2 \nu_1 \ln \frac{1}{(q_{\parallel}r)^2} \frac{(q_{\parallel}l)^2}{(q_{\parallel}l)^2 - (\omega\tau)^2 - i\omega\tau}. \quad (5)$$

The e -ph relaxation rate is calculated as a variation of the collision integral $\tau_{e-ph}^{-1} = -\delta I_{e-ph}/\delta n_e$. In equilibrium, $N_{\omega} = N_{\omega}^{\text{eq}}(T)$ and $n_{\epsilon} = n_{\epsilon}^{\text{eq}}(T)$, and the relaxation rate of electrons at the Fermi surface ($\epsilon = 0$) is

$$\frac{1}{\tau_{e-ph}} = 4\tau \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\gamma^2}{|\epsilon^R|^2} (N_{\omega_q}^{\text{eq}} + n_{\omega_q}^{\text{eq}}) \Re \frac{\zeta_n(\omega_q)}{1 - \zeta_n(\omega_q)}. \quad (6)$$

We also calculate the heat flux from hot electrons with the temperature θ to phonons with the temperature T . The heat flux may be presented through the energy control function $F(T)$ as

$$P(\theta, T) = \nu_n \int d\epsilon \epsilon I_{\epsilon}(\theta, T) = F(\theta) - F(T), \quad (7)$$

$$F(T) = 4\tau \nu_n \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\gamma^2}{|\epsilon^R|^2} \omega_q^2 N_{\omega_q}^{\text{eq}} \Re \frac{\zeta_n(\omega_q)}{1 - \zeta_n(\omega_q)}. \quad (8)$$

First we calculate the relaxation rate in a bulk semiconductor. Substituting ζ_3 [Eq. (3)] and $\epsilon_3^R(q)$ [Eq. (4)] into Eq. (6) we find

$$\frac{1}{\tau_{e-ph}} = \frac{D_0^2 \nu_3}{\rho u^2} \frac{T^3}{(p_F u)^2} F(q_T l, q_T/\kappa_3), \quad (9)$$

$$F(y, z) = \int_0^{\infty} dx x^2 \frac{xy \arctan(xy)}{xy - \arctan(xy)} \times \left(\frac{(xz)^2}{(xz)^2 + 1} \right)^2 (N_x^{\text{eq}} + n_x^{\text{eq}}). \quad (10)$$

These formulas in limiting cases are summarized in Table I. In the pure limit, $Tl/u \gg 1$, we reproduce well-known results [17]: in the case of weak screening, $T > u\kappa_3$, the relaxation rate is proportional to T^3 ; for screened DP, $T < u\kappa_3$, the relaxation rate changes as T^7 . In the impure limit the relaxation rate is proportional to T^2/l for unscreened DP and to T^6/l for the screened DP. Thus, contrary to the Pippard ineffectiveness condition in metals [5–7], the relaxation rate in semiconductors is enhanced by a factor of $u/(Tl)$ due to elastic electron scattering. The energy control function may be estimated as $F(T) \simeq C_e T/\tau_{e-ph}$, where C_e is the electron heat capacity. In Table I we present $F(T)$ with exact coefficients,

TABLE I. Electron-phonon energy relaxation time and energy control function in a bulk semiconductor.

	$T > u\kappa_3$ (weak screening)		$T < u\kappa_3$ (strong screening)	
	$T > u/l$	$T < u/l$	$T > u/l$	$T < u/l$
τ_{e-ph}^{-1}	$\frac{7\pi\zeta(3)}{4} \frac{D_0^2\nu_3}{\rho u^2} \frac{T^3}{(p_F u)^2}$	$\frac{3\pi^2}{4} \frac{D_0^2\nu_3}{\rho u^2} \frac{T^2}{p_F^2 l u}$	$\frac{5715\pi\zeta(7)}{8} \frac{D_0^2\nu_3}{\rho u^2} \frac{T^7}{p_F^2 \kappa_3^4 u^6}$	$\frac{3\pi^6}{4} \frac{D_0^2\nu_3}{\rho u^2} \frac{T^6}{p_F^2 \kappa_3^4 l u^5}$
$F(T)$	$6\pi\zeta(5) \frac{D_0^2\nu_3^2}{\rho u^2} \frac{T^5}{(p_F u)^2}$	$\frac{\pi^4}{10} \frac{D_0^2\nu_3^2}{\rho u^2} \frac{T^4}{p_F^2 l u}$	$10080\pi\zeta(9) \frac{D_0^2\nu_3^2}{\rho u^2} \frac{T^9}{p_F^2 \kappa_3^4 u^6}$	$\frac{4\pi^8}{5} \frac{D_0^2\nu_3^2}{\rho u^2} \frac{T^8}{p_F^2 \kappa_3^4 l u^5}$

because measurements of $F(T)$ are widely used to obtain D_0 .

Now we consider the e -ph relaxation in two-dimensional electron gas. Using Eqs. (3) and (4), we find that the relaxation rate [Eq. (6)] may be presented as

$$\frac{1}{\tau_{e-ph}} = \frac{D_0^2 T^3}{2\pi^2 \rho v_F u^4} \Phi(q_T l, q_T / \kappa_2), \quad (11)$$

$$\Phi(y, z) = \int_0^\infty dx x^2 \int_0^{\pi/2} d\theta \frac{xy}{\sqrt{1 + (xy \sin \theta)^2} - 1} \times \frac{(xz)^2 \sin^3 \theta}{(xz \sin \theta + 1)^2} (N_x^{\text{eq}} + n_x^{\text{eq}}). \quad (12)$$

These formulas in limiting cases are summarized in Table II. In the pure limit, we reproduce well-known results [15]. In the temperature range $T \gg \kappa_2 u$, where DP is weakly screened, the relaxation rate is proportional to T^3 ; for strongly screened DP the relaxation rate changes as T^5 . In the impure limit, in the case of weak screening, the relaxation rate is proportional to $T^2 \ln T$ and inversely proportional to l . At low temperatures, where DP is strongly screened, the relaxation rate is proportional to T^4/l . Thus, in heterostructures, elastic electron scattering significantly enhances the e -ph interaction.

Finally, we consider the e -ph interaction in the multichannel 1D system. Channels may be associated with wires, shells, and electron subbands. Variations of the multichannel model are applied to one-dimensional organic conductors, CuO chains in high- T_c superconductors, and multiwall carbon nanotubes [20]. For simplicity we consider identical channels and neglect the Coulomb interaction between channels. We suggest that electrons are scattered between channels and interchannel scattering prevails over backscattering in the same channel, so the system is in the conducting state. Electron-phonon scattering keeps an electron in the same channel and, therefore, it is screened by electrons in this channel. Using Eqs. (3) and

(6), we find that without screening the relaxation rate in the pure conductor is given by

$$\frac{1}{\tau_{e-ph}} = \frac{7\zeta(3)}{8\pi} \frac{D_0^2 T^3}{\rho v_F u^4}. \quad (13)$$

Calculating the integrant in Eq. (6) in the general case, note that for a 1D conductor $q_{\parallel} = q \cos \phi = qx$ (ϕ is the angle between \mathbf{q} and a wire) and within the logarithmic accuracy the integral over the direction of \mathbf{q} is given by

$$\int_0^1 dx \frac{(qlx)^2}{2 [(1 - 2e^2 \nu_1 \ln q r x)(qlx)^2 - (\omega\tau)^2]^2 + (\omega\tau)^2} = \frac{1}{2ql(1 - 2e^2 \nu_1 \ln q r)^{3/2}} \begin{cases} \pi/2 & \omega\tau \gg 1; \\ \pi/\sqrt{8\omega\tau} & \omega\tau \ll 1. \end{cases} \quad (14)$$

Equation (14) shows that the crossover to the impure limit is described by the parameter $\omega\tau$, which is of the order of $T\tau$. In the impure limit, $T\tau \ll 1$, the characteristic value of the transferred momentum $q_c \simeq \sqrt{\omega\tau}/(l\sqrt{1 + e^2 \nu_1}) \simeq \sqrt{T\tau}/(l\sqrt{1 + e^2 \nu_1})$ is much smaller than q_T . In this case, the relaxation rate and energy control function for the one-dimensional multichannel conductor are

$$\frac{1}{\tau_{e-ph}} = \frac{3(8 - \sqrt{2})\zeta(5/2)}{64\sqrt{2}\pi[1 - e^2 \nu_1 \ln(q_c r)^2]^{3/2}} \frac{D_0^2 T^{5/2}}{\sqrt{\tau} \rho v_F u^4} \quad (15)$$

$$F(T) = \frac{105\zeta(9/2)}{128\sqrt{\pi}[1 - e^2 \nu_1 \ln(q_c r)^2]^{3/2}} \frac{D_0^2 \nu_1 T^{9/2}}{\sqrt{\tau} \rho v_F u^4}.$$

As seen from Eq. (15), screening substantially changes values of τ_{e-ph} and $F(T)$, but just weakly affects the temperature dependencies. Comparing Eqs. (13) and (15), we find that in the impure limit, the electron-phonon interaction is enhanced by the factor of $1/\sqrt{T\tau}$.

The electron-phonon-impurity interference in metals and semiconductors may be qualitatively understood in the following way. First, elastic electron scattering effec-

TABLE II. Electron-phonon energy relaxation time and energy control function in two-dimensional electron structures.

	$T > u\kappa_2$ (weak screening)		$T < u\kappa_2$ (strong screening)	
	$T > u/l$	$T < u/l$	$T > u/l$	$T < u/l$
τ_{e-ph}^{-1}	$\frac{7\zeta(3)}{4\pi} \frac{D_0^2 T^3}{\rho v_F u^4}$	$\frac{D_0^2 T^2}{\rho l v_F u^3} \ln \frac{T}{\kappa_2 u}$	$\frac{93\zeta(5)}{8\pi} \frac{D_0^2 T^5}{\rho \kappa_2^2 v_F u^6}$	$\frac{\pi^2}{4} \frac{D_0^2 T^4}{\rho \kappa_2^2 l v_F u^5}$
$F(T)$	$\frac{6\zeta(5)}{\pi} \frac{D_0^2 \nu_2 T^5}{\rho v_F u^4}$	$\frac{2\pi^2}{15} \frac{D_0^2 \nu_2 T^4}{\rho l v_F u^3} \ln \frac{T}{\kappa_2 u}$	$\frac{90\zeta(7)}{\pi} \frac{D_0^2 \nu_2 T^7}{\kappa_2^2 v_F u^6}$	$\frac{8\pi^4}{63} \frac{D_0^2 \nu_2 T^6}{\rho \kappa_2^2 l v_F u^5}$

tively averages DP over the Fermi surface. Second, the diffusive motion holds an electron in the interaction region and increases the interaction time. In metals the Fermi surface average of the deformation potential equals to zero [17,18]. As a result of this averaging, the effective e -ph vertex is substantially decreased (see Ref. [6]). In metals, this effect prevails over the modification of the interaction time and strongly suppresses the e -ph relaxation. In semiconductors, DP weakly depends on the electron momentum and the DP tensor is usually approximated by a constant. Therefore, elastic scattering in semiconductors enlarges the interaction time, which in turn enhances the e -ph relaxation.

Recently, the e -ph relaxation rate has been directly measured in 2D electron gas in Si with a molecular-beam-epitaxy-grown Sb δ layer [12,13]. Because of lack of the theory for semiconducting materials and structures, the observed T^4 dependence was associated with the Pippard concept of the ineffectiveness of the e -ph interaction. According to our results, the T^4 dependence in 2D structures originates from *disorder-enhanced* screened DP coupling (see Table II). Analogous data with T^4 dependence have been obtained in heavily doped quasi-two-dimensional Si films at subKelvin temperatures [14]. Note that e -ph relaxation rate is often evaluated from the electron dephasing rate. Such data also give evidence in favor of significant enhancement of e -ph coupling in disordered semiconductors. For example, in 3D Si:P layers with $l \sim 5$ nm, the relaxation time at 4.2 K was found to be 10 ps [21], which is significantly shorter than that in pure materials.

To conclude, we calculate the e -ph relaxation rate in disordered semiconductors [Eq. (9), Table I], two-dimensional [Eq. (11), Table II], and one-dimensional [Eq. (15)] semiconducting structures. Our results show that the e -ph relaxation is strongly enhanced due to disorder.

The research was supported by the ONR and MRCAF grants. We are grateful to D. Maslov for helpful discussions.

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