Stimulated decay of nonselectively pumped optical phonons in GaAs

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We have considered the processes determining the steady-state population of longitudinal optical (LO) phonons being intensively produced in GaAs by electrons optically excited or accelerated in strong external electric field. The emitted LO phonons pass their energy to acoustic phonon systems through anharmonic decay into longitudinal acoustic (LA) phonons. The latter emerge as almost monoenergetic and concentrated in a narrow layer in the reciprocal space. Had the occupation numbers of $\boldsymbol{k}$ states within this layer grown to unity, the decay process would have become *stimulated*, effectively desolating the built-in LO states and giving rise to various nonlinear phenomena. We show that this is actually the case by tracing LA phonon kinetics and calculating the number of phonons in each $\boldsymbol{k}$ state available for feedback. The results obtained substantially clarify the physical picture of energy transfer processes mediating heat removal in GaAs.
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I. INTRODUCTION

During recent decades, the fundamental problems of energy transformations in highly nonequilibrium states of electron and phonon systems in a crystal continue to be current topics of interest in solid state physics. This interest is also stimulated by the progress in solid state electronics, where the necessity for heat removal imposes severe restrictions on the further miniaturization of semiconductor devices.

When the mean energy of the electrons sufficiently exceeds the thermal energy, the main channel of their energy exchange with the crystal lattice in polar semiconductors is known to be emission (and absorption) of longitudinal optical (LO) phonons.[1] Having low group velocity and being prone to getting confined in inhomogeneous structures, the LO phonons themselves are not ideally suited for transporting excess energy to a remote heat sink. Naturally enough, of primary importance are mechanisms leading to the transfer of energy from this optical branch to fast-propagating acoustic phonons. This conversion of LO phonons to acoustic phonons is commonly accepted to proceed mainly through the Klemens channel of the anharmonic decay into two longitudinal acoustic phonons $\mathrm{LO} \rightarrow \mathrm{LA}_1 + \mathrm{LA}_2$.[2] First suggested for the case of bulk diamond, this mechanism was then confirmed to play a leading role in LO phonon decay in other materials[3,4] and heterostructures.[5] Since the characteristic time of this weak process, $\tau_{\mathrm{sp}} \sim 7$ ps in GaAs,[4] is at least an order of magnitude longer than that of LO phonon emission by electrons, the process is known as the bottleneck for heat removal in high-field electronic devices.[6] In this paper, we focus on this dominant channel of LO phonon decay. Another recently reported and controversial mechanism where the LO decays into zone-edge LO and TA phonons[7] will be considered elsewhere.

Being produced by hot electrons, the LO phonons of frequency $\sim \omega_0 \approx 5.3 \times 10^{13}\ \mathrm{s}^{-1}$ are concentrated in a small region of the Brillouin zone near $\boldsymbol{q}=0$, $q \leqslant q_0 \sim \sqrt{2\omega_0 m^*/\hbar} \sim 10^6\ \mathrm{cm}^{-1}$, where m^* is the electron effective mass. The consequent LO phonon decay through the Klemens channel results in LA phonons that are nearly monoenergetic, with frequency $\omega_0/2$. The corresponding wave vectors $k \sim k_0 = \omega_0/2s \approx 5.1 \times 10^7\ \mathrm{cm}^{-1}$ lie in the middle of the Brillouin zone ($s = 5.2 \times 10^5$ cm/s is the longitudinal sound velocity). Thus, nearly all the produced LA phonons are distributed over very few modes lying in a thin spherical layer of radius k_0 in the reciprocal space. The population numbers of these modes, $n_{\boldsymbol{k}}$, may well become comparable to or even greater than unity. If this occurs, the LO phonon decay becomes a *stimulated* process, effectively depopulating the LO states and giving rise to various nonlinear phenomena.

A somewhat similar situation was discussed once in the literature regarding selective optical pumping of LO phonons in bulk diamond crystals due to Raman scattering[8] or intentional beats of two laser beams of close frequencies.[9] The population numbers of the acoustic phonon modes resulting from LO phonon decay in that situation have been shown to depend strongly on the pumping radiation linewidth and estimates of their values vary in different papers from 10^{-2} to 10.[8,10–12] In the case under consideration, LO phonon states are produced in a much greater phase space region, which makes it valid not to take into account the uncertainty in the momentum and energy conservation laws for the LO phonon decay. Consequently, the possibility of and restrictions on stimulated decay are determined by processes of quite different physical nature. Note also that the nonselective generation of LO phonons by hot electrons corresponds to the much more commonly encountered experimental situation.

In the present paper we analyze theoretically the influence of a highly nonequilibrium distribution of the product LA phonons on the LO phonon decay process. We obtain the population numbers of the LA phonon modes, $n_{\boldsymbol{k}}$, demonstrate that the phase volume occupied by LA phonons effectively shrinks under high intensity of generation, and show that this effect modifies the LO phonon decay process.

The steady-state population of LA phonons is determined by the competition between two processes: their production by the decaying LO phonons, and their removal as a result of anharmonic decay. This means that to solve the problem we need to know the lifetime of the LA phonons produced by the decaying LO phonons. As we mentioned previously, the wave vectors of the LA phonons under consideration lie in the middle of the Brillouin zone. In order to calculate the lifetime of such phonons one needs to take into account the peculiarities of the acoustic phonon spectrum in the short-wavelength range. This is done in Sec. II of this paper. Then, in Sec. III, we derive coupled kinetic equations for LA and LO phonons. In Sec. IV, we present results of the numerical solution of kinetic equations and asymptotical formulas for the case of a very high level of LO phonon generation. Finally, in Sec. V, we present a discussion of the results.

II. DECAY RATE OF LA PHONONS

In this section we calculate the anharmonic decay rate of high wave vector LA phonons. The energy and momentum conservation laws permit two kinds of decay processes:

$$\mathrm{LA}(\boldsymbol{k})\rightarrow\mathrm{LA}(\boldsymbol{k}')+\mathrm{TA}(\boldsymbol{k}'') \tag{1}$$

and

$$\mathrm{LA}(\boldsymbol{k})\rightarrow\mathrm{TA}(\boldsymbol{k}')+\mathrm{TA}(\boldsymbol{k}''). \tag{2}$$

A factor which plays a crucial role is the densities of the final phonon states. The dispersion curve for LA phonons deviates weakly from a straight line up to the very boundary of the Brillouin zone. In contrast, the TA phonon dispersion curve bends at relatively moderate $\boldsymbol{k}$. As a result, the density of TA states has a sharp divergence at $\omega\rightarrow\omega_t$, where ω_t is the frequency of TA phonons at the boundary of the Brillouin zone. This divergence substantially affects the decay of LA phonons.

In this paper, we adopt the following model dispersion laws for LA and TA phonons, which reflect the features mentioned:

$$\omega_{\mathrm{LA}}=sk,\qquad \omega_{\mathrm{TA}}=\omega_t^*\tanh(k/k^*), \tag{3}$$

where s is the longitudinal sound velocity, k^* is a fitting parameter which determines the degree of bending of the TA phonon dispersion curve, $\omega_t^*=\omega_t/\tanh(k_b/k^*)$, and k_b is the wave vector at the boundary of the Brillouin zone. For GaAs we take $k^*=4.43\times10^7\ \mathrm{cm}^{-1}$, $\omega_t=1.48\times10^{13}\ \mathrm{s}^{-1}$, $s=5.2\times10^5$ cm/s, and $k_b=2.51k^*$.

Within the framework of the Fermi golden rule, the partial probabilities of decay processes are described by

$$w_a(j,\boldsymbol{k};j',\boldsymbol{k}';j'',\boldsymbol{k}'')=\frac{2\pi}{\hbar^2\omega_t^*}|V(j,\boldsymbol{k};j',\boldsymbol{k}';j'',\boldsymbol{k}'')|^2 \times\delta(\tilde{\omega}_j(\boldsymbol{k})-\tilde{\omega}_{j'}(\boldsymbol{k}')-\tilde{\omega}_{j''}(\boldsymbol{k}'')), \tag{4}$$

where $j=t,l$ indicates the phonon branch, $\tilde{\omega}=\omega/\omega_t^*$. Equation (4) is written in terms of the matrix element $V(j,\boldsymbol{k};j',\boldsymbol{k}';j'',\boldsymbol{k}'')$ of the Hamiltonian of the three-phonon interaction between initial and final states.

In calculating the matrix element $V(j,\boldsymbol{k};j',\boldsymbol{k}';j'',\boldsymbol{k}'')$, we follow the method suggested in Refs. 13 and 14, i.e., we use the Hamiltonian of third-order anharmonic interaction derived in the framework of the theory of elasticity. The spatial density of this Hamiltonian is given by the expression

$$\begin{aligned}\hat{H}_a=&(\tfrac{1}{2}c_{12}+3c_{155}-6c_{456})\zeta_{ii}\zeta_{ik}^2\\&+(\tfrac{1}{2}c_{11}-\tfrac{1}{2}c_{12}+6c_{456})\zeta_{ij}\zeta_{ik}\zeta_{kj}\\&+(3c_{155}-6c_{456})\zeta_{ii}\zeta_{jk}\zeta_{kj}+(c_{111}-6c_{155}+4c_{456})\\&\times\zeta_{ii}\zeta_{jj}\zeta_{kk}+2c_{456}\zeta_{ij}\zeta_{jk}\zeta_{ki}.\end{aligned} \tag{5}$$

Here c_{ij} and c_{ijk} are second- and third-order elastic constants and ζ_{ij} are components of the strain tensor.

Calculating the matrix element of this Hamiltonian for the process of Eq. (1), making use of the dispersion relations of Eq. (3), and taking numerical values of the elastic constants for GaAs,[15] we finally obtain the following expression for the decay probability:

$$w_a(l,\boldsymbol{k};l,\boldsymbol{k}';t,\boldsymbol{k}'')=G_a\frac{qq'(q^2-q'^2)^2}{q''^2\tanh q''}\times\cos^2\theta\sin^2\theta\,\delta(q-q'-\alpha\tanh q''). \tag{6}$$

Here θ is the angle between vectors $\boldsymbol{k}$ and $\boldsymbol{k}'$, so that the wave number k'' is given as

$$k''^2=k^2+k'^2-2kk'\cos^2\theta. \tag{7}$$

The numerical value for constant G_a is

$$G_a=1.1\times10^8\ \mathrm{s}^{-1}. \tag{8}$$

The resulting TA wave is polarized in the normal direction to plane $(\boldsymbol{k},\ \boldsymbol{k}')$.

The corresponding decay rate is obtained as

$$\begin{aligned}\Gamma_{llt}(\tilde{\omega})=&\int d^3k'\ w_a(l,\boldsymbol{k};l,\boldsymbol{k}';t,\boldsymbol{k}'')\\=&\frac{\pi G_a\alpha^5}{8\tilde{\omega}^4}\int_0^{\tilde{\omega}_b}d\tilde{\omega}'\frac{\tilde{\omega}'(2\tilde{\omega}-\tilde{\omega}')^2}{(\tilde{\omega}-\tilde{\omega}')^2(1-\tilde{\omega}'^2)K(\tilde{\omega}')}\\&\times[\tilde{\omega}^2+(\tilde{\omega}-\tilde{\omega}')^2-K^2(\tilde{\omega}')]^2\\&\times[(2\tilde{\omega}-\tilde{\omega}')^2-K^2(\tilde{\omega}')][K^2(\tilde{\omega}')-\tilde{\omega}'^2]\\&\times\Theta(\tanh[\alpha(2\tilde{\omega}-\tilde{\omega}')]-\tilde{\omega}').\end{aligned} \tag{9}$$

Here $\tilde{\omega}_b=sk_b/\omega_t^*$ and $\alpha\approx0.59$ is the ratio of transverse and longitudinal sound velocities; $K(\tilde{\omega})$ is determined by the equation

$$K(\tilde{\omega})=\frac{1}{2\alpha}\ln\left(\frac{1+\tilde{\omega}}{1-\tilde{\omega}}\right), \tag{10}$$

which is just the inverse function with respect to the TA dispersion law and is for convenience taken to have the factor $1/\alpha$.

For the process of Eq. (2) there exist, in principle, two possible combinations of the resulting TA phonon polarization vectors: (1) both vectors lie in the plane $(\boldsymbol{k}, \boldsymbol{k}')$; and (2) both vectors are normal to this plane. However, for the case of GaAs the numerical values of the elastic constants make the matrix element representing the second combination very small compared to that representing the first combination. Neglecting this second possibility, we obtain the probability of the process of Eq. (2) as

$$w_a(l,\boldsymbol{k};t,\boldsymbol{k}';t,\boldsymbol{k}'')$$
$$=\frac{G_a}{\alpha}\frac{kk'^2}{k''^2\tanh k'\tanh k''} \tag{11}$$
$$\times[k^2\cos^2\theta-0.74k^2+0.26(k'^2-2kk'\cos\theta)^2]$$
$$\times\delta(k-\alpha\tanh k'-\alpha\tanh k''), \tag{12}$$

where q'' and G_a are still given by expressions (7), (8), and the coefficients with numerical values of 0.74 and 0.26 are produced by complicated combinations of elastic constants.

The corresponding decay rate for the process of Eq. (2) is found as

$$\Gamma_{ltt}(\tilde\omega)=\int d^3k'\,w_a(l,\boldsymbol{k};t,\boldsymbol{k}'';t,\boldsymbol{k}'')=\frac{\pi G_a\alpha^5}{8}\int_0^{\tilde\omega_t}d\tilde\omega'\frac{\{[\alpha^2\tilde\omega^2-K^2(\tilde\omega')-K^2(\tilde\omega-\tilde\omega')]^2-2.96K^2(\tilde\omega')K^2(\tilde\omega-\tilde\omega')\}^2}{\tilde\omega'(\tilde\omega-\tilde\omega')[1-(\tilde\omega-\tilde\omega')^2](1-\tilde\omega'^2)K(\tilde\omega')K(\tilde\omega-\tilde\omega')}$$
$$\times\Theta\left(\tilde\omega'-\tilde\omega+\frac{\tilde\omega'+\tanh(\alpha\tilde\omega)}{1+\tilde\omega'\tanh(\alpha\tilde\omega)}\right)\Theta\left(\tilde\omega-\tilde\omega'-\left|\frac{\tanh(\alpha\tilde\omega)-\tilde\omega'}{1-\tilde\omega'\tanh(\alpha\tilde\omega)}\right|\right), \tag{13}$$

where $\tilde\omega_t=\omega_t/\omega_t^*$. We present $\Gamma_{llt}(\tilde\omega)$ and $\Gamma_{ltt}(\tilde\omega)$ in Figs. 1 and 2, respectively. The arrows indicate the position of LA phonons initially produced by the LO phonon decay. By comparing these functions, it is evident that while the decay channel of Eq. (2) is open, i.e., for $\tilde\omega<2$, the rate of this decay completely dominates over the contribution of the processes of Eq. (2), and leads to a LA phonon lifetime as low as ~ 10 ps. This means that we can neglect any secondary LA phonons being generated as a result of the process of Eq. (1).

III. KINETIC EQUATIONS FOR ACOUSTIC AND OPTICAL PHONONS

The phonons being subject to the Bose-Einstein statistics, the integral rate of LA phonon production in the decay process, LO $\rightarrow \mathrm{LA}_1+\mathrm{LA}_2$, Γ_{LO}, is given by the expression

$$\Gamma_{\mathrm{LO}}=\frac{1}{2}\sum_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}\gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}N_{\boldsymbol{q}}(1+n_{\boldsymbol{k}})(1+n_{\boldsymbol{k}'}), \tag{14}$$

where $\gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}$ is the probability of the transition of the LO phonon having wave vector $\boldsymbol{q}$ into two LA phonons with wave vectors $\boldsymbol{k}$ and $\boldsymbol{k}'$, found in the framework of the Fermi golden rule; $N_{\boldsymbol{q}}$ are the population numbers of LO phonons. The factor 1/2 is introduced here in order not to count the same process twice. The value γ takes into account energy and momentum conservation rules upon the phonon decay. The rate of inverse process $\mathrm{LA}_1+\mathrm{LA}_2\rightarrow\mathrm{LO}$, Γ_{LA}, is

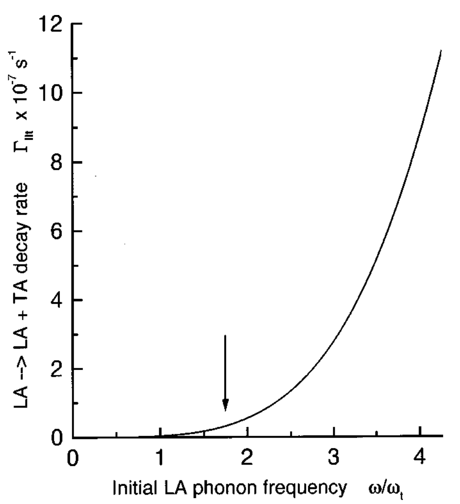

FIG. 1. Total decay rate due to LA→LA+TA process. Arrow indicates the point of initial LA phonon injection.

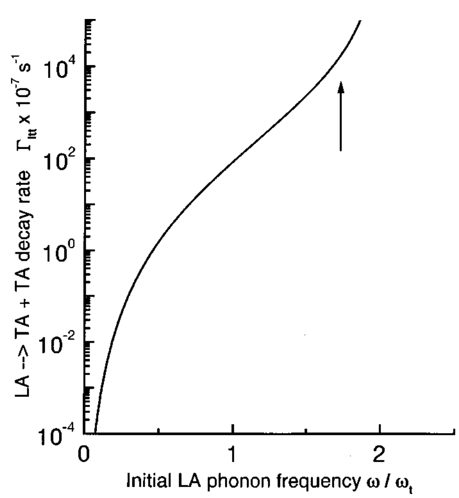

FIG. 2. Total decay rate due to LA→TA+TA process. We use a log scale because of the divergence of Γ_{ltt} at LA phonon frequencies twice the cutoff frequency of the TA spectrum. Arrow indicates the point of initial LA phonon injection.

$$\Gamma_{\mathrm{LA}}=\frac{1}{2}\sum_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}\gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}(1+N_{\boldsymbol{q}})n_{\boldsymbol{k}}n_{\boldsymbol{k}'}. \tag{15}$$

The competition of these two processes makes the LO phonon population change with the rate of

$$\Gamma_{\mathrm{LO}}-\Gamma_{\mathrm{LA}}=\frac{1}{2}\sum_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}\gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}N_{\boldsymbol{q}}(1+n_{\boldsymbol{k}}+n_{\boldsymbol{k}'})-\frac{1}{2}\sum_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}\gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}n_{\boldsymbol{k}}n_{\boldsymbol{k}'}. \tag{16}$$

The second term in Eq. (16) is essential for establishing equilibrium in the phonon system. However, in a highly nonequilibrium situation (like that under consideration) we can neglect this term, as was pointed out by Levinson.[11] Though in the steady-state process the ratio of the total number of LA phonons to the total number of LO phonons is just twice the ratio of their characteristic lifetimes, the volume in the reciprocal space occupied by LO phonons is much less than that of LA phonons. The ratio of these volumes is $\sim(q_0/k_0)^2$; this small parameter provides $n_{\boldsymbol{k}}\ll N_{\boldsymbol{q}}$ and allows one to disregard the second term in Eq. (16) and to introduce the characteristic time of LO phonon decay as

$$\frac{1}{\tau_{\boldsymbol{q}}}=\frac{1}{2}\sum_{\boldsymbol{k},\boldsymbol{k}'}\gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}(1+n_{\boldsymbol{k}}+n_{\boldsymbol{k}'}). \tag{17}$$

Later, upon obtaining formulas for $n_{\boldsymbol{k}}$ in Sec. V, we will be able to estimate the value of the neglected term and thus justify the validity of our approximation.

We assume that generation of optical phonons is isotropic, in other words, there is no special direction in the reciprocal space. This is obviously the case for the optical excitation of electrons. If LO phonons are produced by the electrons accelerated in the high electric field, then we can treat the generation of LO phonons as isotropic with a high degree of accuracy.[6] Upon making these approximations, the population numbers of LO and LA phonons depend only on absolute values of wave vectors, q and k, and Eq. (17) transforms to

$$\frac{1}{\tau_q}=\Gamma\int dk\,d\theta\sin\theta k^2(1+n_k+n_{k'})\,\delta(\hbar\omega_0-\hbar s(k+k')). \tag{18}$$

Here we assumed that the dependence of the value γ on phonon wave vectors is restricted by the conservation rules only. The constant Γ will be expressed below via the rate of spontaneous decay of LO phonons. Here θ is the angle between $\boldsymbol{q}$ and $\boldsymbol{k}$, and k' is determined from the momentum conservation rule:

$$k'=\sqrt{k^2+q^2-2kq\cos\theta}.$$

From Eq. (18) we obtain

$$\frac{1}{\tau_q}=\frac{1}{\tau_{\mathrm{sp}}}\left(1-\frac{1}{12}\frac{q^2}{k_0^2}+\frac{1}{k_0^2q}\int_{\Delta_q}dk\,k(2k_0-k)(n_k+n_{2k_0-k})\right), \tag{19}$$

where the characteristic time of spontaneous decay of the LO phonon for $q=0$ is

$$\frac{1}{\tau_{\mathrm{sp}}}=\frac{\Gamma k_0^2}{\hbar s}, \tag{20}$$

and the region of integration in Eq. (19) is

$$\Delta_q=(k_0-q/2,k_0+q/2). \tag{21}$$

The population number of a LA phonon is determined by the rate of generation from the decay of LO phonons multiplied on the time of LA phonon decay $\tau_a\approx[\Gamma_{ltt}(\tilde{\omega}_0/2)]^{-1}$:

$$n_k=\tau_a\sum_{\boldsymbol{q},\boldsymbol{k}'}\gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}(1+n_{\boldsymbol{k}}+n_{\boldsymbol{k}'})N_q, \tag{22}$$

where N_q is the population number of the LO phonons. Analogous to the case of calculation of τ_q, one gets

$$n_k=\frac{2\tau_a(1+n_k+n_{2k_0-k})}{k_0^2\tau_{sp}}\frac{2k_0-k}{k}\int_{\Delta_k}dq\,qN_q, \tag{23}$$

where the interval of integration Δ_k is

$$\Delta_k=(2|k-k_0|,2k_0). \tag{24}$$

Below we take into account that the characteristic value of q is much less than k_0. This allows one to rewrite Eqs. (19) and (23) in the following form:

$$\frac{1}{\tau_q}=\frac{1}{\tau_{\mathrm{sp}}}\left(1+\frac{1}{q}\int_{\Delta_q}dk(n_k+n_{2k_0-k})\right), \tag{25}$$

$$n_k=\frac{2\tau_a(1+n_k+n_{2k_0-k})}{k_0^2\tau_{\mathrm{sp}}}\int_{\Delta_k}dq\,qN_q. \tag{26}$$

Combining Eqs. (25) and (26), we obtain a nonlinear integral equation for the LA phonon population numbers:

$$n_k=\frac{2\tau_a(1+n_k+n_{2k_0-k})}{k_0^2}\int_{\Delta_k}\frac{dq\,qG_q}{1+(1/q)\int_{\Delta_q}(n_k+n_{2k_0-k})dk}. \tag{27}$$

The solution of Eq. (27) determines both the population numbers of LA phonons and the lifetime of LO phonons via Eq. (25).

IV. SOLUTION OF THE KINETIC EQUATIONS

To describe a particular situation one needs to know the differential generation rate of LO phonons G_q. We model G_q as being constant G for q less than some value q_0 and zero outside that region. The constant G is related to the integral generation rate of LO phonons P as

$$G=\frac{6\pi^2P}{q_0^3}. \tag{28}$$

This choice allows one to rewrite Eq. (27) in dimensionless form

$$n_\zeta=\beta(1+2n_\zeta)\int_{\Delta_\zeta}\frac{\xi\,d\xi}{1+(2/\xi)\int_{\Delta_\xi}n_\zeta\,d\zeta},\tag{29}$$

where $\zeta=(k-k_0)/q_0$, $\xi=q/q_0$, $\Delta_\zeta=(2|\zeta|,1)$, $\Delta_\xi=(-\xi/2,\xi/2)$, and β is a dimensionless parameter, which characterizes the intensity of the LO phonon generation:

$$\beta=\frac{12\pi^2 P\tau_a}{q_0k_0^2}.\tag{30}$$

In Eq. (29) we took into account that the LA phonon population obeys the condition $n_\zeta=n_{-\zeta}$, as is seen from Eq. (27).

It is easy to obtain an asymptotical solution of Eq. (29) for the case of large β. In this case the LA phonon distribution shrinks to the region of small ζ and one can express τ_q^{-1} via the total concentration of LA phonons. Detailed analysis shows that the LA phonon distribution has Lorentzian shape with the width that is determined by β:

$$n_\zeta=\frac{n_0}{1+4\beta\zeta^2},\tag{31}$$

where n_0 is the population number at $\zeta=0$, which equals

$$n_0=\frac{2}{3\pi}\beta^{3/2}.\tag{32}$$

The asymptotics for the LO phonon lifetime are as follows. At $\xi\gg 1/(2\sqrt{\beta})$ we get

$$\frac{1}{\tau_\xi}=\frac{1}{\tau_{\rm sp}}\left(1+\frac{2\beta}{3\xi}\right).\tag{33}$$

For small $\xi\ll 1/(2\sqrt{\beta})$ one has

$$\frac{1}{\tau_\xi}=\frac{1}{\tau_{\rm sp}}\left(1+\frac{4\beta^{3/2}}{3\pi}\right).\tag{34}$$

Based on these results we can easily estimate the value of the last term in Eq. (16) and thus justify our approximation; see the Appendix.

For arbitrary β we solve Eq. (29) numerically with the use of an iteration procedure. In Figs. 3(a) and 3(b) we present the results of this solution for the LA phonon distribution and the LO phonon lifetime for $\beta=1$, 5, and 10. These results clearly demonstrate that even for moderate LO phonon generation, the LO phonon lifetime is modified considerably due to the process of stimulated decay. Another interesting feature is that due to the stimulated LO phonon decay, τ_q^{-1} is q dependent, though the rate of spontaneous decay does not depend on q.

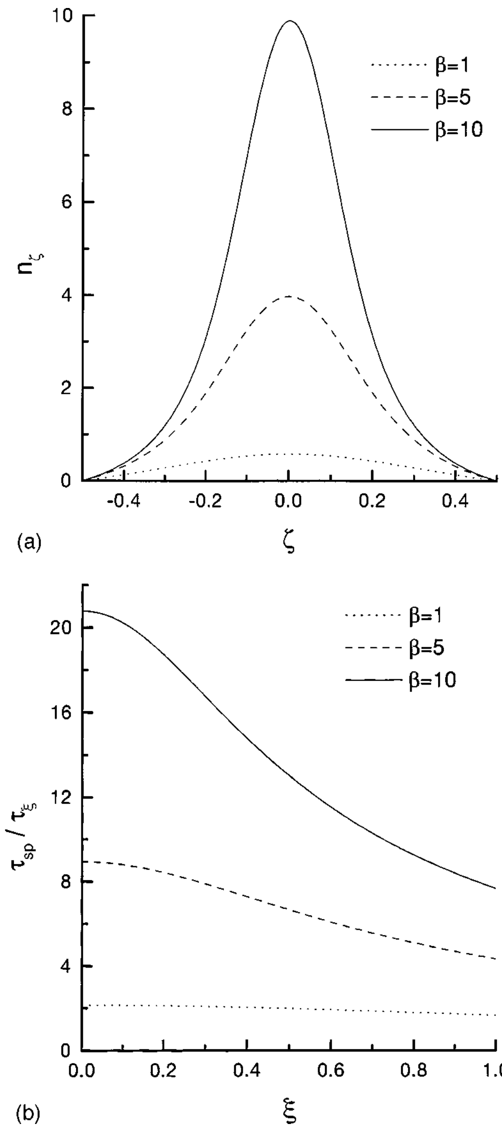

FIG. 3. LA phonon population numbers (a) and LO phonon lifetime (b) as functions of their dimensionless wave vectors at different LO phonon generation rates.

V. DISCUSSION

Let us briefly discuss the consequences of these results.

The result having the most obvious experimental implications is the considerable enhancement of the rate of LO phonon decay in situations where characteristic parameter β exceeds unity. To estimate the value of this parameter in the case of high-field electron transport in GaAs, we use GaAs transport parameters given in Ref. 18. For electron density 10^{18} cm^{-3} and electric field 3 kV/cm we obtain $\beta\sim 2$, provided all the input power dissipates through the emission of LO phonons. This value of β is in good agreement with the data of power consumption in typical semiconductor devices.[19] Higher electric fields do not lead to further increase of β, because substantial growth of the population of X and L valleys lessens the role of emission of long-wavelength LO phonons in electron energy relaxation. In the case of a high-intensity pulse optical excitation of the carriers the values of $\beta\sim 5$ can be achieved. Though in this latter case one deals with a transient process, our estimates of the stimulated LO phonon decay are still valid, because the buildup of the LA phonons is very fast, given their relatively short lifetime we found in Sec. II. Note that a decrease of the LO phonon lifetime with an increase of pumping was observed experimentally by Kash and Tsang.[16] In that paper, this effect was assumed to occur as a result of the LO phonon absorption upon the transitions between light and heavy hole states. As we see, those experimental results can also be due to the stimulated decay of LO phonons. It is worth mentioning also that the authors of Ref. 17 needed to take the LO phonon lifetime as small as 3 ps in order to match satisfactorily the results of their simulations of semiconductor laser operation and experimental data.

The stimulated decay of LO phonons implies that the generation of high wave vector LA phonons takes place. This is manifested through the shrinking of the LA phonon distribution around the value of maximum generation, $k=k_0$. This situation is similar to the case of optical lasers. Note, however, that if one considers an anisotropic situation (say, generation of LO phonons in a spatially confined region, anisotropy of the LO phonon generation and LA phonon decay, etc.), the picture of stimulated decay can be modified considerably with respect to the laser case, since for the LO phonon each decay process results in the generation of *two* different LA phonons.

Our results for the LA phonon decay rate, ~ 10 ps, result in a small value of the LA phonon mean free path of ~ 50 nm. This means that if LO phonons are excited in a layer with a thickness greater than the mean free path (though relatively thin), one can use the results obtained for the bulk case. Very often, the conducting channels of contemporary electronic devices have a thickness of the order of 100 nm. This means that the picture described in this paper can be adapted to the study of stimulated decay of LO phonons in such devices.

Note also that for some other polar semiconductors the effect of stimulated LO phonon decay can be more pronounced. For example, in InP the side valleys of the electron spectrum are situated at energies higher than those of GaAs. So, higher rates of LO phonon emission can be achieved, because the electrons remain mainly in the Γ valley at much higher electric fields. Then, since the LA phonon lifetime is very sensitive to the details of the acoustic phonon spectrum, greater values of τ_a, and, correspondingly, higher LA populations, might be obtained in other materials.

Our results show that stimulated LO phonon decay really occurs in semiconductor structures and devices. This stimulated process should substantially change the LO phonon population numbers and, eventually, the electron transport properties of the system. To describe properly the coupled electron and phonon systems we now have to solve kinetic equations for electrons, LO phonons, and LA phonons self-consistently. Only in this way can we specify the actual q dependence of the LO phonon generation rate G_q, to use it in place of the model function chosen arbitrarily in Sec. III. We plan to present such self-consistent consideration in future publications.

In conclusion, we have considered the phenomenon of the stimulated LO phonon decay process in light of the intensity of the production of LO phonons by the electrons, subject to a strong electric field or to photoexcitation by the light pulse. Stimulated decay results in a considerable decrease of the LO phonon lifetime, which becomes strongly dependent on the LO phonon wave vector, and in narrowing of the distribution of the high wave vector LA phonons, produced by the decaying LO phonons. Estimates show that the decay of LO phonons, produced in modern devices, is stimulated. This should substantially influence the process of heat removal in such devices.

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APPENDIX: MAGNITUDE OF THE SECOND TERM IN EQ. (16)

We use the asymptotic expressions for the LA phonon distribution function of Eqs. (31), (32) to estimate the value of the term

$$\frac{1}{2}\sum_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'} \gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'} n_{\boldsymbol{k}} n_{\boldsymbol{k}'} = \sum_{\boldsymbol{q}} R_{\boldsymbol{q}} \tag{A1}$$

that we have neglected in our consideration of the stimulated LO phonon decay.

For the reciprocal-space density of this term,

$$R_q = \frac{1}{2}\sum_{\boldsymbol{k},\boldsymbol{k}'} \gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'} n_{\boldsymbol{k}} n_{\boldsymbol{k}'}, \tag{A2}$$

we perform summation in straight analogy to what we did in Eq. (17), and obtain

$$R_q = \frac{1}{\tau_{\mathrm{sp}}}\frac{1}{q}\int_{\Delta_q} dk\, n_k n_{k'}. \tag{A3}$$

Making use of Eqs. (31), (32), for long-wavelength LO phonons we can estimate

$$R_q \lesssim R^* = \frac{2\beta^3}{9\pi\tau_{\mathrm{sp}}}. \tag{A4}$$

We compare this with the partial rate of LO phonon production by the electrons, G, and obtain

$$\frac{R^*}{G} = \frac{4}{9\pi}\frac{q_0^2}{k_0^2}\frac{\tau_a}{\tau_{\mathrm{sp}}}\beta^2. \tag{A5}$$

As is usual for systems with stimulated emission, this ratio increases with increase of effective pumping characterized by β and with the lifetime of the decay products. However, it contains also a small factor $(q_0/k_0)^2 \sim 10^{-3}$, which appears due to a substantial difference in the characteristic wave numbers of LO and LA phonons. As a result, the parameter R^*/G is small even at quite high values of β.

The small partial rate R_q does not automatically provide the total rate of Eq. (A1) to be small. In fact, at high β this overall rate might well be comparable to or even greater than the rate of LA phonon anharmonic decay. If this were the case, the LA phonon population would drastically drop and stop providing the stimulated LO phonon decay. However, this is not the case in real semiconductors, due to finite dispersion of LO phonons, $\omega(q)$. With this dispersion taken into account, the energy conservation law forbids the daughter LA phonons to merge into short-wavelength LO phonons.

The frequencies of the daughter LA phonons lying in the band of $\sim sq_0$ width centered at $\omega_0/2$, the estimate for maximum wave number of the inversely produced LO phonon, q_m, can be found from the obvious relation

$$|\omega(q_m)-\omega_0|=sq_0. \tag{A6}$$

This condition gives $(k_0/q_m)^2\sim 10^2$ for GaAs.

Simple calculations give the following expression for the ratio of the rates of LA phonon removal due to the coalescence and anharmonic decay, ε:

$$\varepsilon\sim\frac{\beta^{3/2}}{6\pi}\frac{\tau_a}{\tau_{sp}}\frac{q_m^2}{k_0^2}. \tag{A7}$$

This ratio remains small even at quite high values of the parameter β, which means that the coalescence process does not practically affect the kinetics of LA phonons.

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