Contribution of effective mass variation to electro-acoustic phonon interaction in semiconductor nanostructures

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The finite-size effects of the electron–acoustic phonon of the deformation–potential interaction in semiconductor heterostructures are studied. Modification of the interaction originates from the phonon induced changes of the interface spacing and electron effective mass. Calculations of the electron mobility for a quantum well (QW) show that for narrow GaAs–based QWs, the contribution of the additional mechanisms to the intrasubband scattering depends on the sign of the deformation potential constant and can exceed that from the usual deformation potential.

\section{1. PROBLEM FORMULATION}

The accepted procedure for calculating the electron–acoustic phonon interaction via the deformation potential (DP) in semiconductor heterostructures is to use the bulk expression for the energy of interaction. In a cubic crystal with a conduction band minimum at the $\Gamma$-point, the interaction Hamiltonian is given by

$$H_{\text{int}}^{\text{DP}} = D \text{ div } u,$$

where $D$ is the deformation potential constant, and $u$ is the acoustic displacement. There exists extensive literature on electron–acoustic phonon scattering in semiconductor heterostructures of different types where bulk expressions, similar to Eq. (1), are used (see, e.g., Ref.[1] and references therein). Also, there is uncertainty in the literature concerning the value of the DP constant, $D$, when the bulk interaction (1) is applied to a low-dimensional electron gas. For GaAs/AlGaAs heterostructures, the $D$ value ranges mainly from 6 eV to 11 eV (when screening is neglected).
It has been shown in [2–4] in systems with interfaces that one has to take into account additional mechanisms of electron–acoustic phonon coupling. A macroscopic deformation potential (MDP) [2] (or ripple mechanism [3]) arises when acoustic waves cause the interface spacing to change. This interaction is described by

$$H^{MDP}_{int} = - (u(r) \cdot \nabla) V(r),$$

(2)

where $V(r)$ is a confinement potential for electrons in the undeformed crystal. The mechanism considered in Ref.[4] originates from the phonon induced change of the electron effective mass, $m^*$, in the direction perpendicular to the interfaces. In contrast to the case of bulk semiconductors where this contribution is small compared with that from the DP for energies near the band edge, for the case of heterostructures the minimum of electron energy is reduced to the finite energy of spatial quantization, and the change of $m^*$ can yield a perturbation comparable with that described by Eq. (1). In a cubic crystal, the tensor for the inverse effective mass, $m^{-1}_{ij}$, is expanded in terms of the strain tensor, $u_{ij}$, as $m^{-1}_{ij} = m^*(1 + \chi u_{ij}) \delta_{ij}$ where $m^*$ is the effective mass in the absence of deformation and $\chi$ is a phenomenological parameter. For the case of a position–dependent mass, using the kinetic energy operator of the form $(-\hbar^2/2) \nabla(m^{-1}(r) \nabla)$, gives

$$H^{m}_{int} = - \frac{\hbar^2}{2} \sum_i \nabla_i \left( \frac{\chi}{m} u_{ii} \nabla_i \right),$$

(3)

to the lowest order in the displacement. For narrow–gap semiconductors, the parameter $\chi$ is related to the coefficient $dE_g/dP$ of the pressure bandgap energy dependence as [4], $\chi \simeq (3K/E_g)(dE_g/dP)$, where $K$ is the modulus of the hydrostatic compression. Using data [5] for GaAs, we obtain $\chi \simeq 17$. The condition $\chi \gg 1$ implies that interaction of Eq. (3) dominates over interaction of Eq. (2).

2. RESULTS AND DISCUSSION

In order to illustrate the roles of additional mechanisms defined by Eqs. (2) and (3), we have calculated the mobility, $\epsilon \tau/m^*$, of a degenerate two–dimensional electron gas for scattering due to the total interaction determined by the sum of interactions from Eqs. (1), (2), and (3). Electrons occupy the lowest subband of a rectangular QW ($0 \leq z \leq d$). For simplicity, we take the potential, $V(z)$, outside the QW to be infinite. Note, that for QWs thinner than 30 Å the approximation of infinitely high barriers, as well as the effective mass approximation itself, become invalid, and more sophisticated consideration is necessary. We neglect also in–plane changes of the effective mass, i.e. only $u_{zz} \neq 0$ in Eq. (3). The relaxation rate, $\tau$, is derived from the general form of Ref.[6]. We obtain the following contributions in addition to the bulk interaction of Eq. (1): (i) the electrons interact with transverse phonons and (ii) for carrier scattering with longitudinal phonons.
there is an interference between the DP interaction and the mechanisms of Eqs. (1) and (2). The latter phenomenon leads to a dependence of mobility on the sign of the constant, $D$. Calculations have been carried out using the material parameters of GaAs, for an electron concentration of $2 \cdot 10^{12}$ cm$^{-2}$. The calculated relative changes in the mobility as functions of QW width for several temperatures are given in Fig. 1.

![Graph](image)

Figure 1. Ratio of the electron mobility for scattering via the modified electron–acoustic phonon interaction to that for the scattering due to only the bulk deformation potential interaction for three different temperatures: (1) 1 K; (2) 5 K; (3) 20 K. (a) for $D = 8$ eV, (b) for $D = -8$ eV.

We see that for narrow QWs the contribution of the additional mechanisms defined by Eqs. (2) and (3) prevails over that from the commonly used bulk DP interaction; moreover, their contribution increases as the temperature decreases, and the effect under consideration depends strictly on the interference between the mechanisms as seen from the comparison of (a) and (b) plots which have all parameters the same except the sign of $D$.

Acknowledgement: This work was supported by the US Army Research Office.

REFERENCES