PERGAMON

Solid-State Electronics 43 (1999) 505–513

SOLID-STATE ELECTRONICS

Waves of switching in thyristor-like structures

Z.S. Gribnikov*, I.M. Gordion, V.V. Mitin

Department of Electrical and Computer Engineering, Wayne State University, Detroit, MI 48202, USA

Received 12 October 1997; received in revised form 27 July 1998

Abstract

Differential equations which describe a propagation of switching waves in thyristor-like structures are derived. These waves propagate if a gate current, J_g, differs from a certain value, $J_{g0}(j)$, corresponding to neutral (translationally invariant) equilibrium state for a given current density j in the ON-region. The derivation is based on the initial equation of theory of semiconductor devices. Our consideration is valid only for bases where widths are much less than the width of a transition layer between the ON- and the OFF-regions. For the first time, an explicit analytical formula for the velocity of the switching wave, v, is obtained: v is shown to be directly proportional to $\delta J_g = J_g - J_{g0}(j)$ and inversely proportional to j for $|\delta J| \ll J_{g0}$. The dependence of the velocity v on parameters of the structure is obtained for low injection levels in both bases. The derived expression for the velocity has been applied for calculation of transient and modulation processes in the finite gate controlled thyristor-like structures.

1. Introduction

We refer to a thyristor-like structure (TLS) as a certain p^+npn^+-structure where two outer *pn*-junctions (either homojunctions or heterojunctions) are highly effective emitters. Inner layers (a *p*-base and an *n*-base) provide high enough base transport factors α_I and α_{II} of injected current carriers so that the condition $\alpha_I + \alpha_{II} > 1$ is satisfied. Here we deal with the ON-state of a thyristor and we do not consider the problem of the *S*-shaped characteristic. It is well known [1–3] that for large enough TLS's, an inhomogeneous state of incomplete gate turn off (IGTO) arises for a certain gate current range.

Since the earliest investigations of the gate turning off in the GTO-thyristors, there exists clear comprehension that a process similar to nonstationary squeezing of the current-conducting channel occurs in the TLS. To our knowledge, any analytical approach of squeezing has never used the initial equations of theory of semiconductor devices but has been usually based on comfortable phenomenological speculations. Therefore, results of such approaches contain non-defined parameters or even non-defined functions. This relates not only to the pioneer paper of Wolley [4] but also to later publications (for example, Refs. [2, 5–9]). A detailed and strict analysis of stationary and nonstationary gate control is assumed as accessible only for numerical two-dimensional (2D) simulations which are described in many publications (for example, Refs. [10–14]). Such an opinion is well-founded for silicon controlled rectifiers with very long ungated bases where numerical 2D calculations are inevitable. Here we present the analytical approach which can be of a definite interest for controlling light-emitting devices: photothyristors, optothyristors, light emitting and lasing thyristors. These devices have a completely different design: they do not usually require excessively long ungated bases [15–18]. This allows us to promote a consistent analytical approach for the gated TLS. Such an approach has been demonstrated in works [19, 20] for stationary problems. Using the consideration of the stationary switching waves in the present paper, we develop the self-consistent set of equations and apply them to a number of nonstationary problems.

* Corresponding author. Tel.: +1-313-577-0764; fax: +1-313-577-1101; e-mail: zinovi@besm6.eng.wayne.edu

0038-1101/99/$ - see front matter
PII: S0038-1101(98)00281-0

We consider the simplest structure shown in Fig. 1 with a one-dimensional distribution of the current density, $j(y)$, in the *pn*-junction plane. The gates are in electric contact with one of the bases, which is called the controlling base (or base I). In the IGTO regime, the region of the TLS which is distant from the gates is in a turned on state (ON-region), regions which are adjacent to the gates are in a blocked state (OFF-regions) and the boundary layer between the ON- and the OFF-regions is called the ON/OFF-junction. First we consider an idealized structure where the size of the structure, a_0, is so large that small shifts of the ON/OFF-junction δy ($|\delta y| \ll a_0$) do not affect the current density j in the ON-region. We showed that there is a unique value of the gate current, J_{g0} (per cm), which allows stationary state of a TLS in the IGTO regime for a definite current density j (per cm^2) in the ON-region. (Since the dimension in the direction perpendicular to the plane of the Fig. 1 is not fixed, we use currents per unit length of that size.) If $J_g = J_{g0}(j)$, the ON/OFF-junction is neutrally stable. A deviation of J_g from the stationary value J_{g0} results (after a transient process) in a stationary wave of switching. Based on certain assumptions, we derived the following analytical expression for the velocity of this wave:

$$v = \frac{J_g^2 - J_{g0}^2(j)}{2J_{g0} \cdot \tau j} \approx \frac{\delta J_g}{\tau \cdot j}. \tag{1}$$

The second (approximate) expression for v in Eq. (1) is valid for $|\delta J_g| \ll J_{g0}$, where $\delta J_g = J_g - J_{g0}(j)$. Here τ, an important parameter of speed of operation for the considered TLS, is determined only by geometry and material parameters of the TLS. The direction of the ON/OFF-junction motion depends on the sign of the deviation δJ_g and the value of the wave velocity, v, would remain constant if the condition $|\delta y| \ll a$ is satisfied.

However, in real devices we control the current going through the ON-region, $J_a = 2aj$, where a is the half-width of this region. Hence, the only position of the ON/OFF-junction and the only value of a correspond to the given J_a and J_g if these currents provide the IGTO regime. Thus, we applied Eq. (1) to analyze quasistationary transient and modulation processes of a finite gate controlled TLS. However, the characteristic time, τ_r, of these processes is shown not to be equal to τ but proportional to the anode current and inversely proportional to the gate current. Since $J_g < J_a$ for our consideration, the characteristic time of the transient processes is always more than the effective time of the TLS.

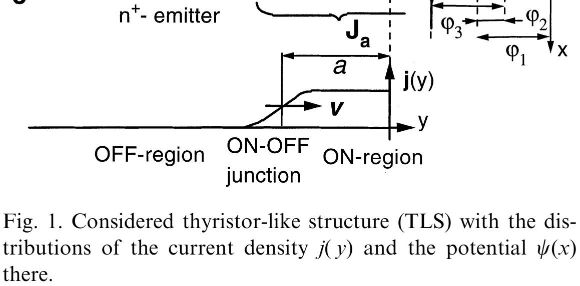

Fig. 1. Considered thyristor-like structure (TLS) with the distributions of the current density $j(y)$ and the potential $\psi(x)$ there.

In this work, we try to reach several goals. First, we derive Eq. (1) and determine its validity range. (Within this range, the effective time τ depends neither on j, nor on δJ_g and is solely defined by the structure.) We calculate τ as a function of base geometry and material parameters for the simplest TLSs. Second, we calculate transient and modulation processes for the small gate current deviations in the finite TLS.

2. Basic equations and assumptions

As a typical TLS, we consider a structure in which an applied voltage U is distributed across three *pn*-junction layers of spatial charge. Two outer junctions (1 and 3 in Fig. 1) are called emitters and the inner junction (2) is called a collector. All these junctions are forward biased in the ON-state, and these biases are determined by the current density j. In the blocked state (the OFF-region at the IGTO-regime), two of the junctions, the collector and the adjacent to base I emitter, are reverse biased due to the gate current. As a result, this base is isolated and it becomes a current-conducting channel for the gate current flowing into the ON/OFF-junction (Fig. 1). The length of the channel is varying with the motion of the ON/OFF-junction. This results in the change of capacity charges and leakage currents of the isolating *pn*-junctions. However, we will not take into account the additional current effects related to elongating or shortening the current-conducting channels. We assume that there is a given gate current, J_g, which is used only for holding and moving of the ON/OFF-junction. We neglect a parasitic transistor current in the system of the forward biased emitter 3, the controlled base (base II) and reverse biased collector 2 (see Refs. [19, 20]). The reverse current of reverse biased emitter 1 (Fig. 1) is neglected as well.

It is assumed that outer layers (emitters) of the TLS, cathode and anode, are heavily doped and they are from a wide band gap material. They inject their majority carriers into the adjacent bases with efficiencies which are close to 1. This means that we can neglect recombination in the emitters. Subjects of our thorough study are two middle bases separated from each other by the collector *pn*-junction. We assume that the speed of operation of the structure is determined by inertial processes just in these bases. We have to solve the continuity equations for electron and hole currents in both bases:

$$-\frac{1}{e}\frac{\partial \mathbf{j}_n}{\partial \mathbf{r}} = -R_n - \frac{\partial n}{\partial t}, \qquad \frac{1}{e}\frac{\partial \mathbf{j}_p}{\partial \mathbf{r}} = -R_p - \frac{\partial p}{\partial t}, \tag{2}$$

where n and p are concentrations of electrons and holes, respectively, $\mathbf{j}_n$ and $\mathbf{j}_p$ are their two-dimensional currents with x- and y-components, and $\mathbf{r} = (x, y)$ is the radius-vector. R_n and R_p are the recombination-generation rates and e is the absolute value of electron charge. We assume that conditions of quasineutrality, $n = p \pm N(x)$, are fulfilled in the bases. They allow us to solve only one of the equations. Base I is assumed to be heavily doped with electrons, so that there is a low injection level. This results in a linear recombination relation in this base: $R_p = p/\tau_{\mathrm{I}}$. Since the doping can depend on x, lifetime τ_{I} can be a function of x.

Low injection level and a linear relation $R_n = n/\tau_{\mathrm{II}}$ is considered in base II as well. Since the widths of the bases are not greater than several diffusion lengths there, we can neglect voltage drops across the base quasineutral regions and take into account only voltage drops across the pn-junctions. We introduce three voltage drops: $\psi_{\mathrm{I}}(y)$, between the anode and base I; $\psi_{\mathrm{II}}(y)$, between the anode and base II and ψ, between the anode and the cathode. The voltages across three pn-junctions indicated in Fig. 1 equal:

$$\varphi_1(y) = \psi_{\mathrm{I}}(y); \qquad \varphi_2(y) = \psi_{\mathrm{I}}(y) - \psi_{\mathrm{II}}(y);$$

$$\varphi_3(y) = \psi - \psi_{\mathrm{II}}(y),$$

where signs of $\varphi_{1,2,3}$ are selected positive for forward biased pn-junctions. The voltages ψ, $\psi_{\mathrm{I,II}}$ and $\varphi_{1,2,3}$ are presented in dimensionless units and corresponding dimensional values are $V_{1,2,3} = \varphi_{1,2,3} \cdot kT/e$, $U = \psi \cdot kT/e$, $U_{\mathrm{I,II}} = \psi_{\mathrm{I,II}} \cdot kT/e$.

As we did before for the stationary problem [19, 20], it is assumed that the variables change in the y-direction much slower than in the x-direction in the region of the ON/OFF-junction. The corresponding inequality could be presented in the form:

$$w_{\mathrm{I,II}}\left|\frac{\mathrm{d}\varphi_{1,2,3}}{\mathrm{d}y}\right| \ll 1, \tag{3}$$

where w_{I} and w_{II} are the widths of bases I and II, respectively. The inequality in Eq. (3) allows us to take into account only the components $\partial(_{jp,n})_x/\partial x$ instead of $\partial \mathbf{j}_{p,n}/\partial \mathbf{r}$ in Eq. (2).

The next assumption makes it possible to derive the analytical formula for the velocity of the switching wave and present several dependences explicitly. We consider a structure where the longitudinal (in-plane) conductivity of the gated base, σ_{I}, is much greater than the in-plain conductivity of base II (σ_{II}):

$$\sigma_{\mathrm{I}} \gg \sigma_{\mathrm{II}}. \tag{4}$$

This is fairly true for asymmetric structures where the gated base is doped much heavier than the ungated base.

Equation of continuity of full currents, $J_{\mathrm{I,II}}$ in the bases are written in the form:

$$\frac{\partial J_{\mathrm{I}}}{\partial y} = j_1 - j_2, \qquad \frac{\partial J_{\mathrm{II}}}{\partial y} = j_2 - j_3, \tag{5}$$

where $j_{1,2,3}$ are current densities through the junctions 1, 2 and 3, respectively. The current densities can be found from the solutions of Eq. (2). We try to obtain these solutions in the form of the stationary wave:

$$\begin{aligned} p(x, y, t) &= p(x, y - vt), \\ n(x, y, t) &= n(x, y - vt), \end{aligned} \tag{6}$$

where the wave velocity v is assumed to be the same for both bases. A new variable, $y - vt$, results in a new version of Eq. (2):

$$\frac{1}{e}\frac{\partial \boldsymbol{j}_p}{\partial \boldsymbol{r}} = -\frac{p}{\tau_{\mathrm{I}}} + v\frac{\partial p}{\partial y}, \qquad -\frac{1}{e}\frac{\partial \boldsymbol{j}_n}{\partial \boldsymbol{r}} = -\frac{n}{\tau_{\mathrm{II}}} + v\frac{\partial n}{\partial y}. \tag{7}$$

3. Stationary solution

For $v = 0$, we start with Eq. (7) written for both bases:

$$\frac{1}{e}\frac{\partial j_{p_x}}{\partial x} = -\frac{p}{\tau_{\mathrm{I}}}, \qquad -\frac{1}{e}\frac{\partial j_{n_x}}{\partial x} = -\frac{n}{\tau_{\mathrm{II}}}. \tag{8}$$

They are solved with boundary conditions:

$$\begin{aligned} p(x = 0) &= p_{0_1} e^{\varphi_1} = p_{0_1} e^{\psi_{\mathrm{I}}}, \\ p(x = w_{\mathrm{I}} - 0) &= p_{0_2} e^{\varphi_2} = p_{0_2} e^{\psi_{\mathrm{I}} - \psi_{\mathrm{II}}}, \end{aligned} \tag{9}$$

$$\begin{aligned} n(x = w_{\mathrm{I}} + 0) &= n_{0_2} e^{\psi_{\mathrm{I}} - \psi_{\mathrm{II}}}, \\ n(x = w_{\mathrm{I}} + w_{\mathrm{II}}) &= n_{0_3} e^{\varphi_3} = n_{0_3} e^{\psi - \psi_{\mathrm{II}}}, \end{aligned} \tag{10}$$

where $p_{0_{1,2}}$ and $n_{0_{2,3}}$ are the concentrations of equilibrium holes in base I and electrons in base II at the pn-junctions. Since Eq. (8) for low injection levels are linear diffusion and drift equations (drift may exist in the built-in electrical fields), the current densities $j_{1,2,3}$ can be written in the form:

$$j_1 = j_{11} e^{\psi_{\mathrm{I}}} - \tilde{\alpha}_{\mathrm{I}} j_{22}{}' e^{\psi_{\mathrm{I}} - \psi_{\mathrm{II}}}, \tag{11}$$

$$j_2 = j_2{}' + j_2{}'',$$
$$j_2{}' = \alpha_I j_{11} e^{\psi_I} - j_{22}{}' e^{\psi_I - \psi_{II}},$$
$$j_2{}'' = -j_{22}{}'' e^{\psi_I - \psi_{II}} + \alpha_{II} j_{33} e^{\psi - \psi_{II}}, \tag{12}$$

$$j_3 = -\tilde{\alpha}_{II} j_{22}{}'' e^{\psi_I - \psi_{II}} + j_{33} e^{\psi - \psi_{II}}. \tag{13}$$

In Eqs. (11)–(13), we neglect 1 in comparison with e^{ψ_I}, $e^{\psi_I - \psi_{II}}$ and $e^{\psi - \psi_{II}}$ because these values are assumed large. Expressions for coefficients $\alpha_{I,II}$, $\tilde{\alpha}_{I,II}$, j_{ik} and $j_{22}{}^{','''}$ can be calculated for a specified base structure (see example in Section 5). Coefficients $\alpha_{I,II}$ and $\tilde{\alpha}_{I,II}$ are usually referred to as forward and reverse base transport factors. As a result of substitution of the current densities from Eqs. (11)–(13) into Eq. (5) and of the equations for the currents through the bases:

$$J_I = -\sigma_I \frac{kT}{e} \frac{d\psi_I}{dy}, \qquad J_{II} = -\sigma_{II} \frac{kT}{e} \frac{d\psi_{II}}{dy}, \tag{14}$$

we obtain:

$$\frac{kT}{e} \sigma_I \frac{d^2\psi_I}{dy^2} = R_I$$
$$= j_{11}(1-\alpha_I) e^{\psi_I} + [j_{22}{}'(1-\tilde{\alpha}_I) + j_{22}{}''] e^{\psi_I - \psi_{II}}$$
$$- \alpha_{II} j_{33} e^{\psi - \psi_{II}}, \tag{15}$$

$$\frac{kT}{e} \sigma_{II} \frac{d^2\psi_{II}}{dy^2} = R_{II}$$
$$= \alpha_I j_{11} e^{\psi_I} - [j_{22}{}' + j_{22}{}''(1-\tilde{\alpha}_{II})] e^{\psi_I - \psi_{II}}$$
$$- j_{33}(1-\alpha_{II}) e^{\psi - \psi_{II}}. \tag{16}$$

The system of two nonlinear equations, Eqs. (15) and (16), can be solved numerically [20]. Here we consider an asymmetric structure where the inequality in Eq. (4) allows us to use the approximate solution of Eq. (16): $R_{II} = 0$ to express $e^{-\psi_{II}}$ through $e^{\psi - \psi_I}$ and obtain from Eq. (15):

$$\frac{kT}{e} \sigma_I \frac{d^2\chi}{dy^2} = j_{11} \frac{\alpha_I + \alpha_{II} - 1}{1 - \alpha_{II}} e^{\psi_I(\infty)} \frac{e^{\chi}(e^{\chi} - 1)}{1 + e^{\chi}/A}, \tag{17}$$

where $\chi = \psi_I - \psi_I(\infty) < 0$,

$$e^{\psi_I(\infty)} = e^{\psi} \frac{j_{33}(\alpha_I + \alpha_{II} - 1)}{j_{22}{}'(1 - \alpha_I \tilde{\alpha}_I) + j_{22}{}''(1 - (1-\alpha_I)\tilde{\alpha}_{II})}, \tag{18}$$

$$A = \left(\frac{1 - \alpha_{II}}{\alpha_I + \alpha_{II} - 1} \right)$$
$$\times \frac{j_{22}{}'(1 - \alpha_I \tilde{\alpha}_I) + j_{22}{}''(1 - (1-\alpha_I)\tilde{\alpha}_{II})}{j_{22} - \tilde{\alpha}_{II} j_{22}{}''}. \tag{19}$$

Here Eq. (18) determines ψ_I in the depth of the ON-region. The value of $\psi_I(\infty)$ corresponds to the unperturbed homogeneous conducting state (ON-region) of the TLS with a certain current density j through the structure. We calculate the first integral of Eq. (17) taking into account this condition (i.e. $\chi(\infty) = 0$):

$$\frac{kT}{e} \sigma_I \left(\frac{d\chi}{dy} \right)^2 = j_{11} \frac{\alpha_I + \alpha_{II} - 1}{1 - \alpha_{II}} e^{\psi_I(\infty)} A$$
$$\left[(1 + A) \ln\left(\frac{1 + A}{e^{\chi} + A} \right) - 1 + e^{\chi} \right]. \tag{20}$$

The conditions on the left of the ON/OFF-junction should be fairly complicated because of the existence of the electric field $-(kT/e)d\psi_I/dy \sim J_g/\sigma_I$ in the OFF-region of base I and a similar electric field in the same region of base II. These fields lead to an increase of the voltage drop, ψ_I–ψ_{II}, in the depth of the OFF-region and to strong field effects (such as a depletion of base current conducting channels, a saturation of the current J_g, etc). Therefore, we assume that the conditions on the left side of the structure are given not too far from the ON/OFF-junction (both stable and moving), so we can neglect the field effects on the conductivities. Assuming that the electrical potential goes to infinity in the depth of the OFF-region (i.e. $\chi(-\infty) = -\infty$), we can derive the expression for J_{g0}^2 from Eq. (20):

$$J_{g0}^2 = \left(\frac{kT}{e} \sigma_I \left. \frac{d\psi_I}{dy} \right|_{y=-\infty} \right)^2 = \frac{2kT}{e} \sigma_I j_{11}$$
$$\times \frac{\alpha_I + \alpha_{II} - 1}{1 - \alpha_{II}} A[(1+A)\ln(1+A^{-1}) - 1] e^{\psi_I(\infty)}. \tag{21}$$

For $J_g = J_{g0}$, there exists an inhomogeneous steady-state solution of Eq. (17), which corresponds to the steady-state IGTO regime: the right hand side of the TLS is in the unperturbed conducting state, the left hand side of the TLS is in the blocking state due to the gate current (see Fig. 1). The detailed consideration of this problem was discussed in Ref. [19].

The explicit dependence $J_{g0}(j)$ is important for the future consideration. Keeping in mind that in the depth of the ON-region $j = j_1 = j_2 = j_3$, we obtain:

$$j = \tilde{j}_{11} e^{\psi_I(\infty)}, \text{ where } \tilde{j}_{11} =$$
$$j_{11} \frac{j_{22}{}'(1 - \alpha_I \tilde{\alpha}_I) + j_{22}{}''(1 - \alpha_{II}\tilde{\alpha}_{II})}{j_{22}{}'(1 - \tilde{\alpha}_I + \tilde{\alpha}_I \alpha_{II}) + j_{22}{}''(1 - \alpha_{II}\tilde{\alpha}_{II})}, \tag{22}$$

$$J_{g0}(j) = \sqrt{j I_1}, \text{ where } I_1 = \frac{2kT}{e} \sigma_I \frac{j_{11}}{\tilde{j}_{11}}$$
$$\times \frac{\alpha_I + \alpha_{II} - 1}{1 - \alpha_{II}} \cdot A[(1 + A)\ln(1 + A^{-1}) - 1]. \tag{23}$$

4. Motion of the ON/OFF-junction

If J_g differs from $J_{g0}(j)$, the ON/OFF-junction moves with a finite velocity v. Here the boundary conditions in Eqs. (9) and (10) for Eq. (7) remain the same. In the frame of the linear approximation on v, we obtain new expressions for the current densities through the pn-junctions:

$$j_1 = j_{10} + v\left(\lambda_{11}\frac{d}{dy}e^{\psi_I} - \lambda_{12}\frac{d}{dy}e^{\psi_I-\psi_{II}}\right), \quad (24)$$

$$j_2{}' = j_{20}{}' + v\left(\lambda_{21}\frac{d}{dy}e^{\psi_I} - \lambda_{22}{}'\frac{d}{dy}e^{\psi_I-\psi_{II}}\right), \quad (25)$$

$$j_2{}'' = j_{20}{}'' + v\left(-\lambda_{22}{}''\frac{d}{dy}e^{\psi_I-\psi_{II}} + \lambda_{23}\frac{d}{dy}e^{\psi-\psi_{II}}\right), \quad (26)$$

$$j_3 = j_{30} + v\left(-\lambda_{32}\frac{d}{dy}e^{\psi_I-\psi_{II}} + \lambda_{33}\frac{d}{dy}e^{\psi-\psi_{II}}\right), \quad (27)$$

where $j_{10,30}$ and $j_{20}{}^{','' }$ are given by the RHS expressions of Eqs. (11)–(13) for $j_{1,3}$ and $j_2{}^{','' }$, respectively. The coefficients λ_{ik}, $\lambda_{22}{}^{','' }$ have the dimensionality of surface charge density and they have to be calculated from Eq. (7) for a specified base structures. An example of the calculation of λ-coefficients is presented in Section 5. The substitution of Eqs. (24)–(27) into Eq. (5) gives us:

$$\frac{kT}{e}\sigma_I\frac{d^2\psi_I}{dy^2} = R_I + vS_I, \quad (28)$$

$$\frac{kT}{e}\sigma_{II}\frac{d^2\psi_{II}}{dy^2} = R_{II} + vS_{II}, \quad (29)$$

where

$$S_I = (\lambda_{11} - \lambda_{21})\frac{d}{dy}e^{\psi_I} - (\lambda_{12} - \lambda_{22})\frac{d}{dy}e^{\psi_I-\psi_{II}} - \lambda_{23}\frac{d}{dy}e^{\psi-\psi_{II}},$$

$$S_{II} = \lambda_{21}\frac{d}{dy}e^{\psi_I} - (\lambda_{22} - \lambda_{32})\frac{d}{dy}e^{\psi_I-\psi_{II}} + (\lambda_{23} - \lambda_{33})\frac{d}{dy}e^{\psi-\psi_{II}},\ \lambda_{22} = \lambda_{22}{}' + \lambda_{22}{}''.$$

We restrict our consideration by the inequality in Eq. (4) which allows us further analytical approach. As in the stationary problem, the RHS of Eq. (29) assumed to be 0. Substituting $e^{-\psi_{II}}$ from the solution $R_{II} + vS_{II} = 0$ into Eq. (28) and taking into account just linear on v terms, we obtain:

$$\frac{kT}{e}\sigma_I\frac{d^2\chi}{dy^2} = j_{11}\frac{\alpha_I + \alpha_{II} - 1}{1 - \alpha_{II}} \times e^{\psi_I(\infty)}\frac{e^{\chi}(e^{\chi} - 1)}{1 + e^{\chi}/A} + v\cdot\left(S_I + \frac{\gamma e^{\chi}/A - \delta}{1 + e^{\chi}/A}S_{II}\right), \quad (30)$$

where $\gamma = (j_{22} - j_{22}{}'\tilde{\alpha}_I)/(j_{22} - j_{22}{}''\tilde{\alpha}_{II})$, $\delta = \alpha_{II}/(1 - \alpha_{II})$.

We have modified our problem to the differential nonlinear equation of the second order. Multiplying Eq. (30) by $d\chi/dy$, integrating it on y from $-\infty$ to $+\infty$ and taking into account Eq. (21) for the steady-state problem, we calculate v:

$$v = [J_g^2 - J_{g0}^2(j)]\cdot\left[2\sigma_I\frac{kT}{e}\int_{-\infty}^{0} d\chi\left(S_I + \frac{\gamma e^{\chi}/A - \delta}{1 + e^{\chi}/A}S_{II}\right)\right]^{-1}, \quad (31)$$

Since we believe that slowly propagating stationary wave does not change its form, we can use the steady-state solution to calculate the integrand of Eq. (31) which can be rewritten in the form:

$$S_I + \frac{\gamma e^{\chi}/A - \delta}{1 + e^{\chi}/A}S_{II} = e^{\psi_I(\infty)}\frac{d\chi}{dy}F(e^{\chi}), \quad (32)$$

where

$$\begin{aligned} F(z) = {} & z\{\lambda_{11} - \lambda_{21} + (\lambda_{22} - \lambda_{12})(s + s^2)BA^{-1}z \\ & - \lambda_{23}Cs^2 + (\gamma A^{-1}z - \delta)\cdot s[\lambda_{21} - (\lambda_{22} - \lambda_{32}) \\ & (s + s^2)BA^{-1}z - (\lambda_{33} - \lambda_{23})Cs^2]\}, \\ & s = s(z) = (A^{-1}z + 1)^{-1}, \\ & B = \alpha_I j_{11}/(j_{22} - \tilde{\alpha}_{II}j_{22}{}''), \\ & C = \alpha_I j_{11}/(j_{33}(1 - \alpha_{II})). \end{aligned} \quad (33)$$

Since $(d\chi/dy)^2$ is proportional to $e^{\psi_I(\infty)}$ in accordance with Eq. (20) and $e^{\psi_I(\infty)}$ is proportional to j (see Eq. (22)), we can obtain Eq. (1) from Eq. (31). The effective time τ in Eq. (1) is given by the expression:

$$\tau = \frac{1}{\tilde{j}_{11}}\int_0^1 dz\frac{F(z)}{z} \times \left(\frac{\ln[(1 + A)/(z + A)] - (1 - z)/(1 + A)}{\ln(1 + A^{-1}) - (1 + A)^{-1}}\right)^{1/2}. \quad (34)$$

5. Analysis of the effective time

Parameters A, B, C, γ and δ in Eq. (33) are the combinations of the parameters for the stationary problem and parameters λ_{ik} are introduced for the nonstationary problem. We calculate the parameters for the structure with homogeneously doped bases ($p_{0_1} = p_{0_2} = p_0$ and $n_{0_2} = n_{0_3} = n_0$), when only the diffusion transport of minority carriers occurs at low injection levels:

$$j_{11} = j_{22}' = ep_0 D_{\mathrm{I}}\beta_{\mathrm{I}}/\tanh z_{\mathrm{I}}, \quad j_{33} = j_{22}'' = en_0 D_{\mathrm{II}}\beta_{\mathrm{II}}/\tanh z_{\mathrm{II}}, \tag{35}$$

$$\alpha_{\mathrm{I}} = \tilde{\alpha}_{\mathrm{I}} = (\cosh z_{\mathrm{I}})^{-1}, \quad \alpha_{\mathrm{II}} = \tilde{\alpha}_{\mathrm{II}} = (\cosh z_{\mathrm{II}})^{-1}, \tag{36}$$

$$\lambda_{11} = \lambda_{22}' = \frac{ep_0}{2\beta_{\mathrm{I}}}\cdot\frac{\cosh z_{\mathrm{I}}\sinh z_{\mathrm{I}} - z_{\mathrm{I}}}{\sinh^2 z_{\mathrm{I}}}, \quad \lambda_{33} = \lambda_{22}'' = \frac{en_0}{2\beta_{\mathrm{II}}}\cdot\frac{\cosh z_{\mathrm{II}}\sinh z_{\mathrm{II}} - z_{\mathrm{II}}}{\sinh^2 z_{\mathrm{II}}}, \tag{37}$$

$$\lambda_{12} = \lambda_{21} = -\frac{ep_0}{2\beta_{\mathrm{I}}}\cdot\frac{z_{\mathrm{I}}\cosh z_{\mathrm{I}} - \sinh z_{\mathrm{I}}}{\sinh^2 z_{\mathrm{I}}}, \quad \lambda_{23} = \lambda_{32} = -\frac{en_0}{2\beta_{\mathrm{II}}}\cdot\frac{z_{\mathrm{II}}\cosh z_{\mathrm{II}} - \sinh z_{\mathrm{II}}}{\sinh^2 z_{\mathrm{II}}}. \tag{38}$$

Here $z_{\mathrm{I,II}} = \beta_{\mathrm{I,II}} \cdot w_{\mathrm{I,II}}$ and $\beta_{\mathrm{I,II}} = (D_{\mathrm{I,II}} \cdot \tau_{\mathrm{I,II}})^{-1/2}$ are inverse diffusion lengths of minority carriers in bases. $D_{\mathrm{I,II}}$ and $\tau_{\mathrm{I,II}}$ are the diffusion coefficients and lifetimes of minority carriers in bases, respectively. The detailed derivations of Eqs. (35) and (36) can be found in Ref. [19].

An analytical expression for τ in the structure with uniformly doped bases can be found in Appendix A. The effective time τ depends on five parameters; four of them are dimensionless: z_{I}, z_{II}, $\tau_{\mathrm{II}}/\tau_{\mathrm{I}}$, $(p_0 D_{\mathrm{I}}\beta_{\mathrm{I}})/(n_0 D_{\mathrm{II}}\beta_{\mathrm{II}})$ and the fifth parameter is lifetime τ_{I}. It is very difficult to clarify such a complicated dependence on all the parameters simultaneously. Therefore, we start from the symmetric structure:

$$\beta_{\mathrm{I}} w_{\mathrm{I}} = \beta_{\mathrm{II}} w_{\mathrm{II}} = z, \quad \tau_{\mathrm{I}} = \tau_{\mathrm{II}} = \tau_0, \quad \frac{p_0 D_{\mathrm{I}}\beta_{\mathrm{I}}}{n_0 D_{\mathrm{II}}\beta_{\mathrm{II}}} = 1, \tag{39}$$

where we obtain from Eq. (A.3):

$$\tau/\tau_0 = f(z) = \frac{(2\cosh z - 1)(\sinh z\cosh z - z)}{4\sinh^3 z}\left(K_{1,0} - \frac{K_{1,2}}{\cosh z - 1}\right) + \frac{2z\cosh z}{4\sinh z(\cosh z - 1)}\left(K_{1,1} + \frac{K_{1,3}}{2(\cosh z - 1)}\right). \tag{40}$$

Here the functions $K_{1,m}$ described in Appendix A depend on z as well. A graph of $f(z)$ is presented in Fig. 2. The argument z varies from 0 to critical value $z_{\mathrm{cr}} = \ln(2+\sqrt{3}) \simeq 1.3$, which corresponds to the condition $\alpha_{\mathrm{I}} + \alpha_{\mathrm{II}} = 1$. The smallest possible value of τ in such a structure is equal to $\tau_{\min} = \tau(z_{\mathrm{cr}}) \simeq 0.57\tau_0$, τ rises rapidly with shortening of the bases (decreasing of z) and diverges as z^{-2} at $z \to 0$. It is worth to stress that the consideration of this symmetric structure is of only academic interest because the base conductivities should differ greatly (see Eq. (4)). But we have to note that the equations in Eq. (39) do not contradict formally with Eq. (4) and their simultaneous fulfillment is not excluded.

The second example consists of an extremely asymmetric structure where base II is doped much lighter than base I:

$$\frac{p_0 D_{\mathrm{I}}\beta_{\mathrm{I}}}{n_0 D_{\mathrm{II}}\beta_{\mathrm{II}}} \longrightarrow 0, \qquad \frac{p_0\beta_{\mathrm{II}}}{n_0\beta_{\mathrm{I}}} \longrightarrow 0. \tag{41}$$

Here we can write τ as a sum of two terms:

$$\tau = \tau_{\mathrm{I}} f_{\mathrm{I}} + \tau_{\mathrm{II}} f_{\mathrm{II}}, \tag{42}$$

where

$$f_{\mathrm{I}} = \frac{(\cosh z_{\mathrm{I}} - 1)(\sinh z_{\mathrm{I}} + z_{\mathrm{I}})}{\sinh 2z_{\mathrm{I}}} K_{1,0} - (z_{\mathrm{I}}\cosh z_{\mathrm{I}} - \sinh z_{\mathrm{I}})\frac{\cosh z_{\mathrm{II}} K_{1,0} - (\cosh_{\mathrm{II}} + 1)K_{1,1}}{\sinh 2z_{\mathrm{I}}(\cosh z_{\mathrm{II}} - 1)},$$

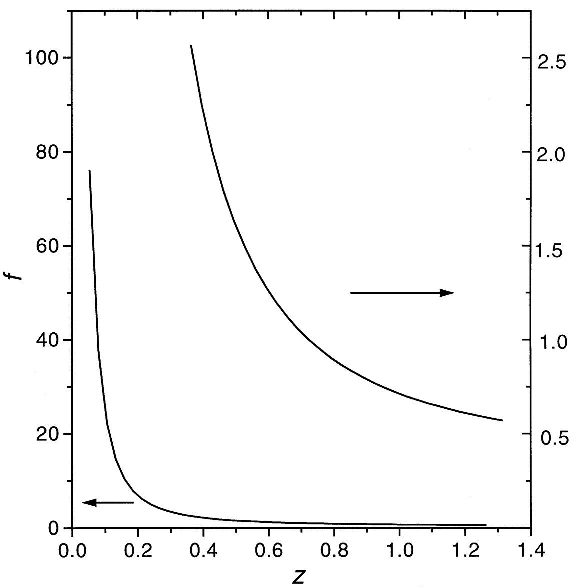

Fig. 2. The dependence of the effective time for the symmetric TLS, τ/τ_0, on the dimensionless width of the bases z.

$$f_{\mathrm{II}} = \frac{(\cosh z_{\mathrm{II}} + 1)[K_{1,1}(\sinh z_{\mathrm{II}} + z_{\mathrm{II}}) - K_{1,0}z_{\mathrm{II}} - K_{1,2}(\sinh z_{\mathrm{II}} - z_{\mathrm{II}})]}{2\cosh z_{\mathrm{I}} \cdot \sinh z_{\mathrm{II}}(\cosh z_{\mathrm{II}} - 1)}, \tag{43}$$

and the argument A of the functions $K_{1,m}$ depends on z_{I} and z_{II} as well. Graphs of f_{I} and f_{II} versus z_{II} for several values of z_{I} and versus z_{I} for several values of z_{II} are presented in Fig. 3(a) and (b). It is worth noticing the features of the obtained results. Shortening of base I leads the function f_{I} to 0 but relatively slightly changes the function f_{II}. On the contrary, a shortening of base II leads to the increase of both f_{I} and f_{II}. They diverge as z_{II}^{-2} at $z_{\mathrm{II}}\to 0$. Therefore, the speed of operation of the TLS is determined mainly by parameters of the base II. Thus, to avoid large inertia we should not shorten base II excessively.

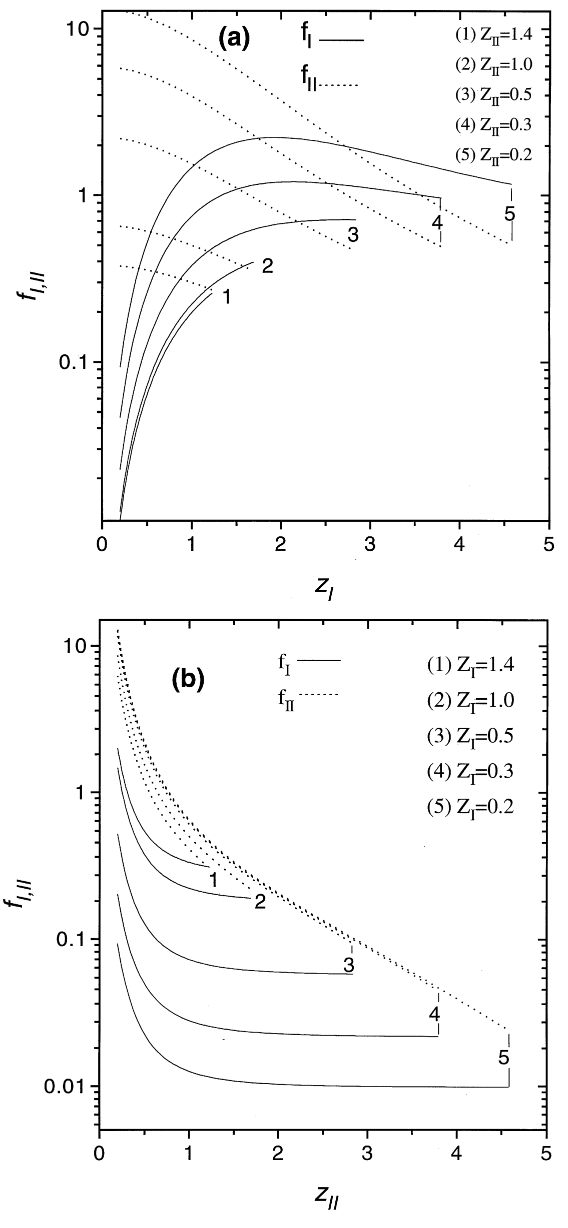

Fig. 3. The contributions of the lifetimes in the bases to the effective time ($\tau = f_{\mathrm{I}}\tau_{\mathrm{I}} + f_{\mathrm{II}}\tau_{\mathrm{II}}$) versus (a) z_{II} for given values of z_{I}; (b) z_{I} for given values of z_{II}.

6. Examples of application in finite structures

In this section, we use Eq. (1) in order to consider the motion of the ON/OFF-junction in finite structures where the half-width of the ON-region, $a(t)$, is time-dependent. The anode current, J_{a}, is constant; thus a current density, $j(t) = J_{\mathrm{a}}/2a(t)$, varies in the ON-region. Eq. (1) can be rewritten (for $|\delta J_{\mathrm{g}}| \ll J_{\mathrm{g0}}$) in the form:

$$-\frac{\mathrm{d}a}{\mathrm{d}t} = 2a\frac{J_{\mathrm{g}}(t) - J_{\mathrm{g0}}(J_{\mathrm{a}}/2a(t))}{\tau J_{\mathrm{a}}}. \tag{44}$$

Taking into account Eq. (23), we can write a general solution of Eq. (44) with the initial condition $a = a(0)$:

$$\sqrt{a(t)} = \sqrt{a(0)}\exp\left(-\frac{1}{J_{\mathrm{a}}}\int_0^{\vartheta} J_{\mathrm{g}}(\vartheta')\,\mathrm{d}\vartheta'\right) + \sqrt{\frac{I_1}{2J_{\mathrm{a}}}} \times \int_0^{\vartheta}\mathrm{d}\vartheta' \exp\left(-\frac{1}{J_{\mathrm{a}}}\int_{\vartheta'}^{\vartheta} J_{\mathrm{g}}(\vartheta'')\,\mathrm{d}\vartheta''\right), \tag{45}$$

where $\vartheta = t/\tau$. Below we consider two examples.

Example 1: if a gate current, J_{g}, is constant, we obtain:

$$\sqrt{a} = \sqrt{a(\infty)} + (\sqrt{a(0)} - \sqrt{a(\infty)})\exp\left(-\frac{J_{\mathrm{g}}}{J_{\mathrm{a}}}\frac{t}{\tau}\right), \tag{46}$$

where $a(\infty) = I_1 J_{\mathrm{a}}/(2J_{\mathrm{g}}^2)$. Since we consider that $|\delta J_{\mathrm{g}}| \ll J_{\mathrm{g0}}$, we have to assume that deflection of $a(0)$ from $a(\infty)$ is not too large. The characteristic time of relaxation which describes the transient process is not equal to the effective time τ introduced in Eq. (1):

$$\tau_{\mathrm{r}} = \tau \cdot J_{\mathrm{a}}/J_{\mathrm{g}}. \tag{47}$$

In contrast to τ, which depends only on structure parameters, τ_{r} depends on J_{a} and J_{g} (i.e. it depends on a regime of operation, even for assumed low injection levels). Since we can consider only relatively small deflections from the stationary regime where $J_{\mathrm{g}} < J_{\mathrm{a}}$, we can obtain only $\tau_{\mathrm{r}} > \tau$.

Example 2: a gate current is a sum of dc and small ac signals: $J_{\mathrm{g}}(t) = J_{\mathrm{g}} + i_{\mathrm{g}}\cos\omega t$. We select $a(0) = I_1 J_{\mathrm{a}}/(2J_{\mathrm{g}}^2)$ and obtain the stationary solution for $i_{\mathrm{g}} = 0$. For

$i_g \ll \omega\tau J_a$, a solution can be written in the form:

$$\sqrt{a} = \sqrt{a(0)}\left(1 - \frac{i_g}{J_a}\left[\frac{\sin(\omega t + \xi)}{\tau\sqrt{\omega^2 + \omega_c^2}} - \frac{\omega_c \exp(-\omega_c t)}{\tau(\omega^2 + \omega_c^2)}\right]\right), \tag{48}$$

where

$$\omega_c = \frac{1}{\tau}\sqrt{\frac{I_1}{2a(0)J_a}} = \tau_r^{-1}, \qquad \tan(\xi) = \frac{\omega_c}{\omega}.$$

Eq. (48) is valid for the following frequency range: $i_g/J_a \ll \omega\tau \ll J_g/J_a$.

Thus the ON/OFF-junction oscillates harmonically after a transient process. At $\omega \ll \omega_c$, Eq. (48) leads to: $\sqrt{a} \simeq \sqrt{a(0)}(1 - i_g/J_g \cos \omega t)$, which shows that the oscillations of the ON/OFF-junction are quasistationary. In the high-frequency limit ($\omega \gg \omega_c$) we obtain $\sqrt{a} \simeq \sqrt{a(0)}(1 - i_g/J_a\omega\tau \sin \omega t)$. Here the ON/OFF-junction oscillation phase is shifted by $\pi/2$ in comparison with the phase of the gate current and the amplitude of oscillations is inversely proportional to ω.

7. Discussion and conclusion

We have considered a behaviour of the TLS with a stationary gate current, J_g, different from the current, J_{g0}, which provides a stationary size of the ON-region. We have modified this problem to the standard problem of propagation of the switching wave in a homogeneous medium.

It is worth noting some restrictions which are important for our approach. First, we assume that the velocity of the ON/OFF-junction motion is comparatively small and does not perturb the stationary distribution of the current density noticeably. The assumption allows us to modify Eq. (7) to the system of Eqs. (28) and (29) instead of a system of integro-differential equations which should be considered generally. Second, we assume that only one of two bases has substantial in-plane conductivity, so we neglect the in-plane conductivity of base II. The restriction allows us to reduce the system of two differential equations, Eqs. (28) and (29), to a single equation, Eq. (30), and to exploit the theory of localized waves [21] for its solution.

The obtained results for asymmetric structures show that the ungated base contributes to the characteristic time τ greater than the gated base. This statement contradicts to a prevalent opinion that base I determines the speed of operation for most cases. Below are several tips which can be used for increase of the speed of operation for a TLS. We can obtain a desired value of the effective time, τ, by varying the parameters of the bases: τ is small for a small length of the gated base and for a moderately long ungated base ($z_{II} \geq 0.4$); τ grows rapidly with shortening of base II. Besides, to decrease τ one can pick up the material with small τ_{II} or dope the ungated base by effective recombination centers. But the characteristic time for the transient processes, τ_r, in the gate controlled TLS is shown to be not equal to τ. This time depends on the regime of operation of the TLS: τ_r is proportional to the ratio of the anode current to the gate current. Hence, for the discussed range of the gate current deflection, the characteristic time of the transient process in gate controlled TLS is always greater than the effective time, τ.

We have to notice that the assumptions mentioned above deprive us of the possibility of considering the fastest processes that could take place for a large amplitude of the gate current. Such gate currents perturb not only the potential form in the ON/OFF-junction but also the homogeneous distribution of current density in the ON-region. These processes are interesting for high-speed dual modulation of light emission of light-emitting and lasing TLSs. However, we would like to point out a number of advantages of the presented calculations. For the first time, an explicit analytical formula, which is derived from the initial equations of semiconductor theory and contents no fitting parameters, is obtained for the velocity of the switching wave. This formula allows us to separate the operation regime dependence from the dependence on the structure parameters. In addition, using Eq. (45), we can calculate transient processes for an arbitrary time-dependent gate current which can depend on the peculiarities and the purpose of the circuit. We do not have to consider just a particular form of the gate current signal (as a step signal in Ref. [4] or a ramp in Ref. [8]). Although, our approach is restricted by the gate current deviations which are not too large or not too fast.

We believe, that the presented approach can be successfully applied also for modulation doped (layered) bases and bases with inner heterojunctions or quantum wells which can be used for real light-emitting devices.

Acknowledgements

These work was supported by the grant from the National Science Foundation. The authors would like to thank O. Melnikova for helpful discussions.

Appendix A

To simplify the right side of Eq. (34), we introduce functions:

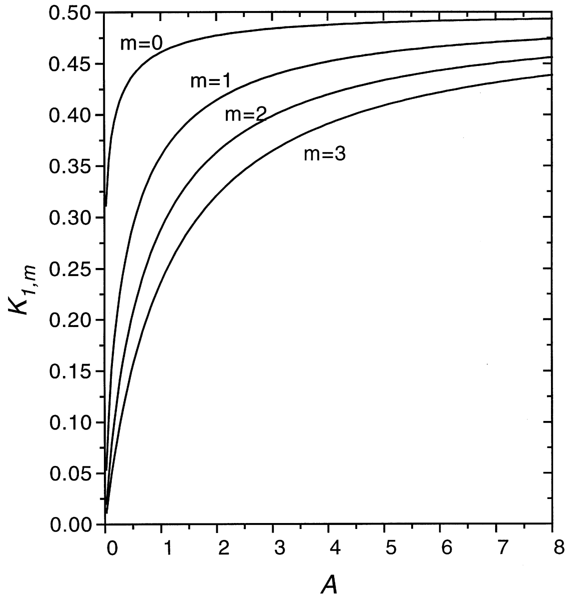

Fig. 4. The functions $K_{1,m}$, $m = 0, 1, 2, 3$ versus A.

$$K_{n,m}(A) = \int_0^1 \frac{z^{n-1}\,\mathrm{d}z}{(A^{-1}z+1)^m} \left(\frac{\ln[(1+A)/(z+A) - (1-z)/(1+A)}{\ln(1+A^{-1}) - (1+A)^{-1}}\right)^{1/2}, \tag{A.1}$$

which are linked to each other by obvious recurrent formulas:

$$K_{n,m}(A) = AK_{n-1,m-1}(A) - AK_{n-1,m}(A). \tag{A.2}$$

These functions allow us to write τ in the form:

$$\tau = \frac{1}{\bar{j}_{11}}\{(\lambda_{11}-\lambda_{21})K_{1,0} + \lambda_{21}[\gamma K_{1,0} - (\delta+\gamma)K_{1,1}] + (\lambda_{22}-\lambda_{12})B[K_{1,0}-K_{1,2}] + (\lambda_{22}-\lambda_{32})B[(\delta+\gamma)(K_{1,1}-K_{1,3}) - \gamma(K_{1,0}-K_{1,2})] + (\lambda_{33}-\lambda_{23})C[(\delta+\gamma)K_{1,3} - \gamma K_{1,2}] - \lambda_{23}CK_{1,2}]\}. \tag{A.3}$$

Fig. 4 shows the dependences of $K_{1,m}$ versus A for $m = 0, 1, 2, 3$. For $A \gg 1$ (or $\alpha_{\mathrm{I}}+\alpha_{\mathrm{II}}-1 \ll 1$) functions $K_{n,m}(A)$ lose their dependence on A and m:

$$K_{n,m}(A) \simeq \frac{1}{n(n+1)} - \frac{m}{A(n+1)(n+2)}.$$

All of these four functions tend to 0 if $A \to 0$ but in different ways:

$$K_{1,0}(A) \simeq \frac{1}{2}\sqrt{\frac{\pi}{(-\ln A)}}, \qquad K_{1,1}(A) \simeq \frac{2}{3}A(-\ln A),$$

$$K_{1,2}(A) \simeq A, \qquad K_{1,3}(A) \simeq \frac{1}{2}A.$$

References

[1] Gribnikov Z, Mitin V, Rothwarf A. Proc 1993 ISDRS 1993;2:603.
[2] Levinstein M. Sov Phys Tech Phys 1981;26:720.
[3] Grekhov IV, Liniichuk IA. Gate turning off thyristors. Leningrad: Energoizdat, 1982 (in Russian).
[4] Wolley E. IEEE Trans Electron Devices 1966;13:590.
[5] D'yakonov M, Levinstein M. Sov Phys Semicond 1978;12:426.
[6] D'yakonov M, Levinstein M. Sov Phys Semicond 1978;12:992.
[7] D'yakonov M, Levinstein M. Sov Phys Semicond 1980;14:283.
[8] Dutta R, Rothwarf A. IEEE Trans Electron Devices 1992;39:1752.
[9] Gribnikov Z, Rothwarf A. Solid-State Electron 1994;37:135.
[10] Nakagava A, Navon DN. IEEE Trans Electron Devices 1984;31:1156.
[11] Nakagava A. Solid-State Electron 1985;28:677.
[12] Bleichner H, Bakowski M, Rosling M, Norlander E, Vobecky J, Lundqvist M, Berg S. Solid-State Electron 1992;35:1683.
[13] Vobecky J, Bleichner H, Rosling M, Norlander E. IEEE Trans Electron Devices 1993;40:2352.
[14] Korobov V, Mitin V, Gribnikov Z. Proc SPIE 1995;2622 (Part 1):374.
[15] Taylor GW, Evaldsson PA, Kiely PA, Wang T, Claisse PR, Daryanani SL, Docter DP, Sargood SK, Cooke PW. IEEE J Quantum Electron 1993;29:785.
[16] Kazahara K. IEEE J Quantum Electron 1993;29:757.
[17] Heremans P, Kuijk M, Vounckx R, Borghs G. J Phys III France 1994;4:2391.
[18] Heremans P, Kuijk M, Vounckx R, Borghs G. Appl Phys Lett 1994;65:19.
[19] Vagidov N, Gribnikov Z, Korshak A, Mitin V. Semiconductors 1995;29:1021.
[20] Gribnikov Z, Vagidov N, Korshak A, Mitin V. Solid-State Electron 1996;39:915.
[21] Gurevich A, Mints R. Sov Phys Usp 1984;27:19.