Relaxation rates of electrons in a quantum well embedded in a finite-size semiconductor slab

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Abstract. We calculated the momentum and energy relaxation rates of 2D electrons interacting with acoustic phonons in a quantum well situated close to the surface of a semiconductor slab in which the quantum well is embedded. These rates depend on the mechanical conditions at the surface of the slab and deviate substantially from the values corresponding to a quantum well situated in the bulk of an infinite crystal. At low enough temperatures the quantization of the acoustic modes becomes important and has an effect on the relaxation rates.

Recently, it has been established that if a quantum well (QW) is situated close to the surface of a slab in which the QW is embedded, the interaction between 2D electrons in the QW and acoustic phonons takes on new features [1, 2]. These peculiarities arise as a result of the modification of the structure of the acoustic modes caused by the surface and are as follows: (i) interference between the incident and reflected acoustic waves, which gives rise to the formation of the nodes and antinodes of the potential of the electron–phonon interaction; (ii) appearance of a Rayleigh wave; (iii) reflection-induced mutual conversion of LA and TA phonons.

In [2] it has been shown that the effect of the proximity of the surface to the QW depends substantially on the mechanical conditions at free and rigid surfaces, the temperature, the 2D electron concentration and the distance between the QW and the surface. The influence of the vicinity of the surface on the magnitude of the electron relaxation rates is the strongest at low temperatures. At low enough temperature, the finite size of the slab in which the QW is embedded leads to new peculiarities in the properties of the phonons interacting with electrons. In fact, these peculiarities are due to the quantization of the acoustic phonons in the slab. As a result, at low temperatures the transport characteristics of the electrons for the cases of a slab of finite width and a semi-infinite sample are different.

To account for the finite size of the slab, we have calculated the acoustic modes confined in the slab for the cases of free and rigid boundaries. The elastic-continuum approach has been used. We have obtained the momentum, ν_p, and energy, ν_e, relaxation rates for electrons interacting with confined acoustic phonons via the deformation potential mechanism. The electron distribution function has been taken to be a displaced Fermi distribution and only the lowest QW level has been assumed to be populated. The former assumption is valid under conditions of strong electron–electron scattering, which is realized for the high electron concentrations.

The most important peculiarities of the phonon spectrum are the following. In the case of a slab with a free surface, the lowest branch of the phonon spectrum starts at the point $\omega = 0$, $q = 0$, where ω and q are the phonon circular frequency and wavevector. At high q, this branch corresponds to a Rayleigh wave, whose amplitude decays away from the surface. In contrast, for the case of a slab with a rigid surface, the lowest branch starts at $\omega = \omega_m$, $q = 0$. This corresponds to the absence of a Rayleigh wave in a slab with a rigid surface. As an order of magnitude estimate, $\omega_m \approx \pi s_t/b$, where s_t is the transverse sound velocity and b is the thickness of the slab. Another important peculiarity of the phonon mode structure is the surface-induced appearance of the nodes and antinodes of the lattice displacements. This feature arises because of the interference between the incident and reflected waves and leads to the formation of nodes and antinodes in the potential of the electron–phonon interaction. Since electrons usually interact with a great number of phonon modes, whose nodes and antinodes do not coincide, the resulting strength of the electron–phonon interaction does not depend on the QW position far from the surface. However, close to the surface *all* phonon modes behave similarly, because all modes obey the same boundary conditions at the surface. For the deformation potential mechanism of interaction one has nodes of the

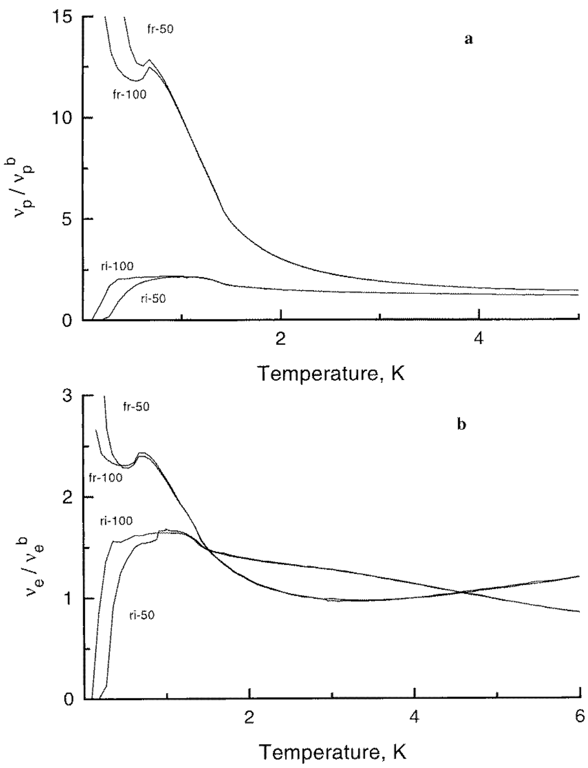

Figure 1. Temperature dependence of the relaxation rates of (*a*) the momentum and (*b*) the energy for a 3 nm QW situated at the surface of a slab. The curves 'fr-100' and 'fr-50' ('ri-100' and 'ri-50') are for the 100 nm and 50 nm slab with a free (rigid) surface respectively.

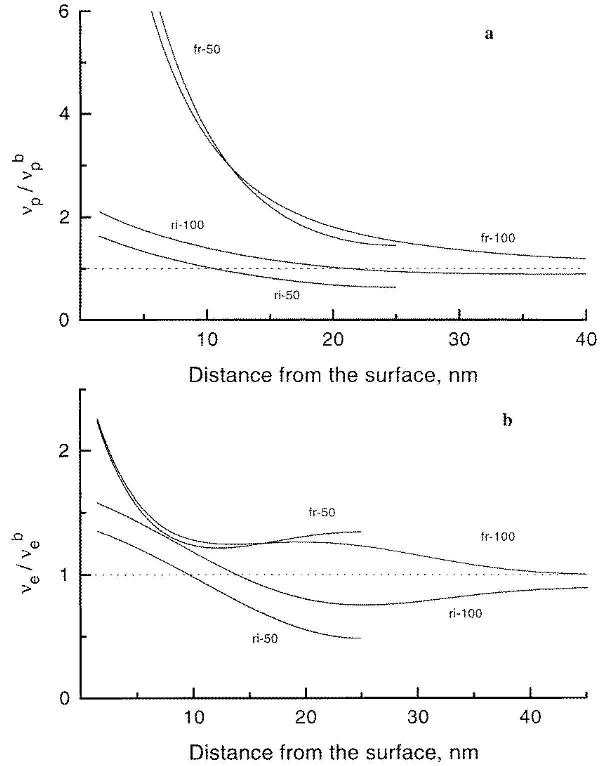

Figure 2. Dependence of the relaxation rates of (*a*) the momentum and (*b*) the energy on the distance between the QW and the surface for a temperature $T = 0.5$ K. The notations used are the same as in figure 1.

electron–phonon interaction potential at the surface for the case of a free surface and antinodes for the case of a rigid surface.

Clearly, many factors influence the electron–phonon interaction near the surface. Their contribution depends substantially on the temperature, which determines which phonons contribute most to the interaction. Two characteristic temperatures can be introduced [3]: $T_1 = 2\hbar s_t/a$ and $T_0 = 2p_F s_t$. Here a is the thickness of the QW and p_F is the Fermi momentum of the electrons. For $T \ll T_0$ the Bloch–Grüneisen regime of scattering occurs; this regime is characterized by the small-angle scattering. For $T \gg T_1$, electrons interact primarily with phonons propagating perpendicular to the QW and have a component of the wavevector perpendicular to the QW that scales in magnitude as $1/a$. Note that the intermediate range $T_0 \ll T \ll T_1$ can be realized for low electron concentrations and for narrow QWs.

In [2] the energy and momentum relaxation rates have been calculated and analysed in these temperature ranges for the case of a QW situated near the surface of a semi-infinite sample. Here, we present the results for the case of a QW embedded in a finite-size slab. In this case a new characteristic temperature should be introduced: $T_s = \pi\hbar s_t/b$. For $T \sim T_s$, the quantization of the phonons influences the values of the electron relaxation rates.

In figure 1 the dependences of the momentum and energy relaxation rates on temperature for a 3 nm QW are shown for 100 nm and 50 nm slabs. Relaxation rates are normalized to the corresponding values, ν_e^b, ν_p^b, for a QW in a bulk crystal . The electron concentration is 3×10^{15} m^{-2} and we use GaAs material parameters. Curves for the 100 nm and 50 nm slabs with free (rigid) surfaces are labelled as 'fr-100' and 'fr-50' ('ri-100' and 'ri-50') respectively. For such a system, $T_0 = 7$ K, $T_1 = 17$ K and $T_s = 1.2$ K for the 100 nm slab and 2.4 K for the 50 nm slab. Note that figure 1 represents results for the case $T < T_0$. As is illustrated, for $T > T_s$ the results do not depend on the slab width. In this case, the temperature dependences obtained are very similar to those obtained for the case of a QW near the surface of the semi-infinite sample [2]. Relaxation rates are enhanced for the slabs with both kinds of mechanical conditions at the surface. For the case of a rigid surface, this occurs as a result of the formation of antinodes in the electron–phonon interaction potential near the surface. For the case of a free surface, one has nodes of the interaction potential near the surface, but at $T < T_0$ scattering due to the Rayleigh wave becomes very important and the resulting relaxation rates are enhanced with respect to the rates for a QW in a bulk crystal.

For $T \sim T_s$, electrons are able to interact with only a few phonon modes. For $T \ll T_s$ and for a slab with a rigid surface, the energy corresponding to the minimum frequency of the acoustic modes, ω_m, is higher than the temperature and the relaxation rates decrease. In contrast, for the case of a semi-infinite sample with a rigid surface, one obtains an increase of the relaxation rate down to very low temperatures. For $T \ll T_s$ and a slab with a free surface, we obtain an increase in the relaxation rates at low temperatures since there is no minimum frequency for the phonon spectrum.

In figure 2 the dependences of the relaxation rates on the distance between the QW and the surface of the slab are shown for $T = 0.5$ K. For such a low temperature, the principal contribution to the scattering comes from a scattering with the lowest phonon mode and the distance dependence qualitatively traces the coordinate dependence of the potential of the electron–phonon interaction of these lowest modes. Note that for low enough temperatures the relaxation rates differ from the bulk values even if the QW is situated far from the surface. We see that, for $T = 0.5$ K and for $b = 50$ nm, the relaxation rates are different from the volume values even if the QW is situated in the centre of the slab. It is worth noting that, in the case of a slab with a free surface, the penetration length of the Rayleigh wave may be estimated by bT/T_s. This means that for low temperature the amplitude of the Rayleigh wave is high even near the centre of the slab.

In conclusion, we emphasize that the relaxation rates of the 2D electrons in a QW situated near the surface of a slab depend on the mechanical conditions at the surface of the slab. At low temperatures, where the effect is particularly strong, the finite width of the slab makes the phonon quantization important and gives rise to new peculiarities in the behaviour of the relaxation rates. Moreover, in this case the relaxation rates are different even if the QW is situated far from the slab surface.

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