Effects of acoustic-mode localization under dimensional crossover of an electron gas

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We have considered the semiconductor heterostructure that localizes acoustic modes within the quantum-well layer. It has been shown that additional confinement of the electrons in the plane of the layer gives rise to a supplementary localization (in a second direction) of acoustic modes. That means, in the many-electron system interacting with the lattice, the crossover from two- to one-dimensional electron-gas behavior leads to the appearance of a collective excitation which consists of lattice vibrations and electron charge-density waves, accompanied by an electrostatic field. The spectrum of this excitation is quite similar to those of acoustic phonons. The excitation is localized in two directions and is one dimensional in character. Splitting of this branch of the excitation from phonon branches of the structure and its additional localization increases with the wave vector of the excitation q. The two-dimensional electron gas forming a conductive channel with submicrometer dimensions completely localizes the excitation within this channel. If the electron gas is confined further to a one-dimensional channel, the excitation reveals maxima in the splitting and localization in the region $q \approx 2k_f$, k_f being the Fermi wave vector. In this region the dispersion curve of the excitation shows rotonlike behavior and the density of states has a peak ($T=0$). [S0163-1829(97)03116-0]

I. INTRODUCTION

Various semiconductor heterostructures are now main objects of study in solid-state physics. The most attention is paid to an electron's quantum confinement, i.e., to peculiarities of two-, one-, and zero-dimensional [(2D), (1D), and (0D)] electrons. The investigation of phonon confinement is much less advanced. Recently researchers began to realize that in the heterostructures there exists a phonon confinement, i.e., a localization of the lattice vibrations within heterolayers, nearby interfaces, etc. The existence of both, the carriers and the phonon modes, within the same narrow spatial domain of the material leads to important consequences for the effects related to electron-phonon interaction. For example, phonon confinement (mainly, optical-phonon confinement) affects the electron transport by modifying the intensity of the electron scattering, etc. Heterolayers can themselves be waveguides for acoustic vibrations, giving rise to a localization of the sound energy into electrically active regions. The latter is important for acoustic and acoustoelectronic phenomena, etc.

The main physical reason for the phonon confinement is a difference in the material characteristics of heterolayers: lattice constants, lattice forces, dielectric functions, etc. Extensive literature exists on the optical phonon confinement within quantum wells (QW's) and wires (see, for example, Refs. 1–8). The electron contribution to the optical-phonon confinement has been also discussed.[9]

Acoustic-phonon localization[10] originating from the lattice nonuniformity of layered heterostructures has been considered in several papers.[11–13] From these studies it follows that a number of widespread heterostructures shows the acoustic-mode localization near QW's: $Al_xGa_{1-x}As/GaAs/Al_xGa_{1-x}As$, GaP/GaAs/GaP (QW's for electrons and holes), $Al_xGa_{1-x}Sb/InSb/Al_xGa_{1-x}Sb$ (QW for electrons), etc. The localization of acoustic-phonon modes within $Al_xGa_{1-x}As/GaAs$ quantum wires has been calculated in Refs. 14,15. The behavior of acoustic phonons in superlattices has been studied in Refs. 16–18. Evidently, the localization due to a difference in the material characteristics should be considerably stronger for strained semiconductor heterostructures where this difference can be larger. The extreme case of a modification of acoustic phonons is demonstrated for free-standing quantum wells.[19]

Another physical reason for the acoustic-mode localization has been pointed out and studied in Ref. 20. It has been shown that even in the case of almost uniform characteristics of the lattice an electron-gas sheet localizes acoustic modes within the sheet due to electron-phonon interaction. Actually, such localized modes are two-dimensional excitations of a coupled electron-phonon system. Their frequencies are close to those of the phonons and are very different from other types of excitations in two-dimensional electron-gas-like plasmons, magnetoplasmons, etc.

In this paper we consider layered semiconductor heterostructure which localize acoustic modes within the QW layer. We suppose that the crossover from a two-dimensional electron gas to a one-dimensional channel is controlled by an external electrostatic field (for example, using the split-gate technique[21]). Formation of quasi-one-dimensional, or one-dimensional electron channels by this method leads to split-off one-dimensional modes, which are one-dimensional collective excitations of the electron-phonon system. In other words, these modes originate in part from the lattice nonuniformity, and are formed and guided in part by the electron channel due to electron-phonon interaction.

The localization of acoustic modes in one more direction could be qualitatively understood recalling two well-known

phenomena. The first follows from the theory of elastic waves.[22–24] a layer buried within another material and characterized by a decreased elastic modulus always splits the bulk acoustic vibrations into bulklike modes and localized modes. The latter are localized within or nearby the embedded layer and propagate along this layer. On the other hand, for bulk crystals one of the results of electron-phonon interaction via deformation potential is a renormalization of elastic modulus and sound velocities. This renormalization always has a fixed sign corresponding to a decrease of elastic modulus. Combining these two phenomena one can see that if there is a spatial area with electrons (an electron channel, for the case in question), the ''effective elastic modulus'' should be decreased within this area. That means the electrons induce an additional nonuniform softening of the lattice. One could expect that this softening causes a supplementary acoustic-mode localization, in a second direction.

In fact, such a localized mode is a collective excitation of the system of interacting low-dimensional electrons and acoustic phonons. The mode also drives waves of the electron density and electrostatic potential (but it differs from well-known plasma waves in low-dimensional systems, see, e.g., Refs. 25–27). This collective excitation is characterized by a one-dimensional wave vector q, because there is only one direction with translation symmetry along the electron channel. Since the electron motion is much faster than that of acoustic phonons ($v_e \gg c$, where v_e is characteristic electron velocity, c is sound velocity), the electrons follow adiabatically the lattice vibrations. As a result, the phonon spectrum experiences much stronger renormalization in comparison with the electron spectrum. As it will be shown, the dispersion curve of the localized mode is split and moved down with respect to that of acoustic modes of an initial layered structure with two-dimensional electrons. The number of created modes depends on electron-phonon coupling, electron density, and width of the electron channel.

For a strictly one-dimensional case, when electron motion is quantized in two directions, there exists only one mode. The mode dispersion relation is no longer a linear function of wave vector q and shows some anomalous behavior around $q=2k_F$, where k_F is the Fermi vector of the electrons. Extraordinary features of different physical quantities in this wave vector region are caused by well-known abnormal polarizations of a one-dimensional Fermi system. For the simplest one-dimensional model this leads to the renormalization of the phonon frequency down to zero (Peierls phase transition[28]). In the case under consideration, when the lattice vibrations are treated as three dimensional and the formation of the charge-density wave is taken into account, the screening effects prevent the Peierls transition (at least for actual material parameters). Below we show that the rotonlike portion of the dispersion curve and corresponding infinite peak(s) in the density of states of the excitations are formed at a low temperature. The characteristic scale of the localization of the mode is also a strong function of q and reaches its maximum in the same region of q.

Note, that a renormalization of the electron spectrum can be negligible, because of the adiabatical character of the electron motion. However, the described collective excitation enhances the energy dissipation of electrons.[29] In this paper we concentrate on the behavior of the excitation for the above described physical situation.

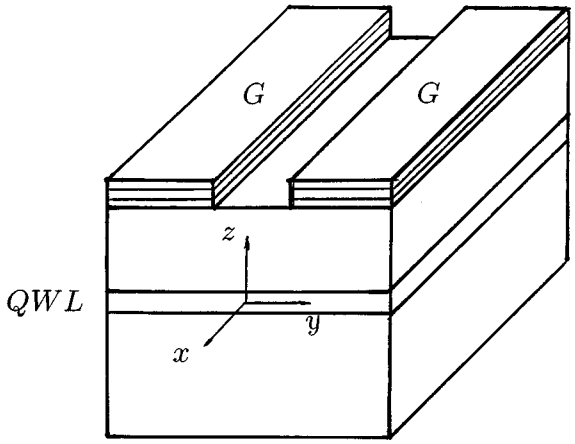

FIG. 1. Sketch of a heterostructure under consideration: embedded quantum-well layer (QWL), split gate (G), and coordinate system.

The lattice vibrations and the electrostatic field of the excitation have complex spatial configurations. We have developed an approach in the spirit of the adiabatical approximation. This approach allows one to find all variables and analyze the excitation spectra.

The paper is organized as follows. In Sec. II the model of the semiconductor heterostructure is described and the necessary equations are formulated. In Sec. III the approach used to analyze the excitation localized in two dimensions is developed and the mode localization near a quasi-one-dimensional electron gas is considered. In Sec. IV the case of strictly one-dimensional electrons is considered. A discussion of the results and numerical estimates are given in Sec. V. Some important, but cumbersome expressions are derived in Appendixes A and B.

II. MODEL AND EQUATIONS

We study the effect of additional localization of the acoustic modes in the heterostructures to model the electron channel shaped on the base of a narrow QW by external potential, which restricts the electron motion in one additional direction (see Fig. 1). A voltage applied to a split gate allows one to control the width of the electron channel. We suppose that the heterolayered medium with a planar embedded layer modulates the lattice properties in the z direction and forms QW's for the electrons.

The phonon subsystem. First, let us formulate the model for acoustic vibrations of the lattice. We describe the long-range acoustic vibrations by the equation for sound waves:[24]

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}, \tag{1}$$

where u_i are components of the displacement vector of medium $\mathbf{u}$, ρ is its density, σ_{ik} is the stress tensor, t and x_k are time and space coordinates, respectively. For simplicity we assume that the heterostructure is composed of isotropic materials. Then, the contribution of the elastic lattice forces to the stress tensor is[24]

$$\sigma_{ik}^{L}=[\lambda(z)+\tfrac{2}{3}\mu(z)]u_{ll}\delta_{ik}+2\mu(z)(u_{ik}-\tfrac{1}{3}u_{ll}\delta_{ik}), \quad (2)$$

where $\lambda(z)$ and $\mu(z)$ are Lamé coefficients which are different for the buried layer and the surroundings; u_{ik} being the strain tensor. In order to avoid overcomplication of the details of the electron and acoustic properties of the heterostructure, we restrict ourselves to the long-wavelength case, then

$$qd_z, \quad \kappa_{\rm ch}d_z \ll 1. \quad (3)$$

Here q is the wave vector, d_z is the thickness of the embedded layer, and $\kappa_{\rm ch}^{-1}$ is the characteristic decay length of the excitation outside the layer. In this case we can write down the modulus $\lambda(z)$, $\mu(z)$ in the form

$$\lambda(z)=\lambda+\lambda' d_z\delta(z), \quad \mu(z)=\mu+\mu' d_z\delta(z), \quad (4)$$

where $\delta(z)$ is δ function. The terms proportional to $\delta(z)$ correspond to the contribution of the embedded layer.

In the absence of electrons, the long-wavelength limit (3) and approximations (4) make our task equivalent to the problem of acoustic modes for the so-called ''flat defect''.[11] Analysis of that problem has shown the existence of localized acoustic modes if λ', $\mu'<0$. The latter just corresponds to the above discussed softening of the lattice at the region of the embedded layer. Another result of the analysis[11] is a mixing of longitudinal and transverse-acoustic vibrations (if $\mu'\neq 0$). Below we will consider the simplest isotropic structure of the electron band, when the electrons interact only with longitudinal acoustic phonons. To make our consideration easier we also assume the inequality $|\mu'|\ll|\lambda'|$, so that a mixing of the longitudinal and transverse elastic waves by the embedded layer is ignored.

For an isotropic electron band the electron-phonon interaction can be described only by one constant of the deformation potential b,[30] and the electron contribution to the stress tensor can be written as

$$\sigma_{ik}^{E}=bn(x,y,z)\,\delta_{ik}=bn_s(x,y)\,\delta(z)\,\delta_{ik}. \quad (5)$$

Here $n(x,y,z)$ is a concentration of the electrons. In the last equality we have taken into account the electron confinement into the narrow QW so that, in vein of criteria (3), we can introduce the ''surface'' electron concentration $n_s(x,y)$. However, the electron motion can possess either a 2D or 1D character dependent on the degree of their additional confinement in the y direction.

For what follows it is convenient to obtain the equation for relative volume change $(\mathrm{div}\mathbf{u})\equiv u_{ll}$. For this quantity from Eq. (1) we find

$$\rho\,\frac{\partial^2 u_{ll}}{\partial t^2}=\frac{\partial^2\sigma_{ik}}{\partial x_i\partial x_k}, \quad \sigma_{ik}=\sigma_{ik}^{L}+\sigma_{ik}^{E}. \quad (6)$$

Outside the layer one can rewrite this equation in a very simple form using the free wave equation

$$\rho\,\frac{\partial^2 u_{ll}}{\partial t^2}-(\lambda+2\mu)\Delta u_{ll}=0, \quad z\neq 0. \quad (7)$$

The solution of Eq. (7) at $z>0$ and $z<0$ should be matched at the plane of the layer. The corresponding boundary condition can be derived from Eq. (6) by integration over the narrow layer around $z=0$:

$$(\lambda+2\mu)\,\frac{\partial u_{ll}}{\partial z}\bigg|_{-0}^{+0}=-\lambda' d_z\Delta_2 u_{ll}-b\Delta_2 n_s, \quad (8)$$

where $\Delta_2\equiv(\partial^2/\partial x^2)+(\partial^2/\partial y^2)$. From Eqs. (7), (8) it can be seen that only relative volume change u_{ll} enters the equations. This means that under the above assumptions only longitudinal acoustic vibrations are coupled with the confined electrons.

The *electron subsystem*. We assume that the electrons follow the lattice vibrations adiabatically and are redistributed in the potential created by the acoustic wave (a necessary condition for the adiabatical approximation always holds for semiconductors). For the electrons the total potential energy induced by the acoustic wave can be written in the form[30]

$$h(r)=bu_{ll}-e\varphi, \quad (9)$$

where bu_{ll} describes a change of the bottom of the electron band due to a lattice deformation, φ is a electrostatic potential arising from the nonequilibrium redistribution of the electrons in space, φ is governed by Poisson's equation

$$\Delta\varphi=\frac{4\pi e}{\varepsilon_0}\,\delta n_s(x,y)\,\delta(z), \quad (10)$$

with ε_0 being the dielectric constant of the crystal.

Besides the electron confinement inside the narrow QW, there is a restriction on the electron motion in the y direction. Let us assume that the conducting channel is along the x axis and that all variables have the following dependences on the x coordinate and time:

$$u_{ll},\varphi,\delta n\sim e^{i(-\omega_q t+qx)}, \quad (11)$$

while the determination of z and y dependences is one of our main goals. Here we can only mention that the characteristics scales of these dependences are q and $\kappa_{\rm ch}$, obeying criteria (3).

Any change in the electron density δn induced by potential (9) can be calculated using ordinary perturbation theory for the density matrix (see, for example, Ref. 20):

$$\delta n(x,y,z)=2e^{iqx}\sum_{n,n'}\sum_{l,l'}\sum_{k}\Psi_{n,l}(y,z)\Psi^{*}_{n',l'}(y,z)\times\frac{f_0(\epsilon_{n'l'k+q})-f_0(\epsilon_{nlk})}{\epsilon_{n'l'k+q}-\epsilon_{nlk}+i\delta}\,\langle nl|h(y,z)|n'l'\rangle. \quad (12)$$

Here ϵ_{nlk} and $\Psi_{nl}(y,z)$ are eigenvalues and wave functions of transverse electron motion, respectively; k is the wave vector of the electrons, $f_0(\epsilon_{nlk})$ being the Fermi distribution function. Let us assume that the wave functions are factored: $\Psi_{nl}(y,z)=\Psi_n(y)\Psi_l(z)$, and the characteristic scale of $\Psi_l(z)$ is d_z. According to criteria (3) the potential $h(r)$ is a smooth function of the z coordinate, so we can write

$$\langle nl|h(y,z)|n'l'\rangle\cong\delta_{ll'}\langle n|h(y,z=0)|n'\rangle. \quad (13)$$

This means that flatten potential $h(y,z)$ does not cause transitions between the electron subbands arising from a quantization in the z direction. Then, using Eq. (13) we find the surface electron concentration

$$\delta n_s(x,y)=2e^{iqx}\sum_{l,n,n'}\sum_{k}\Psi_n(y)\Psi_{n'}^{*}(y)\times\frac{f_0(\epsilon_{n'lk+q})-f_0(\epsilon_{nlk})}{\epsilon_{n'lk+q}-\epsilon_{nlk}}\langle n|h(y,z=0)|n'\rangle. \tag{14}$$

The set of relationships (7)–(11) and (14) self-consistently describes coupled acoustic vibrations of the lattice, an electrostatic field, and a redistribution of the electrons. This set is sufficient to consider the collective excitations localized in two directions.

Spatial scales of the problem. For analysis of this system of equations, it is useful to consider spatial characteristic scales of the problem. There are two scales of the electron confinement, d_z and d_y. For the case of the quantization of electrons inside the embedded layer and restricting their motion in the second direction by an external potential the inequality

$$d_z \ll d_y \tag{15}$$

applies. Then, one has to compare d_z, d_y with inverse Fermi vector k_F^{-1}. The width of the QW d_z is determined by the heterostructure fabrication. The confinement in the y direction d_y is mainly controlled by the gate voltage, while k_F^{-1} is considerably dependent on the doping. These three spatial scales can be controlled and changed almost independently. As a result, different situations can occur

$$k_F^{-1}<d_z<d_y, \tag{16a}$$

$$d_z\leqslant k_F^{-1}<d_y, \tag{16b}$$

$$d_z<d_y\leqslant k_F^{-1}. \tag{16}$$

First inequalities (16a) simply correspond to a conductive channel with three-dimensional electrons. This channel is narrow in z and y directions in a classical sense. Second inequalities (16b) refer to a quasi-one-dimensional case or simply a two-dimensional channel narrow in the classical sense (only one z subband and many y subbands are occupied by the electrons). The last case (16) corresponds to strictly one-dimensional electron channel, where only the lowest electron subband is filled. This case is quite different from the first two and requires special consideration.

Such a classification of possible physical cases allows one to simplify the set of basic equations according to inequalities (16a), (16b), (16) and analyze the problem in detail. We now study the localization effect and behavior of the excitations from quasiclassical cases (16a), (16b).

III. CASES OF NARROW CLASSICAL CHANNELS

In the limiting case of classical narrow channels expressions (12), (14) for the redistribution of the electron density can be substantially simplified and expressed through stationary distributions of the electrons in the potential which determines their confinement.

Under criteria (16a), when a large number of subbands with small intersubband energies contributes to Eq. (12), the concentration of 3D electrons can be presented as

$$\delta n(x,y,z)=-h(y,z)e^{iqx}\frac{\partial n(y,z)}{\partial \nu}, \tag{17}$$

where $n(y,z)$ is the equilibrium spatial distribution of the carriers inside the classical channel, and ν is the chemical potential. Note that formula (17) coincides with obvious form for an electron redistribution in a stationary nonuniform potential $h(x,y,z)$. This result is a consequence of an adiabatical approximation. The electron channel is determined by a given potential $U(y,z)$ formed by both a profile of the band edge of the heterostructure and an external electrostatic field. Thus, for $n(y,z)$ one can write

$$n(y,z)=\int_0^{\infty}\frac{\rho_3(\epsilon)}{1+\exp\left[\dfrac{\nu-[\epsilon+U(y,z)]}{kT}\right]}\,d\epsilon, \tag{18}$$

where ϵ is the kinetic energy of classical electrons, $\rho_3(\epsilon)\equiv\sqrt{\pi}m^{3/2}/\pi^2\hbar^3$ is the 3D density of states per unit volume, m^* is the effective mass, and T represents the temperature. Taking into account the assumed smoothness of the potential $h(y,z)$, after integration over z one can find a variation of the surface electron density in the channel:

$$\delta n_s(x,y)\equiv e^{iqx}\delta n_q(y)=-h(y,0)e^{iqx}P_3(y), \tag{19}$$

where

$$P(y)=\frac{\partial n_s(y)}{\partial \nu}, \tag{20}$$

and $n_s(y)=\int dz\, n(y,z)$. The expressions (19), (20) are valid for three-dimensional electrons at $q<k_F$.

For a narrow two-dimensional channel (quasi-one-dimensional case), under criteria (16b) one can calculate $\delta n_s(x,y)$ and find the same relation (19) at any ratio between q and k_F (see Appendix A). If $P_2(q|\nu)$ is the polarization operator of the two-dimensional electron gas with the chemical potential ν, then the function $P(y)$ can be expressed via P_2 defined by Eq. (A5):

$$P(y)=P_2[q|\nu-U(y)], \tag{21}$$

where $U(y)$ is an external potential restricting the electron motion in the y direction. At $q<k_F$ expression (21) is simplified to the form (20), where the surface electron concentration n_s has to be accounted according to Eq. (18) with the 2D density of states $\rho_2(\epsilon)\equiv m/\pi\hbar^2$ instead of $\rho_3(\epsilon)$.

Now we can write down the final system of equations which should be solved:

$$\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\kappa^2\right]u(y,z)=0,\quad z\neq 0; \tag{22}$$

$$-(\lambda+2\mu)\,\frac{\partial}{\partial z}\,u(y,z)\Big|_{z\to-0}^{z\to+0}$$

$$=\left(\frac{\partial^2}{\partial y^2}-q^2\right)[\lambda' d_z u(y,0)+b\,\delta n_q(y)],\quad z=0; \tag{23}$$

$$\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-q^2\right]\Phi=0,\quad z\neq 0; \tag{24}$$

$$\frac{\partial\Phi}{\partial z}\bigg|_{z\to-0}^{z\to+0}=-\frac{4\pi e^2}{\varepsilon_0}\,\delta n_s(y,0),\quad z=0; \tag{25}$$

$$u(y,z),\Phi(y,z)\to 0,\quad y,z\to\pm\infty; \tag{26}$$

$$\delta n_q=-P(bu+\Phi). \tag{27}$$

Here we introduce the following notations: $u(y,z)\equiv u_{ll}(y,z)$, $\Phi\equiv -e\varphi$, and

$$\kappa=\sqrt{q^2-\omega_q^2/c_l^2}; \tag{28}$$

$c_l=\sqrt{(\lambda+2\mu)/\rho}$ is the velocity of longitudinal sound waves. Note, that boundary conditions (26) correspond to solutions localized within the electron channel.

Solving Eqs. (22), (24), (27) with matching conditions (23), (25) and the boundary conditions (26) gives the spatial distributions of u, Φ, and δn_s, and the dispersion relation $\omega(q)$ for the localized modes. Thus, $\kappa=\kappa(\omega)$ is an eigenvalue of this system of equations.

In order to solve the system we note that spatial scales that characterize the localization in y direction are much larger than those of the z direction. For the electrostatic potential Φ, it follows from criterion (15) combining this criterion with the inequality $q\leqslant k_F$, we get

$$qd_y\gg 1 \tag{29}$$

[see also inequalities (16a), (16b)]. For the acoustic variable u it occurs due to the small renormalization of the elastic modulus by electrons: the localization in the z direction is mainly a result of a finite change of the modulus in the embedded layer, while the localization in y direction is driven by the electron-phonon interaction only. If the characteristic y scale of the function u is $\chi_{\rm ch}^{-1}(q)$, we assume that

$$\kappa_{\rm ch}(q)\gg\chi_{\rm ch}(q). \tag{30}$$

Thus, we assume that all functions vary in the y direction much slower than in the z direction. In this case we can use the approach similar to an adiabatical approximation. We present the unknown functions in the form

$$u(y,z)=A(y)Z_u(y,z)+u_1(y,z), \tag{31}$$

$$\Phi(y,z)=B(y)Z_\Phi(y,z)+\Phi_1(y,z), \tag{32}$$

where the first terms are the leading ones in this approach, Z_u and Z_Φ are adiabatically slow functions of y. The second terms u_1 and Φ_1 give corrections to the first approximation.

To define the first approximation we first ignore all derivatives with respect to y in Eqs. (22)–(24). Then, at fixed y we find

$$u_{(0)}=A(y)e^{-\kappa(y)|z|},\quad \Phi_{(0)}=B(y)e^{-q|z|}, \tag{33}$$

where the relationship between amplitudes of acoustic and electrostatic waves is

$$B(y)=A(y)\,\frac{2\pi e^2 bP(y)/\varepsilon_0}{q+2\pi e^2 bP(y)/\varepsilon_0}. \tag{34}$$

From Eq. (22) for u it follows that the solutions in the form (33) can exist as long as $\kappa(y)$ is

$$\kappa(y)=\kappa_0(q)[1+\alpha(y)], \tag{35}$$

where

$$\kappa_0(q)=-\frac{q^2\lambda' d_z}{2(\lambda+2\mu)}, \tag{36}$$

$$\alpha(y)=-\frac{b^2}{\lambda' d_z}\,\frac{P(y)}{1+2\pi e^2 P(y)/\varepsilon_0 q}. \tag{37}$$

According to Eq. (33), the quantity $\kappa(y)$ characterizes the localization in the z direction. Here $A(y)$ still is an arbitrary function.

Before determining $A(y)$, let us note that in the absence of the electrons κ is always constant and positive, if $\lambda'<0$. The latter is a necessary requirement for the localization of the acoustic modes. For this condition we find the dispersion relation for the *two-dimensional acoustic mode* localized within the embedded layer:

$$\omega_{\rm 2D}^2=c_l^2q^2-c_l^2\kappa_0^2=c_l^2q^2-c_l^2\left(\frac{q^2d_z|\lambda'|}{2(\lambda+2\mu)}\right)^2. \tag{38}$$

Within approximation (4), i.e., for ''a flat defect'' of the lattice, there exists a single localized mode.

Equations (33)–(37) can obviously be applied to the case of homogeneous electron characteristics in the x,y plane, when u_1, Φ_1 are equal to zero and α, P are constant. It is easy to see that signs of the electron contributions to κ and ω [second term in Eq. (35)] correspond to a softening of the lattice and an additional localization of the acoustic mode in the z direction [see Eqs. (35)–(37)]. However, in accordance with the above discussed, these contributions are always smaller than the lattice contribution of the embedded layer.

In Appendix B the equations for subsequent steps of the approximation are presented. In the framework of such an approach one can find the equation for $A(y)$:

$$\frac{d^2A}{dy^2}+[\kappa^2(y)-\kappa^2]A=0. \tag{39}$$

According to Eq. (26), the boundary conditions for $A(y)$ are

$$A(y\to\pm\infty)\to 0. \tag{40}$$

If we denote

$$v(y)\equiv\tfrac{1}{2}[\kappa_0^2-\kappa^2(y)],\quad \epsilon\equiv\tfrac{1}{2}(\kappa_0^2-\kappa^2), \tag{41}$$

we can rewrite Eq. (39) as

$$\left(-\frac{1}{2}\,\frac{d^2}{dy^2}+v(y)\right)A=\epsilon A, \tag{42}$$

i.e., in the form of a ''Schrödinger equation'' with ''a potential'' $v(y)$ and ''an energy'' ϵ. Thus we obtain the well-known eigenvalue problem, where the eigenvalue ϵ determines the dispersion relation and eigenfunction $A(y)$ characterizes y dependences of the excitation.

In this formulation of the problem, the potential $v(y)$ is mostly determined by the electron characteristics: in the absence of the electrons $v=0$, if the electron concentration is independent of coordinate y, $v=\text{const}<0$. An electron contribution makes ''the potential'' v always deeper. A nonmonotonous distribution of electrons creates potential relief. A maximum (minimum) of the electron concentration corresponds to a minimum (maximum) of $v(y)$. Thus, when the electrons are localized in some area, the potential $v(y)$ is negative just in this area. For a one-dimensional Schrödinger equation, such a potential causes at least one discrete energy level and bound state.[31] Thus, one can immediately conclude that there always exists a localization of the excitation mode(s) in the y direction. Actually, a discrete level ϵ_n calculated at fixed q corresponds to the eigenvalue of Eq. (39):

$$\kappa^2=\kappa_0^2+2|\epsilon_n|.$$

The latter gives the frequency of *one-dimensional collective excitation* at fixed q:

$$\omega_{1\mathrm{D},n}^2(q)/c_l^2=q^2-\kappa_0^2(q)-2|\epsilon_n(q)|. \tag{43}$$

We see that the frequency of the collective mode localized in two directions is always less than the frequency of the acoustic mode localized only in one direction due to the embedded layer.

Explicit forms of $\epsilon_n(q)$ and $A_n(y)$ cannot be obtained in the general case. But if ''the potential'' $v(y)$ is shallow, one can write down for the only level that exists in this case

$$A(y)=\text{const}\times e^{-\chi|y|},\quad |\epsilon_0|=\frac{1}{2}\left(\int_{-\infty}^{+\infty}dy\ v(y)\right)^2\equiv\tfrac{1}{2}\chi^2. \tag{44}$$

Here we use the following approximation, which is well known in the quantum mechanics.[24] The localization scale in a shallow potential exceeds the characteristic width of this potential. Thus, the solution has an exponential form everywhere except the potential region. The localization parameter, χ, should be found by substituting the solution into Eq. (44) and integrating it over the entire space. The results are valid if $|\epsilon_0|\ll|v(y)|$.

The described picture of the localization depends on the wave vector q since ''the potential'' $v(y)$ is a parametric function of q. From Eqs. (35), (36), and (41) we see that the depth of this potential well increases for larger q. Thus, if we consider the lowest-energy level ϵ_0 of ''the potential'' we obtain, according to Eq. (43), a dispersion curve for the excitation localized in two directions. Because of increasing $|\epsilon_0(q)|$ with q, we can also conclude that the splitting between two-dimensional acoustic branch (38) and the collective excitation mode increases for greater q, as well as parameters of the localization in both y and z directions. For large q, when at a certain depth of ''the potential'' $v(y)$ an additional discrete level appears, a second localized mode is split, which appears at a certain critical value q_c. Its dispersion curve is located between the two-dimensional branch (38) and the lowest one-dimensional dispersion curve. These general conclusions are valid for both 3D and 2D electrons, which induce the collective excitation.

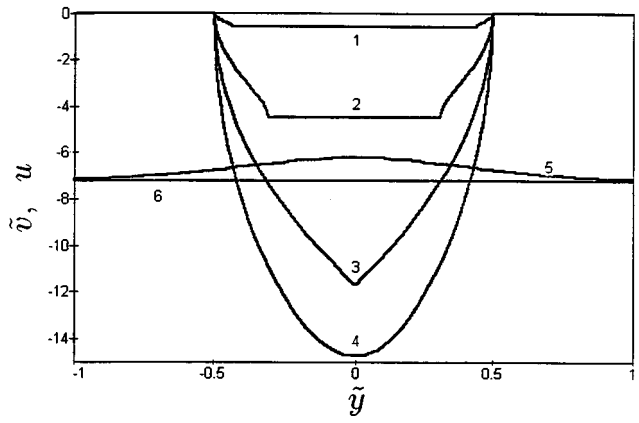

FIG. 2. ''Effective'' potentials $\tilde{v}=v(y)d_y^2$ localizing the excitations versus the dimensionless coordinate $\tilde{y}=yd_y$ (curves 1,2,3,4). The coordinate dependence of the lattice deformation $u(\tilde{y})$ is shown for the lowest mode in ''the potential'' 3 (curve 5, arbitrary units). ''The potentials'' correspond to different q: $q/2q_{F,M}=0.2$ (1), 0.5 (2), 1 (3), 1.2 (4). Particular parameters used in calculations are given in Sec. V.

Now we consider the case of 2D electrons, when their motion is restricted in the y direction in accordance with inequalities (16b) (i.e., the case of the quasi-one-dimensional electron channel). For a completely degenerate electron gas the electron concentration $n(r)$ is determined by the Fermi level ν and an external potential $U(y)$ [see Fig. 2(a)]:

$$n(y)=\frac{m^*}{\pi\hbar^2}[\nu-U(y)]\Theta[\nu-U(y)], \tag{45}$$

where $\Theta(x)$ is the Heaviside step function. From formula (45) it follows that the thickness of the quasi-one-dimensional channel is $d_y=y_1-y_2$, where $y_{1,2}$ are roots of the equation $U(y)=\nu$. For this semiclassical case one can introduce the Fermi vector as the function of the coordinate: $k_F(y)=(1/\hbar)\sqrt{2m^*[\nu-U(y)]}$. Then, one can calculate the polarization operator (21) under assumption $qd_y\gg1$ (see Appendix A):

$$P(y)=\frac{m^*}{\pi\hbar^2}\left\{1-\Theta\left(1-\frac{2k_F(y)}{q}\right)\sqrt{1-[2k_F(y)/q]^2}\right\}, \tag{46}$$

for $y_1<y<y_2$ and $P(y)=0$ at $y<y_1$ and $y>y_2$. Equations (46), (35), and (36) determine ''the potential $v(y)$'' for Eq. (42).

''The potential'' has the simplest form at $q<2k_{F,M}$, where $k_{F,M}$ is a maximum of $k_F(y)$,

$$v(y)\approx\begin{cases}-v_0\equiv-\kappa_0^2\alpha, & y_1<y<y_2;\\ 0, & y<y_1,y_2<y;\end{cases} \tag{47}$$

where

$$\alpha=\frac{b^2m^*}{\pi\hbar^2|\lambda'|d_z\left(1+\dfrac{2e^2m^*}{\varepsilon_0 q\hbar^2}\right)}. \tag{48}$$

For this square ''potential well'' the energy levels are roots of an algebraic equation

$$2\arcsin\sqrt{1-\epsilon_n/v_0}=\pi n-d_y\sqrt{2(v_0-\epsilon_n)}, \tag{49}$$

where $n=1,2,\ldots$. For example, the second mode is split at q_c, which can be found from the equation $v_0(q_c)=\pi^2/2d_y^2$. If ϵ_n is a solution of Eq. (49), outside the electron channel we obtain $A(y)\sim\exp(-y\sqrt{2|\epsilon_n|})$. It is interesting to note that in this case $v(y)$ is independent on the electron concentration, as well as all parameters of the localized excitation(s).

This simple limiting case allows one to estimate the decay length of $A(y)$ outside the channel: $\chi<\kappa_0\sqrt{\alpha}$. Since $\alpha\ll1$, we can see that the above assumption (30) always holds.

Let us analyze the role of the screening effect. A contribution of this effect can be seen from expression (37) for α (second term in the denominator). The screening suppresses the confinement phenomena at small q. Expression (48) gives the characteristic value $q=q_{\rm sc}$ for a two-dimensional electron gas: $q_{\rm sc}=(2e^2m^*/\varepsilon_0\hbar^2)$. If $q<q_{\rm sc}$, from Eq. (48) we get the following result:

$$\alpha=\frac{b^2}{|\lambda'|d_z}\frac{\varepsilon_0 q}{2\pi^2e^2}, \tag{50}$$

which contains only the lattice characteristics and the electron-phonon coupling constant, and does not depend of other electron parameters. For the amplitudes of the waves we get $B(y)\approx A(y)$. If $q>q_{\rm sc}$ the screening effects are small and the amplitude of the electrostatic wave is negligible $B(y)\ll A(y)$. The latter means that one can drop the first inequality from Eq. (3) and extend the results to a region of high q, since the parameters of acoustic localization $\kappa(q),\chi(q)$ still fulfill the second inequality from Eq. (3), and requirement (29).

At finite q only a central part of ''the potential,'' where $k_F(y)>q/2$ holds, is flat. An evolution of the ''potential well'' as a function of q is shown in Fig. 2. From this potential evolution it follows that the power of the ''potential,'' $\int|v(y)|dy$, invariably increases, which means the mode splitting increases in q as well. From Fig. 2 one can see that for presented examples the power is considerably greater at $q\geqslant k_{F,M}$. For such a case one can apply the method developed in quantum mechanics[31] to estimate the number of localized modes: $N\approx(\int\sqrt{|v(y)|}dy)/\sqrt{2}\pi$. As a result we get that N is about from 6 to 10.

At $q\gg2k_F(y)$ the ''potential well'' again takes a simple form, because P is just proportional to n:

$$P(y)\approx\frac{2m^*}{\pi\hbar^2}\frac{k_F^2(y)}{q^2}=\frac{4m^*}{q^2\hbar^2}n_s(y). \tag{51}$$

In this limiting case the electron contribution to $\kappa(y)$ is saturated when q increases. But, accordingly with Eqs. (35), (47), the depth of the ''potential'' still increases. As a result, the splitting and localization of the mode in the y direction increase continuously. This point is not trivial, because the response of the electron gas to an perturbation *decreases* in this region of q. The reason for further localization in the plane of the QW layer is the increase of the localization of two-dimensional phonon in the z direction [see Eqs. (35), (36)]. The latter leads to an enhancement of the electron-phonon interaction and, consequently, to an increase of the localization in the second, y direction. Note that at very large q the analysis of this limiting case is restricted by the first inequality (3) assumed above.

Another important parameter characterizing the system and considerably affecting the results is the width of the electron channel d_y. In terms of the equivalent Schrödinger Eq. (42), d_y determines the width of the ''potential well'' $v(y)$ that immediately gives us the trends of the changes in the results with varying d_y. For very large d_y the dispersion curves of the excitation almost coincides with the two-dimensional phonon (38), where κ_0 has to be replaced by $\kappa_0(1+\alpha)$. In this case, the degree of the phonon localization behaves as d_y^{-1}, i.e., it increases when d_y decreases. For the narrow channel, when only one solution exists in the ''potential'' $v(y)$, we get the ''energy'' $\epsilon_0\propto d_y^2$ and the degree of the localization $\chi\propto d_y$, according to Eq. (44). Thus, the localization is a nonmonotonous effect: it increases with the electron confinement in the second direction, reaches a maximum, and then decreases. The latter occurs at a strong electron confinement.

So far we considered the first approximation of the adiabatical approach. One can derive the equations and boundary conditions for the functions $u_1(y,z)$ and $\Phi_1(y,z)$ giving corrections to u and Φ [see Eqs. (A23)–(A26)]. As usual, for an adiabatical approach the system of equations and boundary conditions for the next level of approximations contain inhomogeneous terms, which generate nonzero corrections and are proportional to derivatives of the functions with respect to the ''slower'' y coordinate, which are calculated in the first approximation. Using the above discussed spatial scales of functions entering those equations (i.e., d_z and d_y for P_y, $\kappa_{\rm ch}$ and d_y for Φ_0, $\kappa_{\rm ch}$ and $\chi_{\rm ch}$ for u_0) one can estimate the order of magnitude of corrections u_1 and Φ_1. The simple analysis shows that relative contributions of the correction terms in expressions (33) and (34) are of the order of $\chi_{\rm ch}/\kappa_{\rm ch}^2d_y$ and $1/qd_y$, respectively. In accordance with inequalities (29), (30) assumed above, these corrections are small. Thus, at the end of this section we can conclude that the quasiclassical electron channel brings about additional localization of the collective mode(s).

IV. CASE OF ONE-DIMENSIONAL ELECTRON GAS

If the width of the two-dimensional electron channel d_y decreases, so that inverse Fermi vector k_F^{-1} becomes comparable with d_y, we cross over to the case of a one-dimensional electron gas. In order to avoid extra details of wave functions and energies of subbands, their occupancies, etc., we consider the case when only one subband is occupied by the electrons [see inequalities (16)].

To further simplify the problem we have to discuss the spatial scales of all three variables u, Φ, and δn. The scale of the electron density, d_y is governed by the external potential $U(y)$ and has to be defined as a characteristic spread of

the square of the wave function $|\Psi_1(y)|^2$ for the lowest subband. The scale of the acoustic deformation $u(y)$ is always much greater than d_y (see estimates below) and greater than the characteristic extension in the z direction. Thus, for $u(y,z)$ we can exploit the same adiabatical approximation (31) that was used in Sec. III. The scale of the electrostatic potential $\Phi(y)$ is of the order of d_y under inequality (29) and is about q^{-1}, in the opposite case. The latter case is rather important if we want to study actual situation $q \sim k_F$. But for this case the adiabatical presentation of $\Phi(y,z)$ in the form (32) is not longer valid, since q^{-1} is the characteristic length of the z dependence.

Neglecting the contributions from the upper subbands, under the above discussed assumptions one can simplify expression (14) for the electron density to the following form:

$$\begin{aligned}\delta n_s &\equiv e^{iqx}\delta n_q(y) \\ &= e^{iqx}|\Psi_1(y)|^2 P_1(q)\langle 1|h(y)|1\rangle \\ &= e^{iqx}|\Psi_1(y)|^2 P_1(q)[bu(0)+\langle 1|\Phi|1\rangle],\end{aligned} \tag{52}$$

where $P_1(q)$ is a one-dimensional polarization operator and the last term is the diagonal matrix element of $\Phi(y,z=0)$ calculated using the wave function $\Psi_1(y)$.

Then, we can write down the exact solution of Poisson's Eq. (24) with boundary conditions (25), (26):

$$\Phi_q(y,z) = -\frac{4\pi e^2}{\varepsilon_0}\int dy' G_q(y-y',z)\,\delta n_q(y'), \tag{53}$$

where

$$G_q(y-y',z) = -\frac{1}{4\pi}\int dQ\,\frac{e^{-i(y-y')Q}}{\sqrt{q^2+Q^2}}\,e^{-|z|\sqrt{q^2+Q^2}}$$

is Green's function of Eq. (24). Note, that $G_q(y,0) = -(1/2\pi)K_0(q|y|)$, where $K_0(x)$ is the modified Bessel function of zeroth order.

From Eqs. (52), (53) we can calculate the matrix element

$$\langle 1|\Phi(y)|1\rangle = -bu(0)\,\frac{4\pi e^2 P_1(q)g(q)/\varepsilon_0}{1+4\pi e^2 P_1(q)g(q)/\varepsilon_0}, \tag{54}$$

where

$$g(q) = \int\int dy\; dy' G_q(y-y',0)|\Psi_1(y)|^2|\Psi_1(y')|^2. \tag{55}$$

Substituting Eq. (54) into Eq. (52) we find the expression for $\delta n_q(y)$ proportional to the product $|\Psi_1(y)|^2 u(0)$. Note that because the characteristic scales for $|\Psi_1(y)|^2$ and $u(y)$ are very different, we can use the approximation[32] $|\Psi_1(y)|^2 u(0) \approx |\Psi_1(y)|^2 u(y)$ and present $\delta n_q(y)$ in the form

$$\delta n_q(y) = bu(y)|\Psi_1(y)|^2\,\frac{P_1(q)}{1+4\pi e^2 P_1(q)g(q)/\varepsilon_0}. \tag{56}$$

One can also find the distribution of the electrostatic potential Φ inside the embedded layer, i.e., in the plane $z=0$:

$$\Phi(y,z=0) = -bu(y)\,\frac{4\pi e^2 P_1(q)/\varepsilon_0}{1+4\pi e^2 P_1(q)g(q)/\varepsilon_0} \times \int dy' G_q(y-y',0)|\Psi_1(y')|^2. \tag{57}$$

Let us turn to the problem of the deformation variable $u(y,z)$. In matching condition (23) one can replace $\delta n_q(y)$ by expression (56). This relationship, Eq. (22), and the first boundary condition from (26) compose a system that contains only the unknown function $u(y,z)$. To find this function one can apply, as discussed above, the adiabatical approximation and seek the solution in form (31). In the first approximation we restrict ourselves to the multiplicative term from Eq. (31). It is easy to show that the function $A(y)$ should satisfy Eq. (39), where $\kappa(y)$ is calculated in accordance to Eq. (35) with $\alpha(y)$ given by

$$\alpha(y) = -\frac{b^2}{\lambda' d_z}\,\frac{P_1(q)|\Psi_1(y)|^2}{1+4\pi e^2 P_1(q)g(q)/\varepsilon_0}. \tag{58}$$

Finally, one can analyze Eq. (39) for $A(y)$ or the equivalent ''Schrödinger'' Eq. (42) with the potential in the form (41). Similar to the previous section, for $\alpha(y)$ given by Eq. (58) the ''potential'' $v(y)$ is attractive, which means that there always is a bound state for the ''Schrödinger'' equation and a localized collective mode for Eq. (39).

The shape of ''the potential'' is given by $|\Psi_1(y)|^2$, its depth substantially depends on q. One can easily calculate the power of the potential $v(y)$:

$$\chi = \int_{-\infty}^{+\infty} dy|v(y)| = \kappa_0^2\,\frac{b^2}{|\lambda'|d_z}\,\frac{P_1(q)}{1+4\pi e^2 P_1(q)g(q)/\varepsilon_0}. \tag{59}$$

Since we expect a relatively weak localization of the acoustic mode in y direction, the value χ completely defines the ''energy'' ε_0 and the function $A(y)$, according to the expression (44) (the estimates for this approximation are given below). Then, it is easy to calculate the dispersion curve (43) for the mode. Thus, we have obtained the solutions for all quantities, u, Φ, δn_q, and derived other parameters characterizing the excitation.

We can juxtapose the localization effects for the two-dimensional classical channel and one-dimensional case by comparing expressions (48), (58) for α. The main qualitative distinctions are caused by a different electron polarization and screening character in these two systems. As for the screening effects, instead of the $1/q$ dependence in the two-dimensional gas, in the latter case a new function $g(q)$ enters the equations. This function is dependent on $|\Psi_1|^2$. It is easy to understand its general behavior: $g(q)$ is a monotonous function of q, so that $g(q)\to\infty$ at $q\to 0$ and $g(q)\sim 1/q$ when $q\to\infty$. If the wave function of the electrons $\Psi_1(y)$ is characterized by only one spatial scale d_y, so $\Psi_1 = \text{const} \times \Psi(y/d_y)$, it can be shown that $g = g(qd_y)$ and two former limits correspond to the following criteria $ql_y \ll 1$ and $ql_y \gg 1$. At $ql_y \sim 1$ the function $g(ql_y)$ is also of the order 1. One can introduce the characteristic value of q_{sc} by the following equation:

$$1=8e^2m^*g(q_{sc})/\varepsilon_0\hbar^2q_{sc}. \tag{60}$$

At $q<q_{sc}$ the screening effects are important in the opposite case the electrostatic effects are suppressed. This criterion is different from that of the quasiclassical case and depends on d_y. The large polarization of one-dimensional electrons is even more essential. For completely degenerate electrons, the polarization operator $P^{1D}(q)$ is given by

$$P^{1D}(q)=\frac{2m^*}{\pi\hbar^2q}\ln\left|\frac{k_F+q/2}{k_F-q/2}\right|, \tag{61}$$

and it has singularities at $q=\pm 2k_F$. If $q\ll k_F,q_{sc}$ then $P^{1D}=4m^*/\pi^2\hbar^2n^{(1D)}$ ($n^{(1D)}$ being the linear electron density in the one-dimensional channel), we obtain for χ

$$\chi=\kappa_0^2(q)\,\frac{b^2\varepsilon_0}{4\pi e^2|\lambda'|d_zg(q)}. \tag{62}$$

This expression is similar to formulas (35) and (50) for the two-dimensional electron channel and contains mainly the lattice parameters and geometrical characteristics d_z, d_y. Because of the divergence of $P_1(q)$ at $q\to 2k_F$, this expression is also valid around $q\approx 2k_F$ at any ratio q/q_{sc}. At $q\gg k_F,q_{sc}$, when $P^{(1D)}=4m^*n^{1D}/\hbar^2q^2$ and $g(q)\to 0$, we find the opposite limit for χ:

$$\chi=\kappa_0^2(q)\,\frac{b^2}{|\lambda'|d_z}\,\frac{4m^*n^{1D}}{\hbar^2q^2}. \tag{63}$$

Both limiting cases, (62) and (63) show that the degree of the localization and splitting of the collective mode increase at larger q, in spite of decreasing $P_1(q)$ under $q>2k_F$.

The abnormal electron polarization around $q=\pm 2k_F$ is responsible for a number of well-known effects in one-dimensional systems. In our case this peculiarity is also important. If the electron charge-density wave and electrostatic field accompanying the lattice vibrations were negligible (formally this case corresponds to the limit $\varepsilon_0\to\infty$), one would obtain α, $\chi\to\infty$ and the square of the excitation frequency $(\omega_{1D}(2k_F))^2<0$. The latter would indicate the existence of a phase transition in the system. This behavior corresponds to the Peierls transition,[28] which has been predicted and is peculiar for entirely one-dimensional system: a one-dimensional chain of atoms with delocalized electrons. For our model in this limit one obtains qualitatively the same results, but for the *three-dimensional elastic medium and one-dimensional electrons*. As it follows from Eq. (62), the formation of the charge-density wave and screening effects prevent the Peierls transition, at least for actual parameters of materials. Nevertheless, the phase transition is possible in principle.

As a rudiment of the phase transition at low temperature our system shows the nonmonotonous behavior of $\omega_{1D}(q)$ in the region around $q=2k_F$: $\omega_{1D}(q)$ has in succession a maximum and a minimum. In order to see that, consider the case $T=0$ and present these quantities in the following form:

$$\chi(q)=\chi(2k_F)\,\frac{1}{1+s/\ln\left|\dfrac{2k_F}{q-2k_F}\right|}, \tag{64}$$

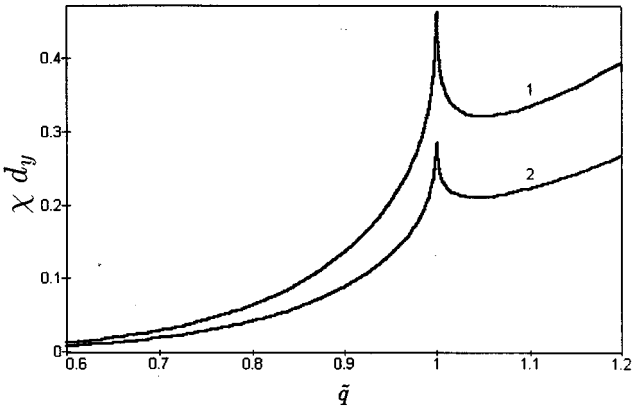

FIG. 3. The dimensionless supplementary localization, κd_y, of the excitation in the QW layer as a function of the dimensionless wave vector $\tilde{q}=q/2k_F$. Two values of the electron confinement are supposed: $1-d_y=150$ Å: $2-d_y=120$ Å.

$$\frac{\omega_{1D}-\omega_{2D}}{c_{2D}2k_F}=\frac{q-2k_F}{2k_F}-\frac{w}{\left(1+s/\ln\left|\dfrac{2k_F}{q-2k_F}\right|\right)^2},$$

$$|q-2k_F|\ll 2k_F, \tag{65}$$

where $\chi(2k_F)$ coincides with (62), and

$$s\equiv\frac{2k_Fg(q_{sc})}{q_{sc}g(2k_F)},\quad w\equiv\frac{c_l^2(\chi(2k_F))^2}{2\omega_{2D}^2}\approx\frac{(\chi(2k_F))^2}{2(2k_F)^2},$$

c_{2D} being phase velocity of the two-dimensional phonons (38) at $q=2k_F$. The formula (65) gives an approximate description of $\omega_{1D}(q)$ in this region of q. Note that in accordance with the above analysis, $w\ll 1$, but s can be greater or less than 1 and depends on the ratio k_F/q_{sc}. Equations (64), (65) show that at $q=2k_F$ the confinement reaches its maximum (see Fig. 3), while the dispersion curve has a minimum ω_m. The dispersion curves are presented in Fig. 4. The position q_M of maximum of $\omega_{1D}(q)$ and its magnitude can also be evaluated analytically in two limiting cases:

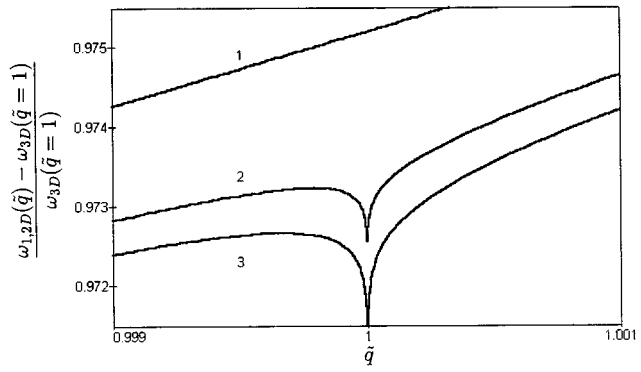

FIG. 4. Dispersion curves of 2D phonons (1) and 1D excitations (2,3) as functions of the dimensionless wave vector $\tilde{q}$. Curves 2,3 correspond to curves 1,2 of Fig. 3.

$$\frac{q_M-2k_F}{2k_F}\approx -2\,\frac{w}{s^2}\,\ln\frac{s^2}{2w},\quad \frac{\omega_M-\omega_m}{\omega_{2D}(2k_F)}\approx w,\quad \left(s\gg\ln\frac{s^2}{2w}\right); \tag{66}$$

$$\frac{q_M-2k_F}{2k_F}=-\frac{2sw}{|\ln 2sw|^2},\quad \frac{\omega_M-\omega_m}{\omega_{2D}(2k_F)}=\frac{2sw}{|\ln 2sw|}$$

$$(s\ll|\ln 2sw|). \tag{67}$$

Thus, one-dimensional collective excitation exhibits a nonmonotonous, rotonlike dispersion curve. The physical reason for this feature is the enhancement of effective electron-phonon coupling at the region near $q=2k_F$ due to the large electron polarization. This enhanced coupling leads to a greater localization effect in this region.

Another interesting behavior provoked by the large electron polarization is the peak(s) in the density of states of one-dimensional excitations. The density of states is an important characteristic of the phonons and is defined as the number of phonon states per unit interval of phonon frequency, per unit volume of the crystal. In our one-dimensional excitation case, we define the density of states $\varrho_{1D}(\omega)$ per unit length of the electron channel. Then, for the dispersion dependence $\omega_{1D}(q)$ the density of states is

$$\varrho_{1D}(\omega)=\frac{1}{\pi}\sum\left|\frac{dq}{d\omega_{1D}}\right|, \tag{68}$$

where the sum is calculated over all of the branches that exist at fixed ω. Outside the range of multivaluedness of $q(\omega)$ (see Fig. 4) there is only one branch of $\omega_{1D}(q)$. In the range $\omega_m<\omega<\omega_M$ there are three branches that have to be taken into consideration. Close to ω_m contributions from two additional branches are negligible [according to Eq. (65), for these branches $|dq/d\omega_{1D}|\to 0$ at $\omega\to+\omega_m$], but their contributions rise, when ω increases. While ω approaches ω_M, the contribution of the branch with $q>2k_F$ are finite, but two others lead to a divergence in the density of states $\varrho_{1D}\sim(\omega_M-\omega)^{-1/2}$. The density of states has a sharp, vertical boundary at $\omega\to+\omega_M$, then there is a finite drop that disappears outside the singular frequency range. The characteristic extension of the singular range is about $\omega_M-\omega_m$.

V. DISCUSSION

We have presented the results of a theoretical consideration of an electron system interacting with the phonons in a semiconductor heterostructures, where a buried layer forms a quantum well for the electrons and, simultaneously, localizes the acoustic waves within the layer. The latter means a splitting of a two-dimensional phonon branch $\omega_{2D}(q)$ from the bulk phonons $\omega_{3D}(q)$. It has been shown that additional confinement of the electrons in the second direction leads to the appearance of one-dimensional collective electron-phonon excitation $\omega_{1D}(q)$. This excitation consists of coupled lattice vibrations and an electron charge-density wave, accompanied by the electrostatic field. Because of the strong difference between phonon and electron velocities, the electrons adiabatically follow the lattice vibrations and, as a result, the coupled excitation has the dispersion curve close to the phonon one. In that sense this collective excitation can be considered as the phonon mode, localized in two directions, i.e., one-dimensional phonon branch.

The phonon localization in the direction perpendicular to the buried layer is mainly due to nonuniform lattice parameters of the heterostructure. The supplementary localization in the second direction is caused by electron-phonon interaction. The characteristic scale of the supplementary localization is always much larger than that originating from the lattice nonuniformity. This has allowed us to employ an adiabatical approximation and to formulate the task of the additional localization as an eigenstate problem of the one-dimensional differential equation. A solution of this problem gives spatial dependences of the deformation, electron density, electrostatic potential, and dispersion relation for the excitation.

The relations between spatial scales of the variables and the wave vector of the excitation q, directed along x axis, play an important role. The electrons are characterized by two geometrical scales d_y, d_z and the inverse Fermi vector k_F^{-1}. The value d_z coincides with the thickness of the buried layer and is considered sufficiently small to quantize the electrons in the layer so, that the electron gas is two dimensional. The scale d_y characterizes the restriction of the electron motion in the second direction by an external potential. It can be changed over a wide range and a crossover from a two-dimensional narrow channel to a one-dimensional electron system can occur. The electron concentration and, consequently, the Fermi wave vector k_F can, in principle, be controlled independently from the other parameters. Different scales result in a variety of possible physical situations.

One of them is the two-dimensional electron channel of a finite width with a quasiclassical motion of 2D electrons [the case of Eq. (16b)]. For this case splitting of the one-dimensional branch from two-dimensional phonons and supplementary localization increases continuously with the wave vector q. The screening parameter q_{sc} and the Fermi wave vector k_F determine q regions with a different behavior of parameters $\kappa(q)$ and $\chi(q)$ giving localization scales for z and y directions, respectively. At $q<q_{sc}$ we have obtained $\kappa\sim q^2$, $\chi\sim q^5$. In the region $q>q_{sc}$, k_F the results are κ, $\chi\sim q^2$. In the region of $q\approx 2k_F$ there are no special features of $\omega_{1D}(q)$, but the subsequent splitting of one-dimensional modes is possible.

Let us perform numerical estimates for the quasiclassical case at high wave vectors q. In order to satisfy the above assumed inequalities we choose a QW with a small width: $d_z=50$ Å. Then, we assume that electron concentration does not exceed $n_M=10^{12}$ cm^{-2}, i.e., $k_F(y)<k_{F,M}=2.5\times10^6$ cm^{-1} (only the lowest z subband is populated). For $q\ll 2k_{F,M}$ one can use ''the potential'' $v(y)$ localizing the excitation in form (47) with a flat bottom. Then, we set $b=15$ eV, $(\lambda+2\mu)=10^{12}$ dyn/cm^2, $|\lambda'|/[2(\lambda+2\mu)]=0.1$, $m^*=0.067m_0$, $\varepsilon_0=15$, $c_l=4.7\times10^5$ cm/s. As a result, for the depth of ''the potential'' one obtains $v_0=2.2\times10^{-15}q^4/(1+1.7\times10^6/q)$, where v_0 is in cm^{-2} and q is in cm^{-1}. For these values, at $q=10^6$ cm^{-1}, when the screening effect is still important, one finds $\kappa=5\times10^4$ cm^{-1}, $\omega_{3D}-\omega_{2D}=0.56$ GHz, $v_0=8.25\times10^7$ cm^{-2}. If the width of the channel $d_y=3.200$ Å, there is the single mode with $\omega_{2D}-\omega_{1D}=0.6$ MHz, and $\chi=4\times10^3$ cm^{-1}. All of the above assumptions are satisfied for these parameters. For larger q

one should specify the shape of the external potential $U(y)$. We set $U(y)-\nu=U_0[\mathrm{ch}(2y/d_y)-1]$, so that the width of the channel is d_y. Corresponding dimensionless ''potentials'' $\tilde{v}=v(y)d^2$ are presented in Fig. 2 for different values of $\tilde{q}=q/2k_{F,M}$. For $\tilde{q}=1$ electrostatic effects are neglected and $\tilde{v}$ localizes up to six modes. The lowest one is localized inside the channel and has the splitting $\omega_{2D}-\omega_{1D}=180$ MHz, while for the two-dimensional phonon we get $\kappa=2.6\times10^5$ cm^{-1}, $\omega_{3D}-\omega_{2D}=17$ GHz.

For the one-dimensional case the main conclusions presented in this section are valid. The supplementary localization of the single excitation is relatively weak and its characteristics are given by the Eq. (44). From Eqs. (43), (44), and (59) it follows that, if the dielectric constant of the material ε_0 is large and electrostatic effects are not essential, the localization parameter $\chi(q)$ increases infinitely in the region of $q\approx2k_{F,M}$ and $\omega_{1D}^2(q)$ can change its sign. The latter indicates well-known Peierls phase transition[28] to the state with periodical deformation of the medium along the axis of the electron channel. The distinction of this case from the usual Peierls model is that this transition would appear in the system of *three-dimensional elastic medium and one-dimensional electrons*. As a result, the periodical deformation would be also dependent on two other coordinates and would decay far away from the electron channel with one-dimensional electrons. Note that there exist quantum heterostructures, based on materials with large ε_0 (for example, A^4B^6 compounds with $\varepsilon_0=300...1000$). For A^3B^5 and A^2B^6 heterostructures the electrostatic effects prevent the phase transition. However, the system shows nonmonotonous, rotonlike behavior of the dispersion curve. This behavior is a rudiment of the Peierls phase transition and exists at low temperatures.

For numerical estimates of the one-dimensional case we need to specify the electron wave function $\Psi_1(y)$. Let us assume that the electrons are confined in the external potential $U(y)$ near its parabolic bottom. Thus, we can set $\Psi_1(y)=(4/\pi d_y^2)^{1/4}\exp(-2y^2/d_y^2)$, in which d_y characterizes a spreading of the square of wave function. Equation (55) gives $g(q)=(1/4\pi)K_0(q^2d_y^2/16)\exp(q^2d_y^2/16)$. In addition to the above parameters, let us assume $n_{1D}=1.5\times10^6$ cm^{-1} and $d_y=150$ Å, then one can obtain $k_F=2.4\times10^6$ cm^{-1}, $q_{sc}=1.4\times10^6$ cm^{-1}. The localization parameter and rotonlike behavior are shown in Figs. 3, 4. The splitting of one-dimensional excitation equals $\omega_{2D}(2k_F)-\omega_{1D}(2k_F)=20$ GHz. According to Sec. IV, in this range there is a peak of the density of states for the excitation. Thus, although the collective excitation of the system with one-dimensional electrons exists at arbitrary q, it is much more pronounced in the narrow region around $q=2k_F$.

One can compare the results of this paper with the effect of the electron-phonon interaction on phonon behavior for the cases of three- and two-dimensional electron subsystems. For the first case the main effect[35] is contained in a renormalization of the phonon frequency $\Delta\omega_{3D}$ proportional to $b^2n_{3D}/\lambda^{\frac{2}{3}}E_F$, where n_{3D}, E_F are the electron concentration and the Fermi energy, respectively. For the case under consideration the splitting down of one-dimensional branch is proportional to the fourth power of the coupling constant b and for certain situations is almost independent on the electron concentration.[36] Then, in bulk material the renormalization is weakly dependent on the phonon wave vector showing anomaly (Kohn anomaly[37,38]) only in derivative of ω_{3D} with respect to q: increasing $d\omega_{3D}/dq$ and diminishing the density of states around $2k_F$. The latter are just the opposite to the above analysis. For two-dimensional electron system the electrons split off the two-dimensional phonon branch and the splitting reaches a maximum at $q=2k_F$. Then, at $q>2k_F$ the branch approaches the bulk dispersion curve[20] which is also in opposition to the results of this paper.

For two cases, the narrow two-dimensional channel and strictly one-dimensional case, the collective excitation should manifest itself in different experiments. For the first case, the excitation can be studied by acoustic methods. The strong anisotropy and localization in a vicinity of the channel can be used to distinguish the one-dimensional branch from bulk and two-dimensional modes. Currently the interaction of the acoustic waves with low-dimensional electrons is actively studied.[39–41] By this method very small changes in the sound velocity caused by the electron system can be measured.[41] A high-frequency region is much more suitable for this acoustic measurements. A special technique was developed[42,43] that allows one to excite and detect short-wavelength acoustic vibrations in nanostructures[44] with frequencies up to several hundred GHz.

Using advanced Brillouin spectroscopy is also possible. By this technique different types of low-energy excitations can be detected. The maximal wave vector of phonons, which participates in the Brillouin process, is $q_{B,M}=4\pi N^R/\lambda_L$ (backward scattering), where λ_L and N^R are the wavelength and the refractive index, respectively. For typical wavelength $\lambda_L=5000$ Å and $N^R=3{,}5$, we obtain $q_B=8{,}8\times10^5$ cm^{-1}. Using the estimates given above, we can see that the two-dimensional phonon can be detected as the Stokes line in the Brillouin spectra (see, for example, Refs. 45 and 46). In the one-dimensional case the excitation will contribute to the red tail of this line. Note that because the scattering signal is expected to be small, multiple-quantum-well structures have to be used in such experiments. Other optical methods, such as high-order light scattering or disorder breakdown of the wave vector conservation, allow one a detection of excitations with $q>q_B$. For the one-dimensional case discussed above, the behavior of the density of states peculiar to a rotonlike dispersion curve can lead to an additional peak in these scattering spectra (by this method the roton feature for low-dimensional magnetoplasmons has been detected[33,34]).

VI. CONCLUSIONS

In a many-electron system interacting with a lattice, a crossover from two- to one-dimensional electron gas leads to the appearance of the collective excitation which consists of the lattice vibrations and electron charge-density wave accompanied by an electrostatic field. The excitation is localized in two directions and is one dimensional in character. The spectrum of this excitation is quite similar to that of the phonons. Therefore, the excitation can be considered as a one-dimensional phonon branch localized by electrons. The splitting of this branch and localization increase with the wave vector q of the excitation. The two-dimensional elec-

tron gas with a submicrometer width conductive channel almost localizes the excitation within the channel. For the one-dimensional electron gas the excitation, in principle, exists at any wave vectors, but the maximal splitting and the localization occur in the region $q \approx 2k_F$. In this region the dispersion curve of the excitation shows the rotonlike behavior and the density of states has a peak.

These effects augment the list of the many-body features peculiar to the dimensional crossover and one-dimensional electron gas of semiconductor heterostructures. The localization of the elastic wave within the region of the electron confinement is important for low-temperature electron transport, acoustic and acoustoelectronic phenomena.

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APPENDIX A

In order to derive Eq. (19) for the case when the electron motion in y direction can be considered as classical, note that in a semiclassical approximation matrix elements $\langle n|h(y)|n'\rangle$ are exponentially small if $n \neq n'$.[31] We assume that only one z subband with $l=1$ is populated. Then one can set $n=n'$ everywhere in Eq. (14), except the explicit dependence on n,n' in the matrix elements. In this case,

$$\delta n_s(x,y) \approx 2e^{iqx} \sum_{l,n,n'} \sum_{k} \Psi_n(y)\Psi_{n'}^*(y) \frac{f_0(\epsilon_{nlk+q}) - f_0(\epsilon_{nlk})}{\epsilon_{nlk+q} - \epsilon_{nlk}} \int dy' \Psi_n^*(y')\Psi_{n'}(y')h(y')$$

$$= 2e^{iqx} h(y) \sum_{n,l,k} |\Psi_n(y)|^2 \frac{f_0(\epsilon_{nlk+q}) - f_0(\epsilon_{nlk})}{\epsilon_{k+q} - \epsilon_k}. \tag{A1}$$

Since at point y the surface density of particles, having a wave vector k along the x axis, is given by

$$n_k(y) = 2\sum_{n,l} |\Psi_n(y)|^2 f_0(\epsilon_{n,l,k}), \tag{A2}$$

we obtain the form (19), where

$$P(y) = \sum_k \frac{n_{k+q}(y) - n_k(y)}{\epsilon_k - \epsilon_{k+q}}. \tag{A3}$$

For the total energy of a two-dimensional classical particle at point y, one can write $\epsilon(k_x,k_y,y) = (\hbar^2/2m)(k_x^2+k_y^2) + U(y)$, where $U(y)$ is an external potential. The number of particles at point y with a kinetic energy, $\hbar^2 k_x^2/2m$ can be presented in another form:

$$n_{k_x} = 2\sum_{k_y} f_{0,\mathbf{k}}(y), \quad f_{0,\mathbf{k}}(y) = \frac{1}{1+\exp[\nu - \epsilon(k_x,k_y,y)]/kT}. \tag{A4}$$

Substituting the last expression into formula (A3) and returning to notation $\epsilon_{\mathbf{k}} \equiv (\hbar^2/2m)(k_x^2+k_y^2)$, one can get formula (21), where $P_2(q|\nu)$ is the polarization operator of the 2D electron gas:

$$P_2(y) = 2\sum_{\mathbf{k}} \frac{f_{0,\mathbf{k}+\mathbf{q}}(y) - f_{0,\mathbf{k}}(y)}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{q}}}. \tag{A5}$$

Contrary to Eq. (A3), the summation in Eq. (A5) is taken over all possible values of the two-dimensional wave vector $\mathbf{k}$. For example, at $T=0$ one can set for the distribution function $f_{0,\mathbf{k}}(y) = \Theta[\nu - \epsilon_{\mathbf{k}} - U(y)]$, then the calculation of sum (A5) gives Eq. (46).

APPENDIX B

In the next step of the adiabatical approximation, substituting Eq. (31) into wave equation (22), we get the differential equation, which contains two unknown functions $A(y)$ and $u_1(y,z)$:

$$\{A''(y) + [\kappa^2(y) - \kappa^2]A(y)\} + \exp(\kappa(y)|z|)\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2} - \kappa^2\right) u_1(x,y)$$

$$= [A(y)|z|^2(\kappa'(y))^2 - 2A'(y)|z|\kappa'(y) - A(y)|z|\kappa''(y)] = 0. \tag{B1}$$

We can define the function $A(y)$ as the solution of the following equation:

$$A''(y) + [\kappa^2(y) - \kappa^2]A(y) = 0. \tag{B2}$$

Then, for $u_1(y,z)$ we can write

$$\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial y^2}-\kappa^2\right)u_1(x,y)=-e^{-\kappa(y)|z|}[A(y)|z|^2(\kappa'(y))^2-2A'(y)|z|\kappa'(y)-A(y)|z|\kappa''(y)]. \tag{B3}$$

The correction to the electrostatic potential Φ_1 can be obtained by substituting Eq. (32) into Eq. (24):

$$\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial y^2}-q^2\right)\Phi_1(y,z)=-B''(y)e^{-q|z|}. \tag{B4}$$

The matching conditions at $z=0$ for u_1 and Φ_1 follow from Eqs. (24), (25):

$$(\lambda+2\mu)\frac{\partial}{\partial z}u_1(y,z)|_{z=-0}^{z=+0}+\left(-\frac{\partial^2}{\partial y^2}+q^2\right)\{\lambda' d_z u_1(y,0)-bP(y)[bu_1(y,0)+\Phi_1(y,0)]\}$$

$$=-\lambda' d_z A''(y)+b\frac{\partial^2}{\partial y^2}P(y)[bA(y)+B(y)], \tag{B5}$$

$$\frac{\partial}{\partial z}u_1(y,z)|_{z=-0}^{z=+0}+\frac{4\pi e}{\varepsilon_0}P(y)[bu_1(y,0)+\Phi_1(y,0)]=0. \tag{B6}$$

The functions u_1,Φ_1 also have to satisfy conditions (26).

Equations (B3)–(B6) compose the system of inhomogeneous linear equations with boundary conditions. The inhomogeneous right-hand sides are determined by the solutions of the first approximation, u_0,Φ_0. One has to seek the particular solution of the system u_1,Φ_1 proportional to those inhomogeneous terms. Using the explicit forms for $A(y)$ and $B(y)$, it is possible to estimate corrections u_1 and Φ_1 induced by the right-hand sides of systems (B3)–(B6) and to find conditions, when these corrections are small.

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