Emission of transverse acoustic phonons by two-dimensional electrons due to heterointerface vibrations

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A mechanism of two-dimensional (2D) electron interaction with acoustic phonons due to heterointerface vibrations is predicted in quantum structures. This mechanism originates from the fact that acoustic phonons induce vibrations of heterointerface boundaries and, hence, the modulation of electron quantization energies. For such a mechanism the electron interaction with transverse acoustic modes is not forbidden. We have calculated the emission rates of transverse acoustic phonons by hot 2D electrons in quantum wells. The energy and angular distributions of emitted phonons via this "macroscopic deformation" mechanism of coupling are calculated. Comparison to the piezoelectric mechanism of phonon emission is presented.

The most important mechanism of electron scattering acoustic phonons is the deformation acoustic (DA) potential interaction.[1,2] In isotropic solid media a DA interaction is characterized by $D\mathrm{div}\mathbf{u}_\mathbf{r}$, where D is the deformation potential, and the divergence of displacement $\mathbf{u}_\mathbf{r}$ determines the relative change of crystal volume due to deformation. Therefore, in isotropic media, electrons can interact only with longitudinal acoustic modes via the deformation potential (the interaction with transverse acoustic phonons is possible only in anisotropic crystals[3]). The piezoelectric acoustic (PA) coupling of electrons with transverse acoustic phonons is relatively weak. The situation is different in heterostructures. Both longitudinal (l-) and transverse (t-) acoustic modes induce vibrations of heterointerfaces. These vibrations in quantum wells (QW's) will cause the time variations of the well width and, hence, the time variation of the quantization energy of two-dimensional (2D) electrons. This mechanism of electron interaction with acoustic phonons, which is due to macroscopic deformations of QW's but does not depend on D, has not yet been considered (see Refs. 1 and 2). Such "macroscopic deformation" acoustic coupling (which we will refer to as the MDA mechanism) between electrons and acoustic phonons is weak in comparison to ordinary DA coupling. Indeed, the variation in 2D electron quantization energy is of the order of $\varepsilon_0\delta u/d$ (where ε_0 is the position of the bottom 2D energy subband, δu is an average displacement of heterointerfaces, and d is the width of the QW), while the variation of the energy band due to deformation is of the order of $D\mathrm{div}\mathbf{u}_\mathbf{r}$. For rough estimates, we will assume that $\mathrm{div}\mathbf{u}_\mathbf{r}$ is of the order of δ/d; then the relative strength of MDA and DA electron-phonon mechanisms is characterized by ε_0 and D, respectively. Since $\varepsilon_0/D \ll 1$, DA interaction of electrons with acoustic phonons is much stronger than via MDA coupling. However, MDA scattering is qualitatively different from DA scattering and may be important, since it permits electron interaction with *transverse* acoustic phonons. Therefore, this additional mechanism should be compared to the PA interaction rather than to the DA interaction. Our estimates show that the MDA mechanism is dominant in narrow QW's and for electron interaction with high-energy (i.e., short-wavelength) acoustic phonons. Transverse acoustic phonons emitted by hot 2D electrons due to the MDA interaction can be detected experimentally, because the propagation conditions for l and t modes in a substrate are different and the spatial separation of l and t phonon fluxes is possible by the time-of-flight technique (see Refs. 4 and 5, and references therein for the description of acoustic-phonon emission by 2D electrons).

In the present paper we propose and analyze the mechanism of electron interaction with acoustic phonons, due to time variation of the QW width. We will consider a rectangular QW, where electrons occupy only the lowest subband which obeys the parabolic dispersion law

$$\varepsilon_0 + p^2/2m, \qquad \varepsilon_0 = (\pi/d)^2/2m, \tag{1}$$

where m is the isotropic effective mass. We assume that $\hbar$ and the normalization volume are equal to unity. We will neglect the modification of elastic parameters in double heterostructures and will use bulklike expressions for the displacement operator $\mathbf{u}_\mathbf{r}$. In the 2D approximation, the energy of the MDA interaction is determined by the shift in the ground state energy level, due to vibration of heterointerfaces:

$$\frac{2\varepsilon_0}{d}(\mathbf{n}\mathbf{u}_\mathbf{r})\Bigg|\begin{matrix} z = d/2, \\ z = -d/2, \end{matrix} \tag{2}$$

where $\mathbf{n}$ is the normal to a 2D layer.

By substituting the Fourier transform of the displacement into Eq. (2), we get the usual form for the operator of electron-acoustic phonon interaction,

$$\sum_{\mathbf{q},j}(\chi_{\mathbf{q},j}b_{\mathbf{q},j} + \text{H.c.}), \tag{3}$$

where $b_{\mathbf{q},j}$ ($b^\dagger_{\mathbf{q},j}$) is the annihilation (creation) operator for acoustic-phonon mode with wave vector $\mathbf{q}$ and polarization j ($j = l$ and $j = t_{1,2}$ for l and t modes, respectively). In the $\mathbf{p}$ representation ($\mathbf{p}$ stands for the 2D momentum), the electron contribution to expression (3) is

$$\langle \mathbf{p}'|\chi_{\mathbf{q},j}|\mathbf{p}\rangle = \delta_{\mathbf{p}'\mathbf{p}+\mathbf{q}_\parallel} \frac{i\sin(q_z d/2)}{\sqrt{2\rho\omega_{q,j}}} \frac{4\varepsilon_0}{qd} \times \begin{cases} -q_z, & j = l \\ q_\parallel, & j = t. \end{cases} \tag{4}$$

Here $\mathbf{q} = (\mathbf{q}_\parallel, q_z)$, ρ is the density of material, and polarization orts $\mathbf{e}_{\mathbf{q},j}$ are chosen in the following form:

$$\mathbf{e}_{\mathbf{q}l} = \mathbf{q}/q, \quad \mathbf{e}_{\mathbf{q}t_1} = (-q_y/q_\parallel, q_x/q_\parallel, 0),$$
$$\mathbf{e}_{\mathbf{q}t_2} = (-q_x q_z/qq_\parallel, -q_y q_z/qq_\parallel, q_\parallel/q). \tag{5}$$

We take into account the fact that the t_1 mode with polarization vector parallel to the heterointerface does not interact with 2D electrons. Thus, we evaluate the additional contribution of the MDA mechanism to the usual[6] matrix elements for electron interaction with l acoustic modes ($j = l$) (Ref. 7) and the new mechanism of electron interaction with t modes, due to heterointerface vibrations.

Following the approach of Ref. 8 for the description of acoustic-phonon emission by hot 2D electrons, we use the phonon kinetic equation

$$\mathbf{v}_{\mathbf{q}j} \frac{\partial N_{\mathbf{q},j}(\mathbf{r})}{\partial \mathbf{r}} = J_{\text{ph-ph}} \tag{6}$$

for semiclassical nonequilibrium phonon distribution $N_{\mathbf{q},j}(\mathbf{r})$ far away from the QW region. Here, $J_{\text{ph-ph}}$ is the phonon-phonon collision integral and $\mathbf{v}_{\mathbf{q},j} = \partial\omega_{\mathbf{q},j}/\partial\mathbf{q}$ is the group velocity of the jth mode. Near the QW region, Eq. (6) must be supplemented by the boundary condition

$$v_\perp N_{\mathbf{q},j}(\mathbf{r})|z = \pm d/2 = I_j(\mathbf{q}), \tag{7}$$

where $v_\perp$ is the normal component of the phonon group velocity and $I_j(\mathbf{q})$ stands for the surface rate of acoustic-phonon emission by the hot 2D electron gas, due to various mechanisms of electron-phonon interaction. In Eq. (7), we have omitted acoustic-phonon absorption, because we consider low-equilibrium lattice temperatures and do not consider phonon propagation along the interface of a QW. The surface rate of phonon emission $I_j(\mathbf{q})$ on the right-hand side of Eq. (7) for any mechanism of electron-phonon interaction is given by

$$I_j(\mathbf{q}) = 4\pi|C_j(\mathbf{q})|^2 \int \frac{d\mathbf{p}}{(2\pi)^2} f_\mathbf{p}(1 - f_{\mathbf{p}-\mathbf{q}_\parallel})\delta(\omega_{q,j} + \varepsilon_\mathbf{p} - \varepsilon_{\mathbf{p}-\mathbf{q}_\parallel}). \tag{8}$$

Here, $f_\mathbf{p}$ is the hot electron distribution function and the strength of the electron-phonon interaction $|C_j(\mathbf{q})|^2$ for the MDA and PA mechanisms can be expressed through the corresponding matrix elements [for the MDA mechanism the matrix element is given by Eq. (4)]. For t modes, we obtain the following form:

$$|C_t(\mathbf{q})|^2 = \frac{\sin^2(q_z d/2)}{\rho\omega_{q,j} d^2} \times \begin{cases} 8(\varepsilon_0 q_\parallel/q)^2, & \text{MDA} \\ a(\frac{ee_{14}}{\kappa})^2[\frac{2/q_z}{1-(q_z d/2\pi)^2}]^2, & \text{PA}, \end{cases} \tag{9}$$

where e is the electron charge, e_{14} is the piezoelectric constant, and κ is the static dielectric permittivity. For the PA mechanism, we do not take into account crystal anisotropy, so that we may use a spherical approximation, which gives $a = 32/35$.[9]

Owing to the cylindrical symmetry of the 2D system, the surface rate of acoustic-phonon emission depends on $q = |\mathbf{q}|$ or phonon frequency $\omega_j = s_j q$ (s_j being the sound velocity for the jth mode) and angle θ between the phonon wave vector $\mathbf{q}$ and the normal to heterointerface $\mathbf{n}$. For low temperatures we may neglect phonon-phonon collisions, so that the distribution of emitted phonons can be expressed through $I_j(\mathbf{q})$. It is convenient[10] to introduce the differential acoustic-phonon flux (i.e., energy per unit solid angle, per unit area, and per unit frequency interval),

$$\delta G_j(\omega, \theta) = (\omega/2\pi s_j)^3 I_j(\omega, \theta). \tag{10}$$

Then the angular distribution of emission intensity $(dG/d\Omega)_j$ is given by the integral $\int_0^\infty d\omega\delta G_j(\omega, \theta)$.

Let us calculate the frequency and angular distributions of emitted transverse acoustic phonons for MDA and PA interactions. We will consider the degenerate 2D electron gas with quasiequilibrium Fermi distribution function $f_\mathbf{p}$ for electron temperature T_e. For quantitative estimates, we will consider a GaAs QW of width $d = 50$ Å with a sheet electron concentration of 5×10^{11} cm^{-2} and electron temperature $T_e = 5$ K and 10 K. We assume that the equilibrium lattice temperature is much less than T_e, so that virtually all phonons emitted by 2D electrons are nonequilibrium.

Figure 1 demonstrates calculated energy dependences of the differential acoustic-phonon fluxes generated due to macroscopic deformation (δG_{MDA}) and piezoelectric (δG_{PA}) electron-phonon interactions. One can see from Fig. 1 that these dependences are considerably different because of the different scattering strengths given by Eq. (9). For the piezoelectric mechanism, $|C_j(\mathbf{q})|^2$ has a nonzero value at $q_z d = 2\pi$ and decreases rapidly for $q_z d \gg 1$. Therefore, nonmonotonic energy dependence is obtained only for the MDA mechanism. The temperature dependences of δG_{MDA} and δG_{PA} are different, so that the MDA contribution to phonon emission decreases when decreasing electron temperature more slowly than the piezoelectric contribution. The ratio of the characteristic intensity parameters ΔG_{MDA} and ΔG_{PA}, which determine the scale of differential acoustic-phonon fluxes in Fig. 1, can be expressed through ε_0 and the characteristic energy of piezoelectric interaction ε_{PA}:

$$\Delta G_{\text{MDA}}/\Delta G_{\text{PA}} = (\varepsilon_0/\varepsilon_{\text{PA}})^2, \; \varepsilon_{\text{PA}} = \sqrt{8/35}\frac{|e|e_{14}s_t}{\kappa T_e}. \tag{11}$$

For the above used parameter values, these energies are $\varepsilon_0 \approx 223$ meV and $\varepsilon_{\text{PA}} \approx 228$ meV. Hence, according to Fig. 1, the MDA mechanism is equally as important as the piezoelectric mechanism for $T_e = 10$ K and is dominant for $T_e = 5$ K in the angle range $\theta \geq 30° - 45°$ and in the photon energy range $\hbar\omega \geq 1 - 1.5$ meV. Parameter $2\varepsilon_0/D$ determines the the contribution of the MDA mechanism, with respect to the contribution of the DA mechanism to the generation of l modes. This parameter

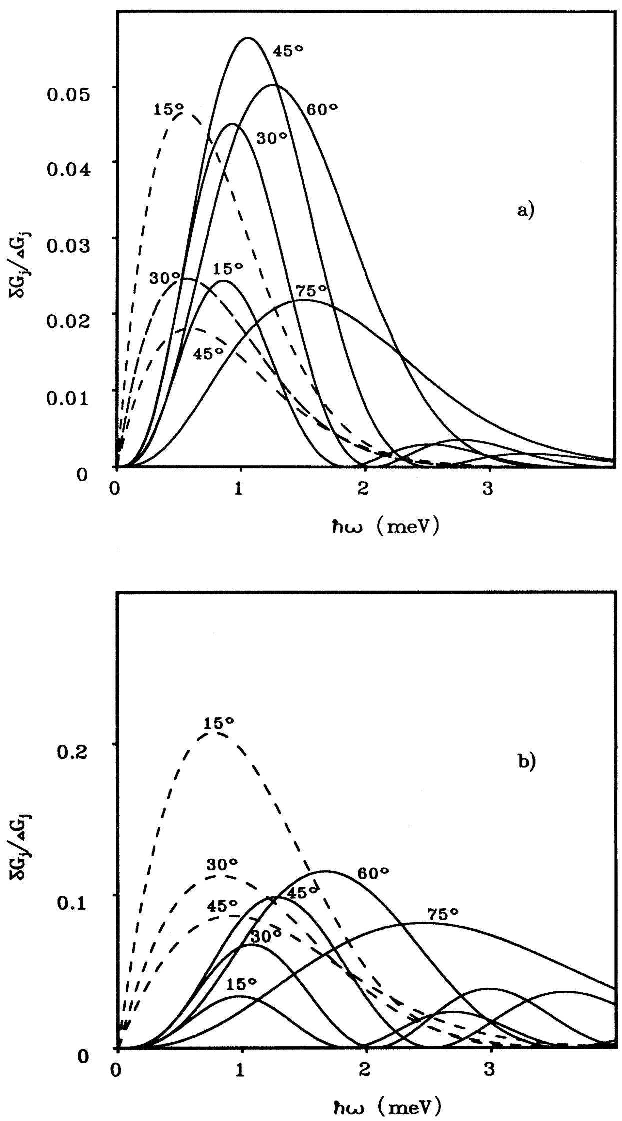

FIG. 1. The frequency dependences of the differential acoustic-phonon fluxes δG_{MDA} (solid curves) and δG_{PA} (dashed curves) for $T_e = 5$ K (a) and $T_e = 10$ K (b). Curves 1 correspond to angle $\theta = 15°$, 2 – 30°, 3 – 45°, 4 – 60°, and 5 – 75°.

is between 0.04 and 0.06, depending upon values of D taken from the literature.[11,12]

The angular dependences of δG_{MDA} and δG_{PA} are essentially different. The δG_{MDA} exhibits nonmonotonic angular dependence. The angular dependences of the total emission intensities $(dG/d\Omega)_{\mathrm{MDA,PA}}$ are plotted in Fig. 2 for two temperatures, $T_e = 10$ K and 5 K. As one can see from Fig. 2, the MDA mechanism exhibits nonmonotonic angular dependence and is more pronounced between 30° and 70°. The characteristic intensity of the MDA mechanism of emission $\Delta G_{\mathrm{MDA}} T_e$ is 0.35 mW/cm^2 for $T_e = 10$ K.

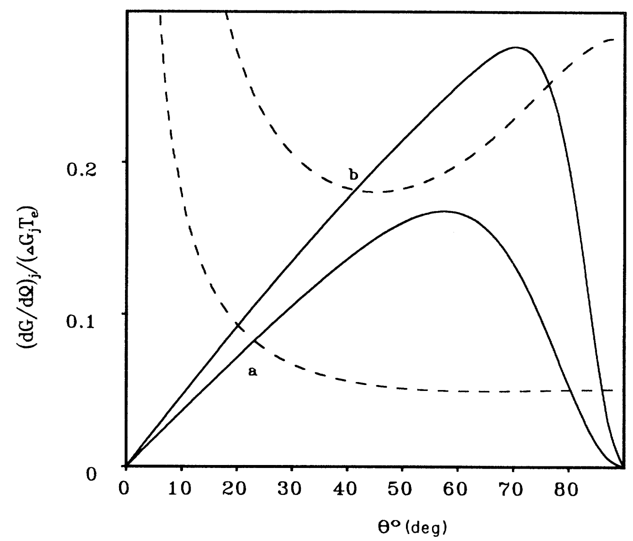

FIG. 2. The angular dependences of the emission intensities $(dG/d\Omega)_{\mathrm{MDA}}$ (solid curves) and $(dG/d\Omega)_{\mathrm{PA}}$ (dashed curves) for $T_e = 5$ K (a) and $T_e = 10$ K (b).

In conclusion, we have proposed a mechanism of electron interaction with acoustic phonons in quantum wells. This mechanism originates from the fact that acoustic phonons induce vibrations of heterointerface boundaries and, hence, the modulation of electron quantization energy. Unlike deformation potential scattering in isotropic crystals, this macroscopic deformation mechanism involves electron interaction with transverse acoustic modes. We have presented comparisons of the MDA mechanism with another mechanism involving transverse acoustic phonons, piezoelectric interaction. Let us list the assumptions made in our calculations. We have considered rectangular quantum well and bulklike acoustic phonons, and neglected the screening of electron-phonon interaction. We have also assumed that $q_{||}d < 1$ and neglected the modification of PA scattering due to crystal anisotropy. In addition, as for the usual consideration of electron mobility due to acoustic-phonon scattering in GaAs/Al$_x$Ga$_{1-x}$As heterostructures, we have neglected the modification of elastic parameters and reconstruction of the displacement field. All these simplifications do not change our results qualitatively. Our estimates demonstrate that the MDA scattering mechanism dominates over PA scattering at low temperatures and in narrow quantum wells and is important for the description of kinetic phenomena (for instance, low temperature mobility[10] in narrow QW's. The emission of acoustic phonons due to the MDA mechanism can possibly be detected by the time-of-flight technique, because the frequency and angular distributions of phonons are considerably different from phonon emission characteristics due to the PA interaction.

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