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Electron streaming caused by inelastic acoustic-phonon scattering in quantum wires

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A nonlinear regime of electron transport in quasi-one-dimensional quantum wires has been predicted and investigated by the Monte Carlo technique. The electron transport in this regime closely resembles electron streaming. However, in contrast to conventional electron streaming caused by periodic emission of optical phonons, the streaming reported here is due to periodic acoustic-phonon emission. This effect originates from strongly inelastic acoustic-phonon scattering in quantum wires. The streaming occurs at low temperatures in a wide range of electric fields and is characterized by an oscillating-velocity autocorrelation function and a nonlinear velocity-field dependence. Both the analytical model and the Monte Carlo simulations yield an $E^{1/5}$ field dependence of the drift velocity and an $E^{2/5}$ dependence of the mean energy as a function of electric field E in the streaming regime.

I. INTRODUCTION

The elasticity of electron scattering by acoustic phonons is a commonly used approximation. A closer look shows[1] that in contrast to bulk materials the electron scattering by acoustic phonons in quasi-one-dimensional (1D) quantum wire (QWI) structures becomes essentially inelastic. The point is that due to the lack of translational symmetry in perpendicular directions to a QWI structure, the momentum conservation for the electron–acoustic-phonon system is preserved only with an accuracy of $2\pi\hbar/L$, where L is the effective thickness of the structure $L^{-2}=L_y^{-2}+L_z^{-2}$. For example, in GaAs QWI with $L_y=L_z=80$ Å and sound velocity $u=5.2\times10^5$ cm/sec the phonon energy $2\pi\hbar u/L$ corresponding to momentum uncertainty is equal to 3.8 meV, i.e., this energy corresponds to the thermal-equilibrium 1D electron energy at $T=88$ K. Thus over a wide range of system parameters an electron can absorb or emit an acoustic phonon with energy comparable to its own. The electron–acoustic-phonon scattering turns out to be essentially inelastic, and becomes an effective mechanism of energy dissipation. The inelasticity of acoustic-phonon scattering not only influences hot-electron relaxation in QWI's (Ref. 2) but can also strongly affect electron transport and noise in QWI's.

The goal of this paper is to demonstrate that the accurate allowance for the inelasticity not only introduces some quantitative corrections to electron transport characteristics but leads to qualitatively new effects, namely the streaming due to emission of acoustic phonons. The paper is organized as follows. In Sec. II we present an analysis of electron streaming due to acoustic-phonon emission based on a simplified model. Simple analytical velocity-field and energy-field relationships are derived. Section III presents results of a Monte Carlo simulation of electron transport controlled by acoustic-phonon scattering in various QWI's. A summary is given in Sec. IV.

II. ANALYTICAL APPROACH

In order to analyze the electron transport, we need to know electron-scattering probabilities in QWI's. Let us first discuss the peculiarities of electron scattering by acoustic phonons revealed in Ref. 1. Figure 1 demonstrates acoustic-phonon-scattering rates versus electron energy for two cross sections of QWI's. Acoustic-phonon emission and absorption rates are plotted separately. As one can see from Fig. 1, the absorption rate depends monotonously on electron energy ϵ. The emission rate, however, has a nonmonotonous dependence on electron energy. It increases when increasing the electron energy from zero to the critical energy $\epsilon_c\approx2\pi\hbar u/L$ determined by the uncertainty of momentum conservation. Thus ϵ_c is defined solely by the thickness of a QWI. Above ϵ_c

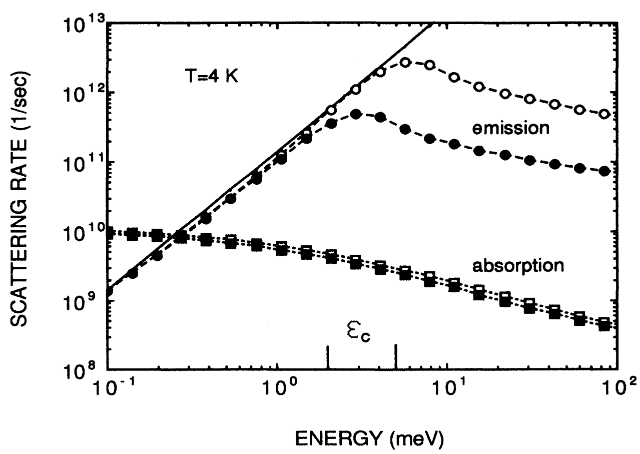

FIG. 1. Acoustic-phonon-scattering rate as a function of electron energy in GaAs QWI of cross sections 40×40 (open dots) and 80×80 Å^2 (full dots) at $T=4$ K temperature. Straight line represents ϵ^2 approximation for emission rate at low energies.

emission and absorption rates decrease nearly as $\epsilon^{-1/2}$. In the present paper we will deal with low electron energies where inelasticity of acoustic-phonon scattering is important. At very low energies ($\epsilon \ll k_B T, \epsilon_c$) the emission rate increases as $\epsilon^{3/2}$,[1]

$$\lambda(\epsilon)=\frac{8E_a^2 k_B T}{9\pi\rho u^4 \hbar^4}\left[\frac{m^*}{2}\right]^{1/2}\epsilon^{3/2}, \quad (1)$$

where E_a^2 is the deformation acoustic potential, ρ is the density of the material, and m^* is the effective mass in the material of a QWI. One can see from Eq. (1) that in this range of electron energies the emission rate does not depend on the cross section of a QWI. In the range of low to intermediate electron energies, $k_B T \leq \epsilon < \epsilon_c$, the power index increases. Overall increase of emission rate below ϵ_c can be well approximated by the ϵ^2 function as shown in Fig. 1:

$$\lambda(\epsilon)=\Lambda\epsilon^2, \quad (2)$$

where Λ is a constant independent of the cross section of a QWI.

Figure 2 demonstrates the acoustic-phonon emission probability in a QWI versus phonon energy for two different electron energies. One can see from Fig. 2 that the probability of acoustic-phonon emission in QWI's for electrons with $\epsilon > \epsilon_c$ has a maximum at acoustic-phonon energies equal to ϵ_c. Therefore, electrons with $\epsilon > \epsilon_c$ predominantly emit acoustic phonons of an energy close to ϵ_c. For electrons with $\epsilon < \epsilon_c$ the emission probability has a maximum at energy ϵ and is equal to zero above ϵ, because electrons cannot emit phonons with an energy greater then their own. Therefore, electrons with $\epsilon < \epsilon_c$ tend to emit phonons with energy ϵ and are scattered by this emission to the subband bottom. The scattering to or close to the subband bottom is a necessary condition for the electron-transport regime which we will consider here.

In an electric field sufficient to break the balance between acoustic-phonon emission and absorption processes, the electrons are accelerated up to a certain energy ϵ_{max} defined by that field. If $\epsilon_{max} < \epsilon_c$, electrons are scattered down to the subband bottom by the emission of acoustic phonon. This cycle repeats periodically with the period defined by the electric field. A similar transport regime, so-called electron streaming, is well known in the case of electron inelastic interaction with optical phonons,[3–6] but has never been observed in the case of electron interaction with acoustic phonons. Note that in higher electric fields, where the electron energy exceeds ϵ_c, the acoustic-phonon emission still prevails over absorption (see Fig. 1), but the emitted phonon energy is not sufficient to scatter electrons to the subband bottom. As a result, scattered electrons are equally distributed over positive and negative velocities, so that their periodic motion and the streaming breaks down.

The conventional streaming due to optical phonons can be realized if certain conditions are met:[4] (i) the temperature must be low enough, generally $k_B T \ll \hbar\omega_0$, where $\hbar\omega_0$ is the phonon energy, so that phonon absorption processes are frozen out; (ii) the phonon emission rate must exceed all other scattering rates near the emission threshold; (iii) the electric field should be strong enough to accelerate an electron up to the phonon emission threshold without scattering, but weak enough to avoid deep electron penetration beyond the emission threshold and thus to assure scattering by phonon emission down to the conduction-band bottom.

In the case of electron streaming due to periodic acoustic-phonon emission the phonon energy is not a fixed parameter and, hence, there is no energy threshold for acoustic-phonon emission. The first two conditions, however, are generally fulfilled for acoustic-phonon scattering if $\epsilon_c \gg k_B T$. Let us estimate the range of electric fields $E_{min} \ll E < E_{max}$, where the streaming due to acoustic-phonon emission may occur. First, we define the "passive region" as the energy range where the acoustic-phonon emission rate is less than the absorption rate (see Fig. 1). By requiring that the electron acceleration time through the "passive region" be much less than the absorption time, we obtain the lower-field limit E_{min}. The condition $\hbar\omega < \epsilon_c$ sets the upper-field limit of E_{max}. The lower-field limit E_{min} weakly depends on the cross section and is approximately equal to 1 V/cm. The upper limit is around 200 V/cm for a 40×40-Å^2 QWI, 35 V/cm for an 80×80-Å^2 QWI, and 4 V/cm for a 250×150-Å^2 QWI, i.e., for the latter case the lower and upper limits almost overlap, and it is not expected to observe the streaming in this thick QWI's. Thus the most favorable (but not unique) conditions for electron streaming due to acoustic-phonon emission can be met in thin QWI's with large values of ϵ_c.

Let us consider an idealized model in order to obtain simple analytical expressions. We neglect acoustic-phonon absorption and assume that electrons are not heated beyond ϵ_c and are scattered exactly to the subband bottom after emission of acoustic phonons. The mean free flight time $\langle\tau\rangle$ generally reads as

$$\langle\tau\rangle=\int_0^\infty d\tau\,\tau\lambda[\epsilon(\tau)]\exp\left[-\int_0^\tau dt\,\lambda[\epsilon(t)]\right], \quad (3)$$

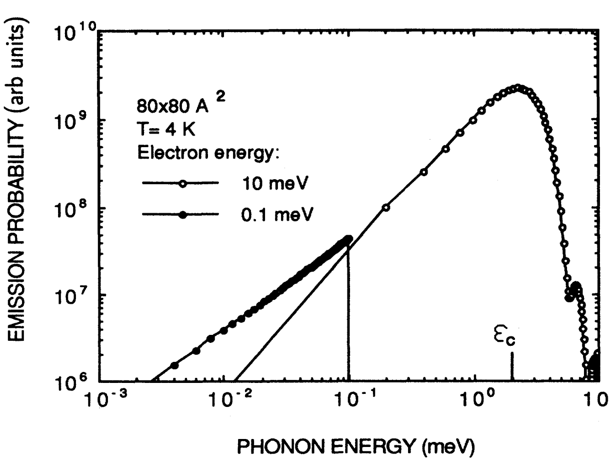

FIG. 2. Acoustic-phonon emission probability as a function of acoustic-phonon energy for a QWI of cross section 80×80 Å^2 and for two electron energies: 10 (open dots) and 0.1 meV (full dots).

where λ is the total scattering rate, in our case equal to the acoustic-phonon emission rate given by Eq. (2). In an electric field, electron momentum during free flight is governed by $dp/dt = eE$. Substituting energy expressed through momentum $\epsilon = p^2/2m^*$ and assuming that an electron after a free flight is scattered exactly to the subband bottom, we obtain the solution in the form of $\epsilon = (eEt)^2/2m^*$. Substituting it into Eq. (2) and then Eq. (2) into Eq. (3), and performing integration, we obtain

$$\langle \tau \rangle = \left[\frac{5}{\Lambda \mathcal{C}^4 E^4} \right]^{1/5} \Gamma , \tag{4}$$

where $\mathcal{C} = e/\sqrt{2m^*}$, and const $\Gamma = \Gamma(\frac{6}{5}) \approx 0.9182$ is the value of the gamma function. Then averaging energy $\epsilon(t)$ over the mean free flight $0 - \langle \tau \rangle$, we find the mean electron energy

$$\langle \epsilon \rangle = \frac{\hbar\omega}{3} , \tag{5}$$

where $\hbar\omega$ is the average acoustic-phonon energy emitted by electrons:

$$\hbar\omega = \left[\frac{5\mathcal{C}}{\Lambda} \right]^{2/5} \Gamma^2 E^{2/5} . \tag{6}$$

Similarly, averaging instant electron velocity over $\langle \tau \rangle$, the drift velocity is obtained:

$$v_d = \left[\frac{\hbar\omega}{2m^*} \right]^{1/2} . \tag{7}$$

Hence the mean electron energy is an $E^{2/5}$ function, and the drift velocity is an $E^{1/5}$ function of the electric field. The drift velocity and the mean electron energy are simply related to each other: $\langle \epsilon \rangle = \frac{2}{3} m^* v_d^2$. Relationships (5) and (7) are the same as for conventional streaming due to optical-phonon emission,[3,6] but the characteristic acoustic-phonon energy (6), unlike the optical-phonon energy, depends on the electric field. Therefore, in contrast to conventional streaming, where v_d and $\langle \epsilon \rangle$ saturate,[3,6] the streaming due to acoustic-phonon emission leads to field-dependent v_d and $\langle \epsilon \rangle$.

III. MONTE CARLO SIMULATIONS

To verify our analytic approach we have carried out Monte Carlo simulations of electron transport in a wide range of electric fields. We have used a simplified model in our simulations. The limitations of this model are discussed in Sec. IV. We have considered rectangular GaAs QWI's embedded in AlAs with infinitely deep potential wells for electrons. We have assumed a nondegenerate electron gas. Electron scattering by confined longitudinal-optical (LO) phonons and localized interface (surface) SO phonons[7,8] as well as by bulklike acoustic phonons[1] has been taken into account in our model. Our model incorporates as many subbands as are actually occupied by electrons. Ionized impurities are assumed to be located sufficiently far from the QWI, so that their influence on the electron motion inside the wire is assumed to be negligible. We have also assumed that the QWI is perfect, and have not taken into account electron scattering on interface roughnesses. We have chosen several different cross sections of a QWI, from rather thick 250×150-Å^2 QWI, where the separation between the two lowest subbands is less than the optical-phonon energy, to an unrealistically thin 40×40-Å^2 QWI, which represents the extreme limit. The inelasticity of acoustic-phonon scattering has been taken into account using a numerical procedure developed in Ref. 6. Simulations have been performed for temperature $T = 4$ K.

Figure 3 demonstrates the electron drift velocity as a function of electric field calculated by the Monte Carlo technique. There are four distinguishable regions on the velocity-field dependence. The near-ohmic velocity-field dependence in a field range below 1 V/cm turns into a sublinear dependence. Then the slope again increases and again decreases approaching saturation at high electric fields. The sublinear dependence extends through two orders of magnitude in electric fields in a 40×40-Å^2 QWI (2–200 V/cm), and appears just as a small kink in a 250×150-Å^2 QWI at around 2 V/cm. Our study will focus on this region of electric fields, where electron transport is controlled by acoustic-phonon emission. In the field range of 5–200 V/cm in 40×40 Å^2, electron drift velocity increases near the $E^{1/5}$ (and mean electron energy near $E^{2/5}$) function of the electric field as is predicted by Eqs. (6) and (7). Note that this field range coincides with the above estimated range 1 V/cm $\ll E < 200$ V/cm for this QWI, where electron streaming due to acoustic-phonon emission occurs. At high electric fields the optical-phonon emission starts to dominate and the drift velocity and mean energy saturate.[3,6] The transition from the acoustic-phonon-controlled electron transport to the optical-phonon-controlled transport occurs at lower electric fields in thick QWI's, where acoustic-phonon-scattering rate is lower.[1]

Our Monte Carlo simulations have confirmed that the emission of acoustic phonons is the sole scattering mechanism in the field range of 10–200 V/cm. Strong emis-

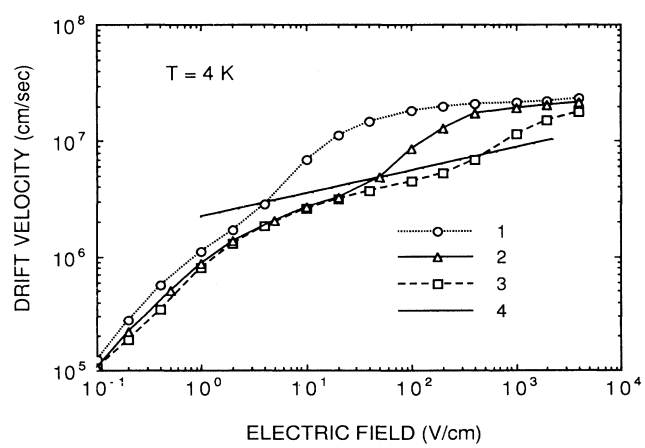

FIG. 3. Electron drift velocity vs applied electric field. Curve 1 represents velocity for 250×150-Å^2 QWI, curve 2 80×80-Å^2 QWI, and curve 3 40×40-Å^2 QWI; curve 4 represents analytical dependence given by Eqs. (7) and (6).

sion of acoustic phonons suggests that QWI's should radiate nonequilibrium acoustic phonons in the streaming regime. The fields of about 300 V/cm and up are strong enough to heat the electrons up to the lowest optical-phonon energy (in our case, the GaAs-like interface mode energy equals 34.5 meV). With further increase of electric field the LO-phonon scattering comes into play, and thus the electron transport in the fields exceeding 400 V/cm is largely controlled by optical-phonon scattering.

IV. SUMMARY

We have presented and investigated a qualitatively new regime of electron transport in a QWI, which closely resembles electron streaming due to acoustic-phonon emission. In the streaming due to optical-phonon emission the velocity-field and energy-field dependences saturate, owing to the fact that electrons cannot exceed an optical-phonon energy which is independent of the electric field. In the case of streaming due to acoustic-phonon emission the maximum energy attainable by electrons increases with the increase of electric field, and so does the drift velocity and the mean electron energy. Our transparent analytical model explains the major trends of velocity-field and energy-field dependences. Although the streaming is well pronounced only in very thin QWI's, the inelasticity of acoustic-phonon scattering is important, and leads to sublinear electron transport even in rather thick QWI's.

Our goal was to demonstrate the principal possibility of electron streaming due to acoustic-phonon emission. Therefore, in our Monte Carlo calculations we have used a simplified model of a QWI with perfect interfaces and an infinitely deep potential well for electrons. We have neglected electron scattering by ionized impurities and electron-electron interaction. Nevertheless, we believe that streaming due to acoustic-phonon emission is possible in real QWI's. There are several reasons for this. Electron energy in the transport regime considered here never exceeds the LO-phonon energy, which is 36.6 meV in GaAs. On the other hand, the barrier height in GaAs/AlAs structures is of the order of several hundreds of millielectronvolts, so that the barriers may be assumed to be infinitely high for electrons in the field range considered here. Ionized impurity scattering and electron pair collisions are important in QWI's with multisubband structures since they cause intersubband transitions of electrons. In thin QWI's such as 80×80 Å^2, only the lowest subband is occupied by electrons. Therefore, electron-electron scattering is not important at all, and the role of ionized impurity scattering is substantially diminished. The electron scattering by interface roughnesses may be very important in real QWI's.[8,9] However, this scattering depends on the way a concrete structure has been fabricated and cannot be quantitatively considered in the general case. The slight variation in the subband positions caused by the variation of the cross section of a QWI (Ref. 10) is not important for the streaming regime considered here, because electrons in this regime will be scattered to the subband bottom by the emission of acoustic phonons. A significant simplification of our model is that we have considered a nondegenerate electron gas. At temperatures as low as 4 K even moderate concentrations of electrons lead to degeneracy. The simulations within more realistic models will be carried out and published elsewhere.

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