Triple-electron collisions in a quantum wire

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The effect of intra-subband triple-electron collisions on the kinetics of a nondegenerate one-dimensional electron gas (1DEG) is investigated since binary collisions give zero contribution to the relaxation. The scattering probabilities are derived from the Lippmann-Schwinger equation for the transition matrix. The corresponding relaxation frequency is of the order of $\eta \nu_T \min[1, \left( e^2 / \varepsilon_L \nu_T \right)^3]$, where $\eta = e^2 n / \varepsilon_L T$ is the plasma parameter, $n$, $T$, and $\nu_T$ are the concentration, temperature, and thermal velocity of the 1DEG, respectively, and $\varepsilon_L$ is the dielectric constant. The relaxation frequency can reach a value of $10^{12}$/s, comparable to that by other scattering mechanisms, and indicates the importance of the effect for the relaxation of a 1DEG.

I. INTRODUCTION

It is well known that electron-electron collisions do not change the total momentum and energy of the electron system, but lead to a redistribution of electrons in phase space, thus controlling the form of the distribution function. This indirect effect has a strong influence on transport, noise, thermal relaxation, and quantum interference of charge carriers in semiconductor structures. The process of electron-electron scattering has been extensively studied in bulk [three-dimensional (3D)] and two-dimensional (2D) systems by experimental, analytical, and Monte Carlo methods.1–7 Recently the problem of the energy and momentum exchange between different electron (hole) subsystems (in general, spatially separated) has received much attention.8–10 In quantum wires the binary collisions can occur between carriers from different subbands11 (inter-subband scattering).

The concept of pair collisions in a system of identical particles is no longer valid for the intrasubband scattering in quantum wires. In one-dimensional (1D) systems the momentum and energy conservation laws do not allow binary collisions in a system of identical particles since the final state is physically indistinguishable from the initial one. So far, this restriction has been removed by considering Coulomb scattering between several different subsystems of the particles: that between different types of carriers (e.g., electron-hole interaction), and between electrons with opposite spins allowing for a small spin splitting of the conduction band due to the lack of inversion symmetry in the III-V group compounds.12

In this paper we consider a different possibility, namely, the triple-electron intrasubband collisions between identical particles. In 3D and 2D systems the collisions between three, four, and higher numbers of particles give a contribution proportional to the corresponding power of the plasma parameter (the ratio of the average potential and kinetic energies), which is usually small. In the 1D limit three-particle collisions give the first nonzero contribution to the kinetics of the system of identical particles.

A similar situation occurs for bulk electrons under a strong ultraquantizing magnetic field, where electron motion becomes essentially one dimensional. For bulk structures, however, an impurity concentration is as a rule larger than or equal to that of the electrons, and the effect of electron-electron-impurity scattering can dominate. The influence of the latter mechanism on the energy redistribution in a 3D electron system was considered in Refs. 13 and 14.

The kinetic equation that describes three-particle scattering, including exchange and degeneracy effects, has been derived in Refs. 15–17 within the formalism of nonequilibrium Green functions and applied to the problem of interaction between free electrons and electron-ion bound states in a dense hydrogen plasma. For a nondegenerate electron gas an identical kinetic equation can be obtained using a physically more transparent formalism of nonrelativistic scattering theory.18

In this paper we study the influence of triple-electron collisions on the relaxation of the nondegenerate one-dimensional electron gas (1DEG). Section II presents the kinetic equation and the transition probabilities and Sec. III the characteristic time of relaxation for two opposite electron beams. A discussion follows in Sec. IV. The Appendix details a convenient coordinate transformation in phase space.

II. KINETIC EQUATION AND TRANSITION PROBABILITY

We consider a system of nondegenerate electrons occupying the lowest subband of a quantum wire directed along the $x$ axis. The Hamiltonian of three interacting electrons is
\[ H = H_0 + V. \]  
(1)

Here $H_0$ is the kinetic energy and $V$ is the interaction

\[ V = \frac{e^2}{\varepsilon L} \left[ \frac{1}{|r_1 - r_2|} + \frac{1}{|r_1 - r_3|} + \frac{1}{|r_2 - r_3|} \right], \]  
(2)

where $r = (x, \rho) = (x, y, z)$, and $\varepsilon L$ is the lattice dielectric constant.

Denoting the transverse one-electron wave functions by $\chi(\rho)$, we present the three-particle wave function of the unperturbed Hamiltonian $H_0$ in the symmetrized form corresponding to the spinless approximation.\(^{19}\)

\[ \langle r_1 r_2 r_3 | \alpha \rangle = \frac{1}{\sqrt{6L^3}} \left[ e^{i(k_1 x_1+k_2 x_2+k_3 x_3)} + \text{perm}(k) \right] \times \chi(\rho_1) \chi(\rho_2) \chi(\rho_3), \]  
(3)

where $\alpha \equiv (k_1 k_2 k_3)$, and $\text{perm}(k)$ stands for the $6 - 1$ remaining permutations in the triad $(k_1 k_2 k_3)$.

The kinetic equation for the one-particle distribution function $f_k$ of a homogeneous nondegenerate 1DEG has the form\(^{15-17}\)

\[ \frac{\partial f_{k_1}}{\partial t} = \sum_{k_2 k_3 k_1' k_2' k_3'} W_{k_1 k_2 k_3 \rightarrow k_1' k_2' k_3'} \times \left( f_{k_1'} f_{k_2'} f_{k_3'} - f_{k_1} f_{k_2} f_{k_3} \right). \]  
(4)

We normalize $f_k$ by the condition

\[ \sum_k f_k = N, \]  
(5)

where $N$ is the total number of particles.

The transition probability in Eq. (4) is given by\(^{15,18}\)

\[ W_{k_1 k_2 k_3 \rightarrow k_1' k_2' k_3'} = \frac{2\pi}{\hbar} \left| \langle k_1' k_2' k_3' | T | k_1 k_2 k_3 \rangle \right|^2 \times \delta \left( \epsilon_{k_1'} + \epsilon_{k_2'} + \epsilon_{k_3'} - \epsilon_{k_1} - \epsilon_{k_2} - \epsilon_{k_3} \right), \]  
(6)

where $\epsilon_k = \hbar^2 k^2/2m$. The transition matrix $T$ satisfies the Lippmann-Schwinger equation\(^{18}\)

\[ T = V + V G_0 T, \]  
(7)

where

\[ G_0(\epsilon) = \frac{1}{\epsilon - H_0 + i0} \]

is the unperturbed Green's function.

We present the solution of Eq. (7) as an expansion in powers of the perturbation $V$:

\[ T_{\alpha \alpha'} = V_{\alpha \alpha'} + \sum_{\alpha''} V_{\alpha \alpha''} V_{\alpha'' \alpha'} + \cdots. \]  
(8)

Calculation of the matrix elements of $V$ with the symmetrized wave functions (3) gives the result

\[ V_{\alpha \alpha'} = \frac{e^2}{L \varepsilon L} \left[ \delta_{k_1 + k_2, k_1' + k_2'} \delta_{k_3, k_3'} F(k_1 - k_1') + \text{perm}(k, k') \right], \]  
(9)

where $\text{perm}(k, k')$ stands for a total of $36 - 1$ remaining permutations in the triads $(k_1 k_2 k_3)$ and $(k_1' k_2' k_3')$, and $L$ is the length of the wire. The form factor $F(\kappa)$ is given by

\[ F(\kappa) \equiv \frac{1}{2\pi} \int \frac{d^2 q}{q^2 + \kappa^2} |\chi(q)|^2 \]  
with

\[ \chi(q) = \int d^2 \rho |\chi(\rho)|^2 \exp[iq \cdot \rho]. \]

The first term in the expansion (8) contains only terms proportional to the $\delta$ functions which can be obtained by permutations in $\delta_{k_1 + k_2, k_1' + k_2'} \delta_{k_3, k_3'}$, and corresponds to the scattering of two particles with momenta $k_1$ and $k_2$, while the third particle is not involved in the collision. The second term in Eq. (8) also contains a part with the same product of two $\delta$ functions giving the next-order contribution in the two-particle scattering. However, due to the momentum and energy conservation laws and identity of particles, the two-particle interaction, to all orders of perturbation, does not give any contribution to the right hand side of kinetic equation (4).

The second term in Eq. (8) is the lowest one in the perturbation series that contains a part proportional to $\delta_{k_1 + k_2 + k_3, k_1' + k_2' + k_3'}$. The latter corresponds to the true three-particle scattering and gives the first nonzero contribution to the collision integral in (4). With the help of Eqs. (8) and (9) we obtain the following expression for the transition matrix, describing three-particle collisions, to the lowest order of perturbation theory:

\[ T^{(3)}_{\alpha \alpha'} = \delta_{k_1 + k_2 + k_3, k_1' + k_2' + k_3'} \left( \frac{e^2}{L \varepsilon L} \right)^2 \frac{6m}{\hbar^2} \times \left\{ \frac{F(k_1 - k_1') F(k_2 - k_2')}{(k_1 - k_1')(k_2 - k_2')} + \text{perm}(k, k') \right\}. \]  
(11)

Note that the integrations of expressions containing $T^{(3)}_{\alpha \alpha'}$ should be performed in the principal value sense: the imaginary part in Eq. (8) contributes to the two-particle interaction.

In order to obtain a convenient expression for the transition probability $W$ we substitute Eq. (11) into Eq. (6) and apply the transformation of phase space coordinates of the three interacting particles (see the Appendix). We obtain

\[ W_{k_1 k_2 k_3 \rightarrow k_1' k_2' k_3'} = -\frac{\pi}{2\hbar^2} \left( \frac{2\pi}{L} \right)^5 \left( \frac{e^2}{\pi \varepsilon L} \right)^4 \times W(\Delta, \varphi, \varphi') \delta(K - K') \delta(\epsilon - \epsilon'), \]  
(12)

where

\[ W(\Delta, \varphi, \varphi') = \Delta^{-4} \left\{ \frac{F\sqrt{2\Delta}(\cos \varphi - \cos \varphi')}{} F\left\{ \sqrt{2\Delta}[\cos(\varphi - 2\pi/3) - \cos(\varphi' - 2\pi/3)]\right\} \right\} \times (\cos \varphi - \cos(\varphi' - 2\pi/3)). \]  

\[ \left\{ \cos \varphi - \cos(\varphi' - 2\pi/3) \right\} \times \delta(K - K') \delta(\epsilon - \epsilon'), \]  
(12)

where

\[ W(\Delta, \varphi, \varphi') = \Delta^{-4} \left\{ \frac{F\sqrt{2\Delta}(\cos \varphi - \cos \varphi')}{} F\left\{ \sqrt{2\Delta}[\cos(\varphi - 2\pi/3) - \cos(\varphi' - 2\pi/3)]\right\} \right\} \times (\cos \varphi - \cos(\varphi' - 2\pi/3)). \]  
(12)
Here \( \text{perm}(\varphi, \varphi') \) stands for \( 36 - 1 \) remaining permutations in variables \( \varphi \) and \( \varphi' \) in accordance with Eq. (A5); the thermal wave vector \( k_T \equiv \sqrt{2mT}/h \) is introduced to make the new coordinates dimensionless; its physical meaning will be clarified later. It is important that in the new coordinate representation only the angle \( \varphi \) is changed in the process of collision while the total momentum \( K \) and energy \( \epsilon \) remain constant. As expected, the transition probability \( W(\Delta, \varphi, \varphi') \) does not depend on the center-of-mass momentum \( K \). Note the symmetry properties: \( W_{123} \rightarrow 1'2'3' = W_{213} \rightarrow 2'1'3' = W_{123'} \rightarrow 1'2'3' \rightarrow 123 \), etc.

The kinetic equation (4), together with expression (12) for the scattering probability, gives a complete description of relaxation in a 1DEG.

III. BALANCE EQUATIONS AND RELAXATION FREQUENCY

In order to investigate the relaxation in a 1DEG we consider the balance equation for the \( i \)th moment

\[
\mathcal{M}_i \equiv \frac{\int k^i f_k \, dk}{\int f_k \, dk}
\]

of the distribution function \( f_k \). From Eq. (4) and (5) we obtain

\[
\frac{\partial \mathcal{M}_i}{\partial t} = \mathcal{R}_i
\]

\[
= \frac{1}{2\pi n} \left( \frac{L}{2\pi} \right)^5 \int dk_1 dk_2 dk_3 dk_4' dk_5' \ k_1^i
\]

\[
\times W_{k_1,k_2,k_3 \rightarrow k_1',k_2',k_3'} \left( f_{k_1} f_{k_2} f_{k_3} - f_{k_1'} f_{k_2'} f_{k_3'} \right),
\]

(13)

where \( n = N/L \) is the concentration of the 1DEG. Note that for \( i \) = 1, 2 the rate \( \mathcal{R}_i = 0 \) since the interaction inside the system does not change the total momentum \( (i = 1) \) and energy \( (i = 2) \) of the system. Thus, we are interested in the rate of change of the third and higher moments of the distribution function.

Substituting the transition probability (12) into Eq. (13) and changing the variables of integration from \( (k_1, k_2, k_3) \) to \( (K, \Delta, \varphi) \) [see Eq. (A6)], we obtain

\[
\mathcal{R}_i = \frac{\hbar k_T^{i-1}}{18\pi^2 m_B^2 n} \int_0^\infty d\Delta \int_{-\infty}^\infty dK \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \ P_i
\]

\[
\times W(\Delta, \varphi, \varphi') \left( f_{k_1} f_{k_2} f_{k_3} - f_{k_1} f_{k_2} f_{k_3} \right).
\]

(14)

Here \( a_B = \varepsilon_L \hbar^2/me^2 \) is the effective Bohr radius; the momenta \( k_i \) in Eq. (14) can be expressed in terms of the variables \( \Delta, K \), and \( \varphi \) according to Eq. (A4); \( P_i \) is an irreducible part of the expression \( k_1^i + k_2^i + k_3^i - (k_1')^i - (k_2')^i - (k_3')^i \), for example,

\[
P_3 = (3/\sqrt{2}) \Delta^3 C_3,
\]

\[
P_4 = 6\sqrt{2} \Delta^3 K C_3,
\]

(15)

\[
P_5 = (15\sqrt{2}/4) \Delta^3 \left( 4K^2 + \Delta^2 \right) C_3,
\]

\[
P_6 = (3/4) \Delta^6 C_6 + (15/\sqrt{2}) \Delta^3 K \left( 4K^2 + 3\Delta^2 \right) C_3,
\]

where \( C_m = \cos m\varphi - \cos m\varphi' \) and \( m = 3, 6, \ldots \).

To illustrate the use of the expressions obtained above, we consider the relaxation in a system of two opposite homogeneous electron beams with equal concentrations \( n/2 \) and temperatures \( T \), having relative velocity \( \hbar k_T u/m \) \( (u \) is dimensionless). The distribution function of such a system, at time \( t = 0 \), is of the form

\[
f_k = \frac{\sqrt{\pi n}}{k_T} \left[ e^{-(k/k_T-u)^2} + e^{-(k/k_T+u)^2} \right].
\]

(16)

Due to internal collisions the electron gas will thermalize, reaching asymptotically the equilibrium with zero center-of-mass velocity and temperature equal to half the initial kinetic energy of the system, \( T_L = T_1(1+2u^2) \). It is convenient to characterize the relaxation of the electron gas by the change of the lowest nonzero moment of the distribution function \( \mathcal{M}_4 \), since by the symmetry of \( f_k \) the odd moments vanish. One can find that the initial fourth moment corresponding to Eq. (16) is equal to \( \mathcal{M}_4^4 = (3/4) k_T^4 (1+2u^2)^2 \), while the moment of the final equilibrium state \( \mathcal{M}_4^f = (3/4) k_T^4 (1+4u^2 + 4u^4/3) \).

Substituting Eq. (15) and (16) into the balance equation (13) and integrating over \( K \) and \( \Delta \), we obtain the rate of change of the fourth moment of the distribution function

\[
\mathcal{R}_4(u) = \frac{23/2}{9\pi^{3/2}} \frac{\hbar n^2}{ma_B^4} \int_0^{2\pi} d\varphi \exp(-\frac{8}{3} u^2 \sin^2 \varphi)
\]

\[
\times \text{erf} \left( \sqrt{\frac{8}{3}} u \cos \varphi \right) \mathcal{W}(\varphi),
\]

(17)

where \( \text{erf} \) is \( (2/\sqrt{\pi}) \int_0^\infty dt \exp(-t^2) \) is the error function, and

\[
\mathcal{W}(\varphi) = \int_0^{2\pi} d\varphi' (\cos 3\varphi - \cos 3\varphi') W(\varphi, \varphi').
\]

(18)

The function

\[
W(\varphi, \varphi') = \left\{ \frac{\text{ln} |\cos \varphi - \cos \varphi'| \text{ln} |\cos(\varphi - 2\pi/3) - \cos(\varphi' - 2\pi/3)|}{\cos \varphi - \cos \varphi' |\cos \varphi' - \cos(\varphi + 2\pi/3)| + \text{perm}(\varphi, \varphi')} \right\}^2
\]

(19)
is proportional to the scattering probability between the states \((K, \Delta, \varphi)\) and \((K', \Delta', \varphi')\). Note that each individual term in Eq. (19) has two first-order poles and a logarithmic singularity in \(\varphi\) and \(\varphi'\). After the summation of all 36 terms in (19) the poles are cancelled, and the logarithmic singularity is removed by the term \(\cos 3\varphi - \cos 3\varphi'\) in Eq. (18). The function \(W(\varphi, \varphi')\) is periodic with a period \(2\pi/3\) and has the same symmetry properties as the function \(\cos 3\varphi\).

In the derivation of Eqs. (17)–(19) we used the asymptotic expression \(F(\kappa) \sim \ln(1/\kappa a)\) (here \(a\) is the diameter of the wire) for the form factor \((10)\), valid in the limit of \(\kappa a \ll 1\) for an arbitrary cross section of the wire. Terms containing \(\ln \Delta a\) do not enter the transition probability \(W(\varphi, \varphi')\) because they correspond to a pointlike interaction between three particles that occurs with probability zero.

Analysis of Eqs. (17)–(18) gives the following asymptotic expressions for the rate \(R_4(u)\) in the limit of large and small relative beam velocity compared to the thermal velocity:

\[
R_4(u) = c_\infty \frac{\hbar n^2}{m a_B^2} \quad \text{for} \quad u \gg 1, \tag{20}
\]

\[
R_4(u) = 4c_0 u^4 \frac{\hbar n^2}{m a_B^2} \quad \text{for} \quad u \ll 1, \tag{21}
\]

where \(c_\infty = (2\sqrt{3}/9\pi) W(0) \approx 16.9\) and \(c_0 = (8/3\sqrt{2\pi}) J_0^{2\pi} d\varphi \cos 3\varphi W(\varphi) \approx 7.09\).

Equations (17)–(21) for the rate \(R\) together with the balance equation (13) can be used for the modeling of the relaxation of the electron system specified by the distribution function (16).

To obtain a quantitative measure of the intensity of triple collisions we introduce the inverse relaxation time \(1/\tau_{\text{reee}}\), defined as the ratio of the rate of change of the moment \(R_4\) to its actual change \(M_4' - M_4\) in the limit \(u \ll 1:\)

\[
\frac{1}{\tau_{\text{reee}}} \equiv \lim_{u \to 0} \frac{R_4(u)}{M_4' - M_4} = c_0 \eta^2 \omega_B. \tag{22}
\]

Here \(\omega_B = me^4/2e_L^2\hbar^3\) is the effective Rydberg energy divided by the Planck constant. The plasma parameter \(\eta = e^2n/e_L T\) is equal to the ratio of the mean potential and kinetic energies.

Analysis shows that other possible definitions of the relaxation time \(\tau_{\text{reee}}\) (based on other moments of the distribution function or different initial distributions of electrons) produce essentially the same result with the accuracy of a numerical factor of order unity.

IV. DISCUSSION

We start with a discussion of the applicability of the results. For the kinetic equation (4) to be valid, the electron gas should be weakly nonideal, \(\eta \lesssim 1\). Second, the criteria of truncation of the perturbation series (8) for the transition matrix \(T\) must be specified. From physical considerations and results for electron scattering between systems of different dimensionality,\(^9,10,20\) it follows that the maximum transferred momentum during collision, \(q_{\text{max}}\), is limited by the thermal de Broglie wave vector \(k_T\), and the Landau wave vector \(q_\lambda = eL/\hbar = n/\eta\) (where \(q_\lambda^{-1}\) is equal to the minimum impact distance between two Coulombically repelling particles with kinetic energies \(T\)):

\[
q_{\text{max}} = \min(k_T, q_\lambda). \tag{23}
\]

The quantum cutoff at \(q \sim k_T\) is elaborated in our theory and is present in Eq. (14). Thus, the case of \(k_T < q_\lambda\) corresponds to the region of applicability of the perturbation theory and of the results obtained above. This criterion can be rewritten in the form \(\eta \lesssim n/k_T \lesssim 1\). The relaxation time \(\tau_{\text{reee}}\) is given in this case by Eq. (22). Note that in a degenerate 1DEG we have \(n/k_T \sim 1\), where \(k_T\) is the Fermi wave vector, and the perturbation theory is always applicable for \(\eta \lesssim 1\).

The opposite limit of \(k_T > q_\lambda\) (or \(n/k_T > \eta \lesssim 1\)) corresponds to the nonperturbative case of classical colliding hard spheres with a radius of the order of \(q_\lambda^{-1}\). In this case the characteristic time of triple-electron scattering \(\tau_{\text{reee}}\) can be estimated as the time \(1/n\nu_T\) for two spheres to collide, multiplied by the probability \(q^2/\eta\) to find at this point a third sphere of radius \(q_\lambda^{-1}\). The result \((\nu_T = \sqrt{2T/m}\) is the thermal velocity),

\[
\frac{1}{\tau_{\text{reee}}} \sim \eta n \nu_T, \tag{24}
\]

does not contain the Planck constant \(\hbar\).

This estimation, as well as the result (22) for \(k_T \lesssim q_\lambda\), can be derived directly from Eq. (14) by artificially cutting the integration over \(\Delta, \varphi,\) and \(\varphi'\) so that the transfer of momentum during a collision does not exceed \(q_{\text{max}}\), e.g., \(|k_1 - k_1'| = \sqrt{2k_T \Delta} \cos \varphi - \cos \varphi' \lesssim q_{\text{max}}\) [cf. Eq. (A4)]. Then we obtain the following interpolation formula between the two limiting cases for the nondegenerate \((nk_T \lesssim 1)\) weakly nonideal \((\eta \lesssim 1)\) 1DEG:

\[
\frac{1}{\tau_{\text{reee}}} \sim \frac{\eta^2 \nu_T}{q_\lambda^2} \quad q_{\text{max}} \sim \frac{e^2 n^2}{\epsilon_L \sqrt{mT}} = \min \left[1, \left(\frac{e^2}{\epsilon_L \nu_T}\right)^3\right]. \tag{25}
\]

For the sake of comparison we present the relaxation time \(\tau_{\text{ee}}\) due to binary collisions between the nondegenerate carriers from different subsystems\(^9\) (e.g., separated by a thin impenetrable barrier):

\[
\frac{1}{\tau_{\text{ee}}} \sim \frac{\eta \nu_T}{q_\lambda^2} \quad q_{\text{max}}. \tag{26}
\]

It should be noted that in a nondegenerate 1DEG the screening effects are not important (the correction to \(\epsilon_L\), proportional to \(1 + \eta\), is negligible for a weakly nonideal gas\(^9,21\)). This is quite different from the case of bulk electrons in a strong magnetic field,\(^13,14\) where the electron motion is quasi-one-dimensional, but screening is essentially three dimensional (electrons surrounding a test charge from any direction contribute to screening).
Due to the conditions of weak nonideality and nondegeneracy of the 1DEG the relaxation frequency $1/\tau_{seee}$ is limited by the Bohr frequency $\omega_B$, which in GaAs is equal to $8.9 \times 10^{12}/s$. As an example we consider electrons in a GaAs quantum wire with concentration $n = 2 \times 10^{5}/cm$ at temperature $T = 77 K$. Estimation according to Eq. (25) gives the relaxation frequency $1/\tau_{seee} \approx 1.5 \times 10^{12}/s$.

This value should be compared with the momentum and energy relaxation frequencies due to other scattering mechanisms (acoustic phonon scattering for sufficiently pure samples). However, despite a large number of reports on fabrication and photoluminescence observations in thin (70–300 Å in diameter) quantum wires, so far no results are available on diffusive one-subband transport measurements to our knowledge. On the other hand, transport properties of wide quantum wires with a large number of occupied subbands are similar to those of quantum wells. Therefore, for the sake of comparison, we mention the momentum relaxation frequency $1/\tau_{sd} \approx 2 \times 10^{11}/s$ at temperature $T = 77 K$ deduced from mobility measurements in a 200-Å-wide quantum well, and the result of Monte Carlo simulation, $1/\tau_{MC} \approx 3 \times 10^{11}/s$, at the same temperature for a 100 Å × 100 Å quantum wire. Note that for acoustic phonon scattering the frequency of the energy relaxation is even lower. From the comparison with the obtained value of $1/\tau_{seee}$ we can conclude that the triple-electron collisions strongly influence the formation of the distribution function of the 1DEG.

Finally we compare the intensity of triple-electron collisions with that due to the spin splitting of the conduction band. In Ref. 12 only numerical evaluations for the case of degenerate statistics were presented. Since the momentum transferred during a collision is equal to the spin splitting of the conduction band $q_s$, we estimate the corresponding relaxation frequency as $1/\tau_s \sim q_s/q_{max} \tau_{seee}$. The intensity of the triple-electron collisions will be higher provided $\tau_s/\tau_{seee} \sim \eta_{0}^{2}/q_s q_{A} > 1$. In GaAs $q_s \approx 2 \times 10^{4}/cm$ and for the parameters of the example considered above the rate of triple-electron scattering exceeds that due to the spin splitting by a factor of 5. For a low concentration of the 1DEG the effect of binary collisions between electrons with opposite spins will dominate.

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APPENDIX

The scattering event between three interacting particles in one dimension has a natural geometrical interpretation in 3D phase space $(k_1/k_T, k_2/k_T, k_3/k_T)$ (division by $k_T$ makes the coordinates dimensionless). First we introduce the average dimensionless momentum $K$ and energy $\epsilon$ of the three particles

$$K = \frac{1}{3k_T} (k_1 + k_2 + k_3),$$

$$\epsilon = \frac{1}{3k_T^2} (k_1^2 + k_2^2 + k_3^2).$$

In phase space the surfaces of constant $\epsilon$ and $K$ represent a sphere, of radius $\sqrt{3}\epsilon$ with the center at the origin, and a plane which cuts the intervals $3K$ on each coordinate axis (see Fig. 1). The intersection of the plane and the sphere produces a circle of radius $\sqrt{3}\Delta$, where

$$\Delta = \sqrt{\epsilon - K^2}.$$  

To specify a position on the circle we introduce the polar angle $\varphi$ with the polar axis taken along the $k_1$ axis, cf. Fig. 1. The values $(K, \epsilon, \varphi)$ or, alternatively, $(K, \Delta, \varphi)$ uniquely specify the position in phase space and could be considered as the “new” coordinates. The transformation to the “old” coordinate system is described by

$$k_1/k_T = K + \sqrt{2}\Delta \cos \varphi;$$

$$k_2/k_T = K + \sqrt{2}\Delta \cos \left(\varphi - \frac{2\pi}{3}\right);$$

$$k_3/k_T = K + \sqrt{2}\Delta \cos \left(\varphi + \frac{2\pi}{3}\right);$$

Note that six possible permutations of three identical particles can be obtained by the changes

$$\varphi \rightarrow \varphi \pm 2\pi/3, \quad \varphi \rightarrow -\varphi.$$  

We also need the expression for the integral over the whole phase space:
\begin{align*}
\int dk_1 \int dk_2 \int dk_3 \ldots & \nonumber \\
& = \frac{1}{2\sqrt{3}} \int_0^\infty \int_0^{\sqrt{3}e} dK \int_0^{2\pi} d\varphi \ldots \\
& = \frac{1}{\sqrt{3}} \int_0^\infty \Delta d\Delta \int_{-\infty}^{+\infty} dK \int_0^{2\pi} d\varphi \ldots . \tag{A6}
\end{align*}

The rightmost form of this integral is more convenient since the integration limits for each variable do not depend on any of the other variables.

In the process of three-particle collision the total momentum and energy are conserved, i.e., \( K = \text{const} \) and \( \epsilon = \text{const} \) (which implies \( \Delta = \text{const} \)). Therefore the change of the three individual momenta \( k_1, k_2, \) and \( k_3 \) during the collision corresponds to the change of a single variable \( \varphi \) in the new coordinate set.

19. The wave function (3) corresponds to a system of three identical particles with spins 0. In the case of three interacting electrons the properly symmetrized combinations of one-electron wave functions with total spins 1/2 and 3/2 arise according to Young’s scheme (Ref. 20). The spinless approximation is often used to avoid cumbersome calculations and gives results differing from that with the full account of spin by a numerical factor larger than and of the order of unity, i.e., it leads to some overestimation of scattering intensity.
FIG. 1. Coordinate systems in phase space.