

Negative differential conductance under conditions of carrier cooling by an electric field

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It is shown that when carriers are cooled by an electric field, which occurs in semiconductors if carriers that have emitted an optical phonon reach a region of strong impurity scattering, then the probability of impact ionization from shallow impurity levels W_i and the recombination time of carriers at these levels τ_r are functions with a maximum. The fall of W_i gives rise to a negative differential conductance of the N -type (N -NDC) in electric fields E_x of intensities higher than the breakdown value for weakly compensated samples and the decrease of τ_r on increase in E_x gives rise to an N -NDC in the prebreakdown range of E_x in the case of strongly compensated samples. In the latter case the breakdown field for the release from impurities E_i shifts toward very high values of E_x (beyond the cooling region). In the case of samples with moderate compensation there are three (and not one) characteristic fields: two impact ionization fields E_i , of which the second is located at high values of E_x beyond the cooling region, and between them a field E_a in which carriers are frozen out at impurities and a giant N -NDC region is observed. A magnetic field H suppresses carrier cooling so that an N -NDC is reduced, shifts toward higher fields E_x , and disappears on increase in H . A giant negative magnetoresistance is possible when the current rises by several orders of magnitude on increase in H or when the field E_a is suppressed or when the field E_i decreases strongly on increase in H .

It was reported first in Ref. 1 and then in Refs. 2-4 that chaos occurs in p -type Ge at 4 K and lower temperatures. The chaotic behavior of a hole plasma occurs in strong electric field E_x close to the field E_i corresponding to impurity breakdown (impact ionization). Chaos is possible both in the prebreakdown¹ range of fields ($E_x < E_i$) and above the breakdown threshold³ ($E_x > E_i$). An analysis of the equations of conservation of the total current and of the balance equations for the density of free carriers under quasineutral conditions shows² that chaos may occur only if the impact ionization probability W_i is a nonmonotonic function of the electric field. Model dependences of W_i and E_x have been used, but the mechanisms ensuring the reduction in W_i reported in Ref. 2 are not known.⁵⁻⁷

It is shown in Ref. 8 that a reduction in W_i on increase in E_x occurs when carriers are cooled in an electric field. In the case of p -type Ge and several other semiconductors the impact ionization energy ($\epsilon_i \approx 10$ meV)⁹ is less than the optical phonon energy ($\hbar\omega_0 \approx 37$ meV) and the probability of emission of an optical phonon is high, compared with the other scattering probabilities¹⁰ and with the probability of impact ionization. At low temperatures ($kT \ll \epsilon_i < \hbar\omega_0$) the heating of the electron gas is accompanied by an increase in the average energy $\bar{\epsilon}$, as well as in W_i and in the recombination time at impurities τ_r until carriers reach the energy $\hbar\omega_0$. When this happens they emit practically instantaneously an optical phonon and drop to the range of low energies.^{10, 11} If the scattering by ionized impurities predominates at low energies, then carriers are retained in this range by strong scattering, but if they leave this range they rapidly cross the range of intermediate energies where the scattering probability is low and having attained the energy $\hbar\omega_0$ they return again to low energies.¹² The value of $\bar{\epsilon}$ decreases¹²⁻¹⁵ and so do W_i and τ_r (Ref. 9) ($\bar{\epsilon}$ may be even less than $3kT/2$, i.e., absolute cooling may be observed).¹²⁻¹⁵

When an increase in E_x causes the removal of carriers from the range of low energies without

scattering or if carriers begin to penetrate deeply into the range $\epsilon > \hbar\omega_0$, then $\bar{\epsilon}$, W_i , and τ_r increase again on increase in E_x . The nonmonotonic dependences of W_i and τ_r on E_x are the reason for an N -type negative differential conductance (N -NDC). A study of the conditions for the appearance of an N -NDC and of the influence of a magnetic field on this effect are the subject of the present paper.

2. We shall calculate the current-voltage characteristic of an extrinsic p -type semiconductor at low temperatures. We shall consider the steady-state conditions and a homogeneous semiconductor, so that

$$i = epv_d, \quad (1)$$

where i is the current density; e is the absolute charge of an electron; p is the density of holes; v_d is the average drift velocity; p and v_d are nonlinear functions of the electric field. The hole density is usually calculated from the steady-state balance equation^{1, 16, 17}

$$A(N_A - N_D - p) - B_T(N_D + p)p + A_I(N_A - N_D - p)p = 0, \quad (2)$$

where N_A and N_D are the concentrations of acceptors and of donors compensating them; B_T and A_I are the coefficients corresponding to the recombination at impurities and impact ionization from impurities; A is a parameter which although allows for thermal ionization, is governed by the ionization of acceptors due to infrared radiation. The Auger processes are not included in Eq. (2), because relatively pure semiconductors are under consideration. We can calculate v_d and $\bar{\epsilon}$, and the coefficients B_T and A_I if we know the distribution function F_k :

$$B_T = \sum_k v_{Tr} F_k / \sum_k F_k, \quad A_I = \sum_k v_{Ii} F_k / \sum_k F_k, \quad (3)$$

$$\bar{\epsilon} = \sum_k \epsilon F_k / \sum_k F_k, \quad v_d = \sum_k v F_k / \sum_k F_k. \quad (4)$$

The prime in Eq. (3) denotes that the summation is limited to the states k of energy ϵ greater than

the ionization energy ϵ_j ; k is the wave vector; $v = v(k)$ is the carrier velocity; σ_T and σ_I are the recombination and impact ionization cross sections; σ_T was taken from Refs. 5 and 6 and σ_I was calculated from the integral expressions given in Refs. 18 and 19.

The distribution function F_k was calculated by solving the transport equation in the same way as in Ref. 12. The only difference was that in addition to an electric field, we allowed for a magnetic field H applied at right-angles to the current.

The flux of electrons from the active ($\epsilon > \hbar\omega_0$) to the passive ($\epsilon < \hbar\omega_0$) region was allowed for, but the distribution function in the active region is small because of the high probability of emission of optical phonons and it was therefore assumed to be zero.¹⁰⁻¹² In the passive region an allowance was made for the scattering by acoustic lattice vibrations and by ionized impurities, and in this case the distribution function considered in the quasi-elastic approximation is of the form

$$F_k = F_0(\epsilon) + F_1(k), \quad F_1 \ll F_0, \quad (5)$$

where

$$F_0(\epsilon) = f \exp[-I(x)] \int_{-\infty}^{\infty} \exp[I(y) - I(x_0)] dy / |y^{3/2}(y)|, \quad (6)$$

$$F_1 = e\tau^* [v \cdot E + |v \cdot H| E\tau/cm] (-\partial F_0/\partial \epsilon), \quad (7)$$

$$f(x) = 1 + (E/E_0)^2 x / (1 + B) [1 + 3x^2/(x^2 + B)], \quad I(x) = \int_0^x dy/f(y), \quad (8)$$

$$\tau^* = \tau / [1 + 3x^2/(x^2 + B)^2], \quad \tau = \tau_0 / (x^2 + B), \quad (9)$$

$$b = (e\tau_0 H/cm)^2, \quad \tau_0 = 3\sqrt{\pi} m\mu_a/(4e), \quad B = 6\mu_a/\mu_s. \quad (10)$$

Here, H is the magnetic field; m is the effective mass; τ is the momentum scattering time; μ_a and μ_s are the low-field mobilities governed by the scattering on acoustic phonons and ionized impurities, respectively; $x = \epsilon/kT$; $x_0 = \hbar\omega_0/kT$; $E_0 = (4s/(3\pi)^{1/2})/\mu_a$ is the characteristic electric field; s is the velocity of sound; j is the normalization constant found from the condition $2 \sum_k F_k = 1$.

At a fixed temperature the distribution function is controlled by electric and magnetic fields, and by the parameter B representing the contribution of scattering by ionized impurities [Eqs. (9) and (10)]. It should be pointed out that in the case of ionization the concentration of ionized impurities $N_i = N_D + N_A^- = 2N_D + p$ changes, which is allowed for self-consistently in Eqs. (2), (6), and (7).

Equations (3), (4), (6), and (7) were used to calculate the dependences of B_T , A_I , v_d , and $\bar{\epsilon}$ on H and N_i , and on the applied electric field E_X ; if $H \neq 0$, the Hall field E_H appears in a sample and it should be included in the total field $E = (E_X^2 + E_H^2)^{1/2}$ which governs the distribution function.

Figure 1 shows the dependences $A_I/A_I(E=0)$, $B_T/B_T(E=0)$, and $\bar{\epsilon}/(3kT/2)$ on E_X/E_0 for certain fixed values of H and N_i . The calculations were carried out for $T = 4$ K and a semiconductor with the parameters of p-type Ge on the assumption that $\mu_a = 5 \cdot 10^6$ cm²·V⁻¹·sec⁻¹, $A_I(E=0) = 7.43 \cdot 10^{-19}$ cm³/sec deduced from Eqs. (3) and (6), that $B_T(E=0) = 3 \cdot 10^{-5}$ cm³/sec taken from Ref. 20, and also that $E_0 = 0.14$ V/cm. Only heavy holes were allowed for in these calculations.¹⁾

In the range of electric fields where cooling

begins (Fig. 1c) a linear reduction in the average energy when $H = 0$ is accompanied by a linear reduction in A_I (Fig. 1a) and by an increase in B_T changes to a fall in high values of E_X . In the case of an ultrapure material when $N_i \rightarrow 0$ and in the passive region when only the scattering by acoustic phonons is important, there is no cooling effect; $\bar{\epsilon}$, A_I , and B_T show saturation and the saturation region of B_T is preceded by nonmonotonic behavior. This behavior is due to the fact that carriers that have attained the energy $\hbar\omega_0$ drop to low energies facilitating some increase in the rate of recombination even if carriers are not confined to this region by impurity scattering. The maximum values of $\bar{\epsilon}$ and A_I decrease and the minimum value of B_T rises on increase in N_i .

The strongest effect of a magnetic field in the case of an ultrapure sample is an increase in the range of rise of B_T on increase in E_X and of the saturation value of B_T , which is due to a strong influence of a magnetic field on low-energy carriers because τ^* in Eq. (9) is inversely proportional to δ/x in the limit $B \rightarrow 0$ and $\epsilon \rightarrow 0$. On the other hand, the application of a magnetic field to a doped material reduces and then suppresses carrier cooling. The values of $\bar{\epsilon}$ and A_I in the saturation region are then close to the values for an ultrapure material, i.e., when cooling takes place if $H = 0$, then there is a considerable increase in $\bar{\epsilon}$ and A_I on increase in H .

Before discussing the dependences of p and i on E_X , we note that the distribution function of Eqs. (6) and (7) is deduced using the quasi-elastic approximation without allowance for the penetration of carriers to the active region where $\epsilon > \hbar\omega_0$. In the case of p-type Ge at 4 K this is true right up to $E_X = E^+ = 3$ kV/cm (Refs. 10 and 11), where E^+ is considerably higher than the fields considered here, and the former assumption is disobeyed in fields $E > E^-$, where E^- can be estimated for our case as follows^{10,11}:

$$E^- = 1.5 \cdot 10^7 p^{-1} = 1.5 \cdot 10^7 p_0^{-1} (1 + B/6) = 3.6 (1 + 4.75 \cdot 10^{-11} N_i) \text{ V/cm}. \quad (11)$$

If $N_i \approx 1 \cdot 10^{12}$ cm⁻³ and $\mu \approx 5 \cdot 10^4$ cm²·V⁻¹·sec⁻¹, we find that $E^- \approx 400$ V/cm (Refs. 10 and 11). As long as the field E^- is less than E^+ , it represents the limit of validity of the theory presented here. For $N_i \sim 10^{12}$ cm⁻³ this represents practically the whole range shown in Fig. 1, which expands on increase in N_i . Fields $E \geq E^-$ will not be considered because we are interested in cooling of carriers which is suppressed when $E > E^-$ and $\bar{\epsilon}$ and A_I exhibit saturation in the interval of fields (E^- , E^+) when $H = 0$ and this is true even of a doped material.

3. It is usual to distinguish a prebreakdown range of magnetic fields, when the last term in Eq. (2) is unimportant, and the range above the breakdown field E_1 which is found by equating the last two terms in Eq. (2) (see Refs. 5 and 16):

$$B_T(N_D + p) = A_I(N_A - N_D - p). \quad (12)$$

We can simplify Eq. (12) by dropping p , because $p \ll N_D$, $N_A - N_D$. Then, the field E_1 is found from the simple intersection of the B_T and A_I curves shown in Fig. 1 multiplied by N_D and $N_A - N_D$, respectively. In the usual situation studied earlier¹⁶ these curves have a single intersection, whereas we now have further possibilities due to the non-

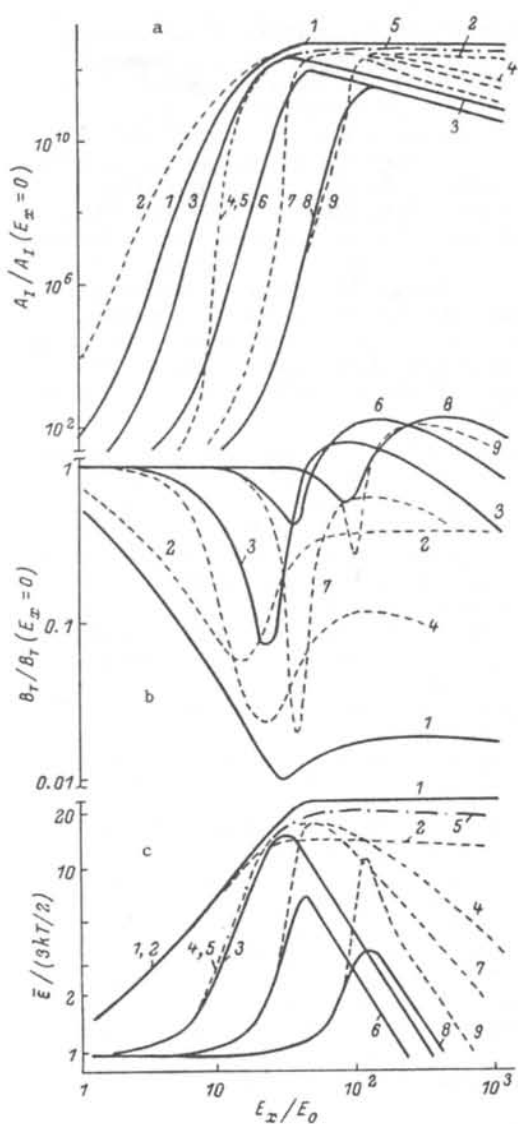


FIG. 1. Dependences of the impact ionization coefficients of impurities (a), of the coefficient of recombination at impurities (b), and of the average energy (c) on the applied electric field E_x/E_0 calculated for different values of the dimensionless magnetic field $\sqrt{\delta}$ and of the ionized impurity concentration N_i (10^{13} cm^{-3}): 1) 0; 2) 0; 3) 0.1; 6) 1; 7) 1; 8) 10. Magnetic field $\sqrt{\delta}$: 1), 3), 6), 8) 0; 2), 4), 7), 9) 100; 5) ∞ .

monotonic nature of the dependences $A_I(E_x)$ and $B_T(E_x)$.

We shall later consider in detail the most interesting case of three intersections, i.e., the case of three solutions of Eq. (12) which, as shown below, occurs for moderately doped samples; however, at this stage we shall deal briefly with two other cases.

If samples are strongly compensated, then breakdown does not occur in the investigated range of electric fields, i.e., Eq. (12) has no solutions in this range. This is true of p-type Ge for $N_D/N_A \geq 0.03, 0.07, 0.2$, and 0.7 corresponding to $N_A = 10^{15}, 10^{14}, 10^{13}$, and 10^{11} cm^{-3} , respectively. The dependence of p on E_x is governed entirely by the dependence of B_T^{-1} on E_x . An N-NDC region is observed in the prebreakdown range of fields. A magnetic field H suppresses cooling, so that an N-NDC decreases and disappears on increase in H . The investigation reported in Ref. 1 was concerned

also with samples with the concentrations N_D and N_A satisfying the above criterion of strong compensation. According to the authors of Ref. 1, chaos was observed in the prebreakdown range of fields, i.e., it could be assumed that this happened in the N-NDC region predicted here.

If a sample is weakly compensated then - as in Ref. 16 - there is only one intersection and one field E_i . Above the breakdown threshold the non-monotonic behavior of $A_I(E_x)$ makes the dependence $p(E_x)$ nonmonotonic and an N-NDC is observed (the condition of weak compensation for p-type Ge is $N_D/N_A \leq 0.003, 0.008, 0.03$, and 0.15 for $N_A = 10^{15}, 10^{14}, 10^{13}$, and 10^{11} cm^{-3} , respectively). An increase in H reduces an N-NDC and shifts it to higher fields E_x and then suppresses it completely. It should be pointed out that the experimental observations reported in Ref. 3 revealed a shift of an N-NDC toward higher values of E_x and its suppression on increase in H ; since samples with $N_A = 10^{14}-10^{15} \text{ cm}^{-3}$ were not deliberately compensated, the condition of weak compensation was obeyed.

In the case of moderate compensation there are two intersections in the investigated range of fields. One of them determines the breakdown field E_i and the other the freezeout field E_a . The next breakdown field is outside the range of fields of interest to us. In the field E_a the samples ionized by $E_x \sim E_i$ are frozen out again at impurities and this gives rise to a giant N-NDC. The current-voltage characteristics shown in Figs. 2a-2d are calculated for the case when the field E_a exceeds slightly E_i and $H = 0$ (Fig. 2a). The calculations were carried out for $N_A = 10^{14} \text{ cm}^{-3}$, $N_D/N_A =$

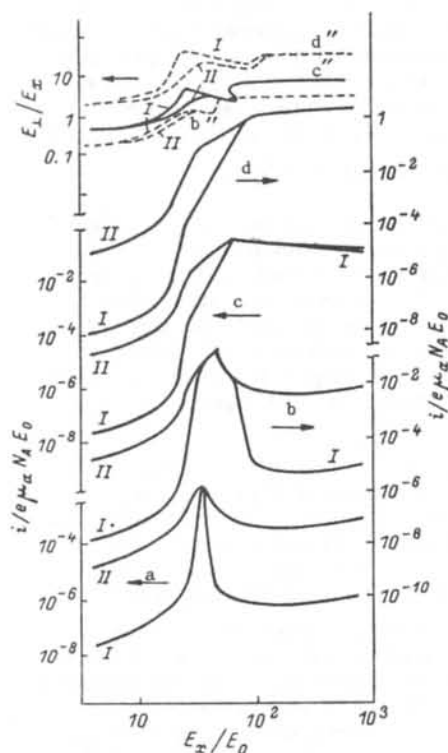


FIG. 2. Dependences of the current (a-d) and of the ratio of the Hall field E_L to the applied field E_x (b''-d'') on the field E_x/E_0 applied to p-type Ge. $T = 4 \text{ K}$. $N_A = 10^{14} \text{ cm}^{-3}$, $N_D/N_A = 0.05$. $A/[B_T(E=0)N_D]$: I) 10^{-6} ; II) 10^{-3} . $\sqrt{\delta}$: a) 0; b), b'') 31.6; c), c'') 100; d), d'') 1000.

0.05, and two degrees of excitation $p(E=0)/N_A \equiv A/[B_T(E=0)N_D] = 10^{-6}$ and 10^{-3} . The former case corresponds to a low degree of excitation (curves denoted by I) and the latter to a high degree (curves denoted by II).

An increase in H reduces the field E_X (Fig. 2b) and then the field E_a is suppressed (Fig. 2c) and finally the N-NDC region disappears (Fig. 2d), i.e., a magnetic field suppresses the freezeout field E_a and causes the current to rise by several orders of magnitude (this gives rise to a giant negative magnetoresistance). Such behavior is associated with the suppression of carrier cooling by a magnetic field (Fig. 1).

The current-voltage characteristics in Figs. 2b and 2c have another special feature. Instead of an N-NDC region there is in fact an N_S -NDC region (for a classification of these regions see Sec. 23 in Ref. 23), i.e., not one value of the current corresponds to three values of the field (N-NDC), but also one field corresponds to three values of the current. A region of the N_S type is not very pronounced in Fig. 2b, but in Fig. 2c it appears as a clear loop. This feature is explained by the dependence of the ratio E_1/E_X on E_X , which are also given in Fig. 2.

The regions of rapid change of the current on increase in E_X are due to rapid variation of p with E_X . The dependence of p on the total field E is simple. When the field reaches E_1 , impact ionization takes place and the density of holes rises strongly at a rate that becomes greater on increase in the magnetic field. The rise of p in the prebreakdown range is due to a reduction in the recombination probability $B_T(E)$. The prebreakdown range of fields in the dependence of p on E_X becomes narrower and then both p and consequently the current (Fig. 2) rise much faster on increase in E_X than on increase in E because heating of carriers occurs in this range of fields. Carriers leave the range of energies with a short characteristic momentum scattering time τ to the region with a high value of τ [Eq. (9)] and the magnetic field effectively increases (i.e., the ratio $H\tau/mc$ increases), which results in a rapid rise of the whole field (Figs. 2b"-2d") and this increases the total field E responsible for the heating [Eqs. (6) and (8)] and causes an increase in $B_T^{-1}(E)$, and $p(E)$. On the other hand, the breakdown region expands strongly on increase in H and in strong magnetic fields the rise of p on increase in E_X may be even gentler than in the prebreakdown range. This is due to the fact that a rapid rise of p on increase in the total field E is accompanied by an equally rapid rise of the concentration of ionized impurities, an effective weakening of the magnetic field, and a reduction of E_1/E_X (Figs. 2b"-2d"). Consequently, in the case of strong magnetic fields when $E_1/E_X \gg 1$, a slight increase in E can be achieved only by a strong rise of E_X . The fall of p above the breakdown threshold has the opposite effect: it reduces N_i and increases E_1/E_X and the rise of E_1 on increase in E may be superlinear, i.e., an increase in E may be achieved by reducing E_X because of the strong rise of E_1/E_X . This gives rise to a pronounced S-type dependence of E_1/E_X on E_X , which is shown in Fig. 2c", and to an N_S -type region in the dependence of p on E_X which after allowance for the dependence of v_d on E_X gives rise to the loop in Fig. 2c. The N_S -NDC region described here is manifested most strongly and the breakdown region widens on reduction in the compensation of a sample.

When the degree of compensation is high, so that $N_D \sim N_A$, the dependence $p(E)$ has practically no effect on the concentration of the scattering centers.

It should be pointed out that in a strongly compensated sample which is close to the condition for moderate compensation an N-NDC region appears in $H = 0$ in prebreakdown electric fields and in this case the breakdown field E_1 lies outside the range of E under consideration. An increase in the magnetic field gives rise to fields E_1 and E_a , and E_a rises rapidly and disappears on increase in H , and then the N-NDC region is suppressed, i.e., the magnetic field shifts the breakdown field toward higher values of E_X to the range under discussion and increases the current by several orders of magnitude.

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¹) It was shown in Ref. 21 that in the quasielastic approximation adopted here the symmetric parts of the distribution function are the same for the heavy- and light-hole bands so that the smallness of the contribution of the light holes to all the effects under consideration can easily be justified (see also the results of calculations in Ref. 22).

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