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A Negative Differential Conductivity Due to Recombination and Impact Ionization in Semiconductors at Low Temperatures

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Abstract. The probability of impact ionization and the recombination time are known to increase monotonically with the electric field E. I show that at low temperatures both functions achieve a maximum and decrease in the electric field range where the emission of optical phonons with subsequent impurity scattering dominate. This nonmonotonicity results in three different types of N-shaped negative differential conductivity (n-ndc). The carrier concentration and the current decrease when E increases due to decreasing of the impact ionization probability for weakly compensated samples and of the recombination time for highly compensated samples. At the antithreshold electric-field impact ionization dies out, which results in a dramatic decrease of the current for intermediately compensated samples. This huge n-ndc could be used in a novel type of the Gunn diode. The essential increase of threshold electric field of impact ionization is also predicted, and the effect could enhance the efficiency of photodetectors.

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The examination of semiconductor instabilities [1, 2] provides a deeper understanding of the operation of practical semiconductor devices in nonlinear regimes. Quite recently nonlinear oscillations and chaos were found [3] in extrinsic far-infrared Ge photoconductors [4] at temperatures from $T = 1.5$ to 4.2 K. The oscillations were observed in the pre-breakdown regimes. Those authors suggested that the oscillations are connected with the impact ionization of shallow donors [3] and that it is due to the decreasing of the impact ionization probability with increasing electric field [5]. Unfortunately, no physical picture of possible instabilities was presented. The nonlinear oscillations and chaos were observed in the post-breakdown regimes [6], too.

The prediction of a new negative differential conductivity at low temperature is reported here. The model explains preconditions for instabilities in pre- and post-breakdown regimes. The breakdown dies away, and a huge (almost rectangular) impulse of current appears in the current voltage characteristic in especially compensated samples. All these peculiarities are due to the cooling of the charge carriers with increasing electric field. Two quite different reasons for the cooling are shown to be important. Firstly, cooling due to emission of optical phonons with subsequent scattering on ionized impurities in the low energy region [7–9]. Secondly, the increase of ionized-impurities concentration under impact ionization enhances the probability of scattering by these impurities. The first contribution is the most essential in our treatment.

Let us calculate the current-voltage (I–V) characteristic in semiconductors at low temperatures. We restrict ourselves to the steady state and to homogenous conditions, such that

$$j = epv_d \tag{1}$$

where j is the current density, e is the electronic charge, v_d is the mean drift velocity, p is the hole concentration (the p-type extrinsic semiconductors is considered

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further). Furthermore, p and v_d are nonlinear functions of the electric field strength E. The hole concentration p is usually calculated from the steady-state rate equation [4, 3, 10]

$$A(N_A-N_D-p)-B_T(N_D+p)p + A_I(N_A-N_D-p)p=0\,, \tag{2}$$

where N_A and N_D are the concentration of acceptors and compensating donors, B_T and A_I denote the recombination and impact ionization. A is considered as a parameter which includes thermal ionization and ionization by external infrared radiation. In (2), the Auger processes are not included because only relatively pure semiconductors are analysed. The B_T and A_I in (2) as well as v_d in (1) are functions of the electric field E and the hole concentration p:

$$B_T=\sum_{\mathbf{k}} v\sigma_T f_{\mathbf{k}}/\sum_{\mathbf{k}} f_{\mathbf{k}}\,; \qquad A_I=\sum_{\mathbf{k}}{}' v\sigma_I f_{\mathbf{k}}/\sum_{\mathbf{k}} f_{\mathbf{k}}\,, \tag{3}$$

$$\mathbf{v}_d=\sum_{\mathbf{k}} \mathbf{v} f_{\mathbf{k}}/\sum_{\mathbf{k}} f_{\mathbf{k}}\,, \tag{4}$$

because the distribution function $f_{\mathbf{k}}$ depends on E and on p. [$f_{\mathbf{k}}=f_{\mathbf{k}}(p)$ since $f_{\mathbf{k}}$ depends on the intensity of the scattering on the ionized impurity $N_A^-=N_D+p$]. The prime in (3) denotes that summation is restricted to states $\mathbf{k}$ with an energy ε greater than the ionization energy ε_I, $\mathbf{k}$ is the wavevector. In (3, 4) v is the carier velocity, σ_T and σ_I are the recombination and impact ionization cross sections. σ_I was taken from [11] as

$$\sigma_I=\sigma_0(\varepsilon/\varepsilon_I-1)/(\varepsilon/\varepsilon_I)^{1.25}\,, \tag{5}$$

where $\sigma_0=2.25\,\pi a_0^2$ and a_0 is the Bohr radius. The dependence of σ_T on the energy ε was tabulated on the basis of the integral expression in [12, 13].

Solution of Boltzmann Equation

Boltzmann's transport equation must be solved for the distribution function $f_{\mathbf{k}}$. We analyse it only in the low-temperature limit, when kT is small in comparison with the ionization energy ε_I and with the energy of optical phonons $\hbar\omega$: ($kT \ll \varepsilon_I < \hbar\omega$). It is confirmed theoretically and experimentally that up to $E=3$ kV/cm in p-Ge at 4 K the penetration of the holes in the region $\varepsilon > \hbar\omega$ (active region) is negligible. We are interested in an essentially smaller electric field, thus it is only necessary to solve the transport equation in the passive region [7, 8, 14, 16] ($\varepsilon < \hbar\omega$) taking into account the carrier flux from the active to passive region due to emission of optical phonons.

Only the acoustic and ionized-impurity scatterings are important in the passive region. Under the assumption that the distribution function has little anisotropy, i.e., that

$$f_{\mathbf{k}}=f_0(\varepsilon)+f_1(\varepsilon)\cos\theta\,, \qquad f_1(\varepsilon) \ll f_0(\varepsilon) \tag{6}$$

(here θ is the angle between $\mathbf{k}$ and $\mathbf{E}$) we have [17, 18]

$$f_1(\varepsilon)=e\tau vE\left(-\frac{\partial f_0(\varepsilon)}{\partial\varepsilon}\right), \tag{7}$$

where τ is the momentum relaxation time [18]

$$\tau=(3\sqrt{\pi}m\mu_a/4e)x^{3/2}/(x^2+B)\,, \qquad B=6\mu_a/\mu_I\,, \tag{8}$$

m is the effective mass, μ_a and μ_I are the low-field mobilities limited by the acoustic phonons and ionized impurities scattering, and $x=\varepsilon/kT$.

The second-order differential equation for $f(\varepsilon)$ is integrated once, and the result is [7, 18]

$$(\tfrac{2}{3}e^2E^2\varepsilon v\tau + kT\varepsilon mv/\tau_\varepsilon)\partial f_0/\partial\varepsilon + mv/\tau_\varepsilon f_0(\varepsilon) = -j_0(\varepsilon)\,, \tag{9}$$

where τ_ε is the energy relaxation time

$$\tau_\varepsilon=\tau(B=0)\cdot kT/(2ms^2)\,,$$

and s is the sound velocity. The presence of optical scattering causes the flux $j_0(\varepsilon)$ to be different from zero [7, 8, 16, 18]

$$j_0(\varepsilon)=\int_0^\varepsilon [mv(\varepsilon')/\tau_0]\,(N_0+1)f_0(\varepsilon'+\hbar\omega_0)d\varepsilon'\,, \tag{10}$$

where N_0 is the number of optical phonons (in the low-temperature limit $N_0 \to 0$). τ_0 is the time of emission of optical phonons, it independent on ε' for $\varepsilon' \ll \hbar\omega$. The τ_0 is small in comparison with τ_ε and that is why penetration of the carrier in the active region is small. The distribution function decreases rapidly in the active region [7, 8, 14, 16] and so the main contribution to the integral (10) is from small ε'; $j_0(\varepsilon)$ depend on ε only for very small ε and saturates. It means that for (9) it is possible to take j_0 as independent of ε in the hole region of ε [7, 8, 14, 16]. Equation (9) is then simply integrated. With the boundary condition that $f_0(\varepsilon)$ tends to zero when ε tends to $\hbar\omega$ the solution is [7]

$$f_0(\varepsilon)=j_1\exp[-I(x)] \cdot\int_x^{x_0}\exp[I(y)-I(x_0)]/[y^2F(y)]dy\,, \tag{11}$$

where

$$I(x)=\int_0^x dy/F(y)\,, \qquad F(x)=1+(E/E_0)^2x/(x^2+B)\,, \tag{12}$$

$x_0=\hbar\omega/kT$, and E_0 is the characteristic electric field

$$E_0=[4s/(3\pi)^{1/2}]/\mu_a\,. \tag{13}$$

The normalization constant j_1 in (11) can be determined from the normalization condition $2\sum_{\mathbf{k}} f_{\mathbf{k}}=1$, but this normalization is not necessary for the calculation of B_T, A_I, and v_d on the basis of (3, 4).

It is necessary to stress here that the approach with j_0 being independent of ε and $f_0(\varepsilon \to \hbar\omega_0) \to 0$, which gives (11), was reliably proved even for stronger electric fields [14, 15] than we consider here. Meanwhile it was tested again. For this we have found the distribution function in the active region as in [16]. The continuity condition for $f_0(\varepsilon)$ at $\varepsilon = \hbar\omega$ is used for the active region as a boundary condition. The flux $j_0(\varepsilon)$ was calculated. and $f_0(\varepsilon)$ in the passive region was found. Such a procedure does not change the values of B_T, A_I, and v, but it requires one more integration in addition to three time integrals in (3, 4, 11).

Results and Discussion

Dependences of $B_T/B_T(E=0)$ and $A_I/A_I(E=0)$ on E/E_0 are shown in Fig. 1 for different concentrations of ionized impurities, $N_a^- = N_D + p$, in p-Ge at 4 K. $E_0 = 0.14$ V/cm was calculated from (13) with $\mu a = 5 \times 10^6\ \mathrm{cm^2/V \cdot s}$, $A_I(E=0) = 7.43 \times 10^{-19}\ \mathrm{cm^3/s}$ was estimated from (3,5) and $B_T(E=0) = 3 \times 10^{-5}\ \mathrm{cm^3/s}$ was taken from [19]. B_T decreases and A_I sharply increases, as E increases in a relatively small electric field region. It is an ordinary situation, which was studied before in (10–13, 19). The exponentially small number of the carriers only reach the energy of optical phonons in this electric-field region. The distribution function (11) coincides with the Davydov's quasi-isotropic distribution function [17, 20], because j_0 in (9) is almost zero. The flux j_0 increases when E increases. The carriers from the high-energy region ($\varepsilon \gtrsim \hbar\omega_0$) return to the very low energy region when j_0 (i.e., emission of optical phonons) becomes essential. In the low-energy region the scattering time is controlled by the ionized impurities (8). The carriers are scattering often here and the distribution function increases in comparison with the high-energy region. The accumulation of the carriers in the low-energy region results in an increase of B_T with increasing E. The number of the carriers in the high-energy region decreases when E increases because phonon emission returns them to the low-energy region where they are arrested by impurity scattering [7]. A_I decreases as a result. The electric field, where B_T and A_I take extreme values, increases, when the concentration of ionized impurities increases. This is due to the fact that a larger electric field is needed for the carriers to achieve the optical phonon energy under the effect of stronger scattering.

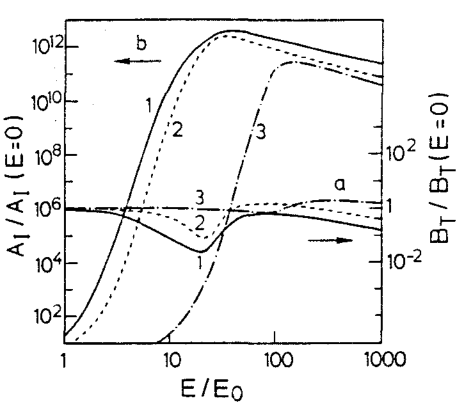

Fig. 1a and b. Dependence of the recombination $B_T/B_T(E=0)$ (a), and ionization $A_I/A_I(E=0)$ (b) coefficients on the normalized electric field E/E_0 in p-Ge at $T = 4$ K for the different ionized impurity concentration $N_A^- = N_D + p = 10^{11}\ \mathrm{cm^{-3}}$ *(1)*, $10^{12}\ \mathrm{cm^{-3}}$ *(2)*, $10^{14}\ \mathrm{cm^{-3}}$ *(3)*

The presence of the extrema of $A_I(E)$ and $B_T(E)$ result in three new negative differential conductivity regims which were not discussed before. We treat them consecutively.

The concentration p must be calculated from the nonlinear equation (2). It is possible to distinguish the pre-breakdown and post-breakdown regimes. The breakdown electric field E_c is normally found from the condition that the last two terms of (2) compensate each other:

$$B_T(E_c)(N_D + p) = A_I(E_c)(N_A - N_D - p). \tag{14}$$

The field E_c may be taken as the field of the intersection of the probability of recombination [left-hand part in (14)] and ionization (right-hand part) probabilities (Fig. 1):

1) Equation (14) has no solution in the range of the electric field considered if the sample is highly compensated ($N_d/N_A \to 1$). (It is very simple to understand this from the Fig. 1). The last term in (2) is unimportant in this case. The dependence of p on E is determined mostly by the dependence of B_T on E. This is why p increases when B_T decreases and p decreases in the opposite case. Dependence of p on E becomes N-shaped and it calls for a N-shaped dependence of j on E, as demonstrated in Figs. 2a and 3a. [We should stress that all results in Figs. 2 and 3 were obtained from (2) taking into account the dependences of B_T and A_I on p (11, 12).] The N-shaped dependences may be observed for any concentration of N_A if the proper N_D is choosen ($N_D/N_A \gtrsim 0.03$ enough for $N_A = 10^{15}\ \mathrm{cm^{-3}}$ but it increases up to $\gtrsim 0.07$ for $N_A = 10^{14}$, $\gtrsim 0.2$ for $N_A = 10^{13}$ and up to almost $\gtrsim 0.7$ for $N_A = 10^{11}\ \mathrm{cm^{-3}}$). The segment of the current-voltage characteristic with negative differential conductivity (n-ndc) is, of course, unstable [1, 2].

2) If the compensation is decreasing, solutions to (14) appear. There is not just one solution as normal [10, 11], but two. [Of course, the third solution is also in the higher electric field, when our approximate solution of Boltzmann's transport equation (11) is not applicable due to strong penetration of carriers in the

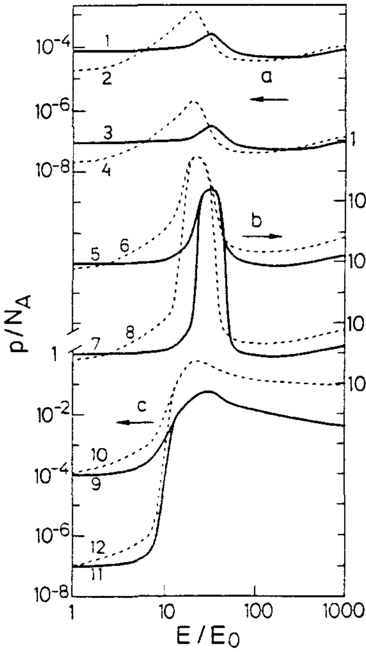

Fig. 2. Dependence of normalized hole concentration p/N_A on normalized electric field E/E_0 for two excitation levels $A/(B_T(E=0)N_D)=10^{-7}$ (curves *3, 4, 7, 8, 11, 12*) and 10^{-4} (*1, 2, 5, 6, 9, 10*), for two different dopings $N_A=10^{14}\,\mathrm{cm}^{-3}$ (solid curves) and $10^{11}\,\mathrm{cm}^{-3}$ (dashed curves) and for different compensation $N_D/N_A=0.08$ (*1, 3*), 0.8 (*2, 4*), 0.03 (*5, 7*), 0.4 (*6, 8*), 0.003 (*9, 11*), 0.1 (*10, 12*)

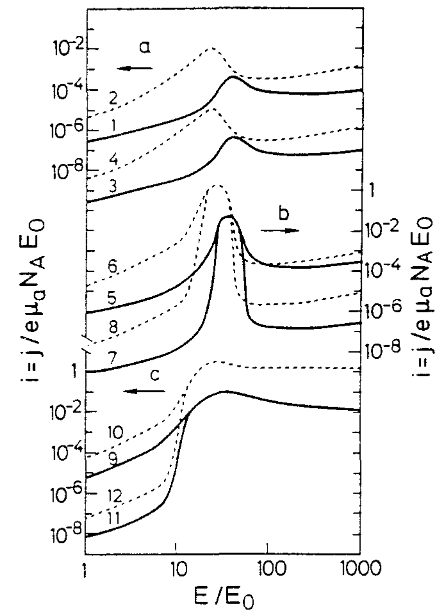

Fig. 3. Dependence of the current on the electric field E/E_0 for the same cases as in Fig. 2

active region.] The first solution is a threshold electric field, but the second refers to as an antithreshold. The concentration sharply increases at the threshold electric field and decreases at the antithreshold (Fig. 2b). This change in concentration causes a huge impulse in the current voltage characteristics (Fig. 3b). The smaller A is, the larger is the increase and subsequent decrease of the current.

The second case is realised only in a certain range of compensation

$(0.02 \gtrsim N_D/N_A \gtrsim 0.0003 \quad \text{for} \quad N_A=10^{15}\,\mathrm{cm}^{-3},$

$0.06 \gtrsim N_D/N_A \gtrsim 0.008 \quad \text{for} \quad N_A=10^{14}\,\mathrm{cm}^{-3},$

$0.3 \gtrsim N_D/N_A \gtrsim 0.03 \quad \text{for} \quad N_A=10^{13}\,\mathrm{cm}^{-3},$

and

$0.6 \gtrsim N_D/N_A \gtrsim 0.15 \quad \text{for} \quad N_A=10^{11}\,\mathrm{cm}^{-3}).$

3) Only one solution of (14) is possible for the low-compensating samples. The decrease of A_I with increasing E is essential even in this case. The decrease of concentration and the N-shaped negative differential conductivity (n-ndc) are realised in the post-breakdown region (Figs. 2c and 3c).

Therefore the n-ndc and nonlinear instabilities may be observed in the prebreakdown region, as it was for highly compensated samples in [3] (the authors had samples even with $N_D/N_A \to 1$) or in the postbreakdown region as in weakly compensated samples in [6]. The antithreshold may be realised in intermediately compensated samples. This results in a huge n-ndc which may lead to device applications similar to the Gunn diode.

The most important consequence of the first two cases is that the current is sensitive to the excitation intensity A in a higher electric field region. It gives one the opportunity to increase the sensitivity of the infrared detectors [4] preparing them from highly compensating p-Ge.

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References

1. H. Hartnagel: *Semiconductor Plasma Instabilities* (American Elsevier, New York 1969)
2. J. Pozhela: *Plasma and Current Instabilities in Semiconductors* (Oxford, Frankfurt 1981)
3. S.W. Teitsworth, R.M. Westervelt, E.E. Haller: Phys. Rev. Lett. **51**, 825 (1983)
4. R.J. Keyes (ed.): Optical and Infrared Detectors, 2nd ed., Topics Appl. Phys. **19** (Springer, Berlin, Heidelberg 1980)

5. S.W. Teitsworth, R.M. Westervelt: Phys. Rev. Lett. **53**, 2587 (1984)
6. J. Peinke, A. Mühlbach, R.P. Huebener, J. Parisi: Phys. Lett. **108A**, 407 (1985)
7. Z.S. Gribnikov, V.A. Kochelap: Zh. Eksp. Teor. Fiz. **58**, 1046 (1970) [Sov. Phys.-JETP **31**, 562 (1970)]
8. R.I. Rabinovich: Fiz. Tekh. Poluprov. **3**, 996 (1969) [Sov. Phys.-Semicond. **3**, 439 (1970)], Fiz. Tverd. Tela, **12**, 577 (1970) [Sov. Phys.-Solid State **12**, 440 (1970)]
9. S.P. Ashmontas, Ju.K. Pozhela, L.E. Subachyus: Pis'ma Zh. Eksp. Teor. Fiz. **33**, 580 (1981) [JETP Lett. **33**, 564 (1981)]
10. S.H. Koenig, R.D. Brown, W. Schillinger: Phys. Rev. **128**, 1668 (1962)
11. V.F. Bannaya, L.I. Veselova, E.M. Gershenson, V.A. Chuenkov: Fiz. Tekh. Poluprov. **7**, 1972 (1973) [Sov. Phys.-Semicond. **7**, 1315 (1974)]
12. V.N. Abamukov, I.N. Yassievich: Zh. Eksp. Theor. Fiz. **71**, 657 (1976) [Sov. Phys.-JETP **44**, 345 (1976)]
13. M. Lax: Phys. Rev. **119**, 1502 (1960)
14. S. Komiyama: Adv. Phys. **31**, 255 (1982)
15. Ju.K. Pozhela, E.V. Starikov, P.N. Shiktorov: Phys. Lett. **96A**, 361 (1983); Fiz. Tekh. Poluprovodn. **17**, 904 (1983) [Sov. Phys.-Semicond. **17**, 566 (1983)]
16. I.I. Vosilius, I.B. Levinson: Zh. Eksp. Teor. Fiz. **50**, 1660 (1966) [Sov. Phys.-JETP **23**, 1104 (1966)]
17. E.M. Conwell: High Field Transp. in Semicond., *Solid State Phys.* **9** (Academic, New York 1967)
18. H.G. Reik, H. Risken: Phys. Rev. **124**, 777 (1961)
19. V.N. Abamukov, V.I. Perel', I.N. Yassievich: Fiz. Tekh. Poluprovodn. **12**, 3 (1978) [Sov. Phys.-Semicond **12**, 1 (1978)]
20. B.I. Davydov, I.M. Shmushkevich: Usp. Fiz. Nauk **24**, 21 (1940)