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Galvanomagnetic Effects in Semiconductors of p-Ge Type

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It is shown that the rate equations for the light and heavy hole distribution functions in p-Ge can be solved analytically using the real anisotropic dispersion law since for acoustic lattice scattering the collision terms can be described by the isotropic relaxation time depending only on the energy of holes, which is proved by numerical computations. The even Hall effect, the transverse and longitudinal magnetoresistance (MR) as a function of the current and the magnetic field orientation with respect to the crystallographic axes in non heating electric fields E as well as the transverse $\gamma_\perp$ and longitudinal $\gamma_{||}$ anisotropy of conductivity arising in the heating fields, and the MR linear in the magnetic field are calculated. The linear MR is determined largely by light holes (without these or with their isotropic spectrum the MR would have the reverse sign for the same direction of the magnetic field). The relative contributions of heavy holes to $\gamma_\perp$ and $\gamma_{||}$ are nearly half of that in the earlier approximate calculations. The even Hall effect, the longitudial MR, and the anisotropy of the transverse MR are almost entirely due to the anisotropy of the heavy hole dispersion law.

Показано, что кинетические уравнения для функций распределения легких и тяжелых дырок в p-Ge могут быть решены аналитически с реальным анизотропным законом дисперсии, потому что при рассеянии на акустических колебаниях решетки столкновительные члены могут быть описаны изотропным временем релаксации, зависящим только от энергии дырок, что было доказано расчетами на ЭВМ. Вычислены четный эффект Холла, зависимость поперечного и продольного магнитосопротивления (МС) от ориентации тока и магнитного поля относительно кристаллографических осей в негреющих электрических полях E, а также возникающие в греющих полях поперечная $\gamma_\perp$ и продольная $\gamma_{||}$ анизотропия проводимости и линейное по магнитному полю МС. Линейное МС определяется в основном легкими дырками (при их отсутствии или при изотропном их спектре МС имело бы противоположный знак для того же направления магнитного поля). Относительный вклад тяжелых дырок в $\gamma_\perp$ и $\gamma_{||}$ уменьшился практически в два раза по сравнению с приближенными расчетами, выполненными ранее, при этом более, чем в два раза уменьшаются и сами $\gamma_\perp$ и $\gamma_{||}$. Четный эффект Холла, продольное МС и анизотропия поперечного МС практически полностью обусловлены анизотропей закона дисперсии тяжелых дырок.

1. Introduction

In semiconductors of p-Ge type the light and heavy hole bands are degenerate at $\boldsymbol{k} = 0$ ($\boldsymbol{k}$ is the wave vector) and warped [1], therefore, they exhibit a variety of effects not found in semiconductors with an isotropic dispersion law $\varepsilon(\boldsymbol{k})$. For the non-heating electric field the transverse magnetoresistence (MR) depends on the orientation of electric E and magnetic H field with respect to the crystallographic axes [2], the longitudinal MR, even Hall effect [3, 4], longitudinal Hall effect [5], etc. were observed. Transverse and longitudinal anisotropy of conductivity [6, 7] and MR linear in magnetic field [8, 9] arise in a heating electric field.

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The anisotropic kinetic effects were calculated either numerically [10], using an exact dispersion law for heavy holes and an isotropic one for light holes, or analytically [5, 11 to 13] with approximate $\varepsilon(\boldsymbol{k})$, when $\varepsilon(\boldsymbol{k})$ is

$$\left.\begin{aligned} \varepsilon_i(\boldsymbol{k}) &= \frac{\hbar^2 k^2}{2m_0}\,\Phi_i, \qquad \Phi_i = |A| + (-1)^{i+1} M, \\ M &= \sqrt{B^2 + \varkappa C^2 + C^2\big(\sum_{\alpha>\beta} k_\alpha^2 k_\beta^2/k^4 - \varkappa\big)} \end{aligned}\right\} \tag{1}$$

and M is expanded in powers of $C^2(\sum_{\alpha>\beta} k_\alpha^2 k_\beta^2/k^4 - \varkappa)$ which essentially simplifies all calculations. In [5] $\varkappa = 1/6$, and in [11 to 13] $\varkappa = 1/5$. In (1) A, B, C are the band parameters [1], $i = 1$ for light holes and $i = 2$ for heavy holes, $\alpha, \beta = 1, 2, 3$ coincide with the $\langle 100\rangle$ crystal axes, m_0 is the free electron mass. The isotropic approximation is obtained from (1) when $\varkappa = 1/5$, dropping $C^2(\sum_{\alpha>\beta} k_\alpha^2 k_\beta^2/k^4 - \varkappa)$.

The expressions for the kinetic coefficients involve the derivatives $\partial^n \varepsilon_i(\boldsymbol{k})/\partial \boldsymbol{k}_\alpha^n$. It is not difficult to see that the discrepancy between the derivatives calculated by the exact equation (1) and the approximate [5, 11 to 13] laws $\varepsilon_i(\boldsymbol{k})$ increases with increasing n, reaching 50% when $i = 2$ for some directions of $\boldsymbol{k}$ even at $n = 2$, provided for $\varkappa = 1/5$ only the two first expansion terms are retained. Therefore, it is of interest to calculate the galvanomagnetic effects using the exact dispersion law (1), and the comparison of the results obtained with the approximate calculations will enable us to determine their validity range.

2. Rate Equations

In order to calculate the galvanomagnetic coefficients it is necessary to solve the set of rate equations for the hole distribution function $\mathcal{F}_{\boldsymbol{p}}^{(i)}$

$$e\left(\boldsymbol{E} + \frac{1}{c}\,[\boldsymbol{v}_i \boldsymbol{H}]\right) \nabla_{\boldsymbol{p}} \mathcal{F}_{\boldsymbol{p}}^{(i)} = \sum_{j=1}^{2} \sum_{\boldsymbol{p}'} \left(W_{\boldsymbol{p}'j,\boldsymbol{p}i} \mathcal{F}_{\boldsymbol{p}'}^{(j)} - W_{\boldsymbol{p}i,\boldsymbol{p}'j} \mathcal{F}_{\boldsymbol{p}}^{(i)}\right), \tag{2}$$

where $\boldsymbol{v}_i = \nabla_{\boldsymbol{p}} \varepsilon_i$, $W_{\boldsymbol{p}'j,\boldsymbol{p}i}$ are the charge carrier transition probability from the state with the quasimomentum $\boldsymbol{p}'$ of the j-th band to the state $\boldsymbol{p}$ of the i-th band. We consider semiconductors with predominant acoustic scattering. $W_{\boldsymbol{p}'j,\boldsymbol{p}i}$ for this case are obtained in [1]. The current density in each of the bands is expressed as usual through $\mathcal{F}_{\boldsymbol{p}}^{(i)}$, which are normalized to the hole concentration $\mathcal{P}$.

$$j^{(i)} = e \sum_{\boldsymbol{p}} \boldsymbol{v}_i \mathcal{F}_{\boldsymbol{p}}^{(i)}, \qquad \mathcal{P} = \sum_{i=1}^{2} \sum_{\boldsymbol{p}} \mathcal{F}_{\boldsymbol{p}}^{(i)}. \tag{3}$$

Let us represent $\mathcal{F}_{\boldsymbol{p}}^{(i)}$

$$\mathcal{F}_{\boldsymbol{p}}^{(i)} = \mathcal{F}_0^{(i)} + \sum_{r=1}^{3} \sum_{m=0}^{2} f_{rm}^{(i)} E^r H^m, \tag{4}$$

where $\mathcal{F}_0^{(i)} = \sum_{\boldsymbol{p}} \mathcal{F}_{\boldsymbol{p}}^{(i)}\, \delta(\varepsilon - \varepsilon_{\boldsymbol{p}}) / \sum_{\boldsymbol{p}} \delta(\varepsilon - \varepsilon_{\boldsymbol{p}})$ is the symmetric part of the distribution function dependent only on energy. The terms of higher order in E and H will not be considered here, since we examine only three particular cases: non-heating E and weak H (Section 3), arbitrary heating E at $H = 0$ (Section 4), magnetoresistance linear in the magnetic field in heating electric fields (Section 5). It is necessary to emphasize that equations (4) are not expansions in powers of E but they are valid for arbitrary electric field when quasi-elastic mechanisms of scattering are predominant as take place here (i.e. $\mathcal{F}_0^{(i)}(\varepsilon)$ are dependent on E and (4) are expansions in powers of $v_d/v_r \ll 1$, where v_d is the drift velocity due to the electric field E, and v_r is the mean random velocity).

3. Non-Heating Electric Field

Here $\mathcal{F}_0^{(i)}(\varepsilon) = f_0(\varepsilon)$ coincides with the equilibrium Boltzmann distribution function and it is necessary to find $f_{10}^{(i)}, f_{11}^{(i)}, f_{12}^{(i)}$. For the light holes we neglect in the right-hand side of (2) the terms describing an arrival from the first and the second bands, and for the heavy holes, the terms describing an arrival from the first band. Such an approach will be proved below after calculating $\mathcal{F}_{\boldsymbol{p}}^{(i)}$ (for similar arguments for the isotropic $\varepsilon(\boldsymbol{k})$, see [14]). Then for $i = 1$ we have the trivial solution

$$f_{10}^{(1)} = e\vec{\mathcal{E}}\boldsymbol{v}^{(1)}\tilde{\tau}_1 f_0/k_0T\,, \qquad f_{1m}^{(1)} = -\frac{e}{c}\,\tilde{\tau}_1[\boldsymbol{v}^{(1)}\vec{\mathcal{H}}]\,\nabla_{\boldsymbol{p}} f_{1m-1}^{(1)}\,, \qquad (m = 1, 2)\,, \quad (5)$$

where $\vec{\mathcal{H}} = \boldsymbol{H}/H$, $\vec{\mathcal{E}} = \boldsymbol{E}/E$, $1/\tilde{\tau}_1 = \sum_{j=1}^{2}\sum_{\boldsymbol{p}'} W_{\boldsymbol{p}1,\boldsymbol{p}'j}$. The relaxation times $\tilde{\tau}_1$ were calculated numerically using the deformation potential constants from [15] (for p-Ge $a = -2.6$, $b = -1.8$, $d = -3.7$ and for p-Si $a = -3$, $b = -2.4$, $d = -5.3$) and the band parameters from [1] (for p-Ge $A = 13.27$, $B = -8.63$, $C = 12.4$ and for p-Si $A = 4.28$, $B = -0.75$, $C = 5.25$). It was found that $\tilde{\tau}_1$ is weakly dependent on the direction of $\boldsymbol{p}$ as compared with $\boldsymbol{v}^{(1)}$ and the other $\boldsymbol{p}$-dependent functions entering (5). Therefore, $\tilde{\tau}_1 = \tau_1\sqrt{k_0T/\varepsilon_1}$ is a good approximation. For the heavy holes, the time describing a departure of carriers from the $\boldsymbol{p}$ state $1/\tilde{\tau}_2 = \sum_{j=1}^{2}\sum_{\boldsymbol{p}'} W_{\boldsymbol{p}2,\boldsymbol{p}'j}$ is also practically isotropic and can be written in the form $\tilde{\tau}_2 = \tau_2\sqrt{k_0T/\varepsilon_2}$. This was also calculated numerically. The term $\sum_{\boldsymbol{p}'} W_{\boldsymbol{p}'2,\boldsymbol{p}2}(\mathcal{F}_{\boldsymbol{p}'}^{(2)} - \mathcal{F}_0^{(2)})$ can be described by the relaxation time only in the case, if $\mathcal{F}_{\boldsymbol{p}'}^{(2)} - \mathcal{F}_0^{(2)} = \boldsymbol{p}'\boldsymbol{\chi}(\varepsilon_2')$, therefore, it is convenient to represent $\boldsymbol{v}^{(2)}$ in the form

$$\boldsymbol{v}^{(2)} = (\boldsymbol{v}^{(2)} - \boldsymbol{\nu}^{(2)}) + \boldsymbol{\nu}^{(2)}\,, \qquad \boldsymbol{\nu}^{(2)} = \frac{\hbar\boldsymbol{k}}{m_2}. \quad (6)$$

We have shown that for $m_2 = m_0/\left(|A| - \sqrt{B^2 + C^2/5}\right)$ the solution of (2) for $i = 2$ is

$$f_{10}^{(2)} = e\left(\vec{\mathcal{E}}\boldsymbol{v}^{(2)}\tilde{\tau}_2 - \vec{\mathcal{E}}\boldsymbol{\nu}^{(2)}(\tilde{\tau}_2 - \tilde{\tau}_2')\right) f_0/k_0T\,, \quad (7)$$

$$\begin{aligned} f_{11}^{(2)} = &-\frac{e^2}{c}\left(\tilde{\tau}_2^2[\boldsymbol{v}^{(2)}\vec{\mathcal{H}}]\,\nabla_{\boldsymbol{p}}(\boldsymbol{v}^{(2)}\vec{\mathcal{E}}) - \tilde{\tau}_2(\tilde{\tau}_2 - \tilde{\tau}_2') \times \right.\\ &\left.\times [\vec{\mathcal{H}}\vec{\mathcal{E}}]\,\boldsymbol{v}^{(2)}/m_2 - \tilde{\tau}_2'(\tilde{\tau}_2 - \tilde{\tau}_2')\,[\vec{\mathcal{H}}\vec{\mathcal{E}}]\,\boldsymbol{\nu}^{(2)}/m_2\right) f_0/k_0T\,, \end{aligned} \quad (8)$$

$$\begin{aligned} f_{12}^{(2)} = &\frac{e^3}{c^2}\,[\tilde{\tau}_2^3[\boldsymbol{v}^{(2)}\vec{\mathcal{H}}]\,\nabla_{\boldsymbol{p}}\left([\boldsymbol{v}^{(2)}\vec{\mathcal{H}}]\,\nabla_{\boldsymbol{p}}(\boldsymbol{v}^{(2)}\vec{\mathcal{E}})\right) - \tilde{\tau}_2^2(\tilde{\tau}_2 - \tilde{\tau}_2') \times \\ &\times [\boldsymbol{v}^{(2)}\vec{\mathcal{H}}]\,\nabla_{\boldsymbol{p}}(\boldsymbol{v}^{(2)}[\vec{\mathcal{H}}\vec{\mathcal{E}}])/m_2 - \tilde{\tau}_2\tilde{\tau}_2'(\tilde{\tau}_2 - \tilde{\tau}_2')\,[\boldsymbol{v}^{(2)}\vec{\mathcal{H}}] \times \\ &\times [\vec{\mathcal{H}}\vec{\mathcal{E}}]/m_2 - (\tilde{\tau}_2')^2\,(\tilde{\tau}_2 - \tilde{\tau}_2')\,[\boldsymbol{\nu}^{(2)}\vec{\mathcal{H}}]\,[\vec{\mathcal{H}}\vec{\mathcal{E}}]/m_2^2]\, f_0/k_0T\,. \end{aligned} \quad (9)$$

The time $\tilde{\tau}_2'$ is given by the expression $\boldsymbol{p}/\tilde{\tau}_2' = \boldsymbol{p}/\tilde{\tau}_2 - \sum_{\boldsymbol{p}'} \boldsymbol{p}' W_{\boldsymbol{p}'2,\boldsymbol{p}2}$ with $\tilde{\tau}_2' = \tau_2'\sqrt{k_0T/\varepsilon_2}$ and is also almost independent of the direction of $\boldsymbol{p}$. The distribution function for $i = 2$ has a more complicated form than that for $i = 1$, since for $i = 2$

there are two specified relaxation times, the time $\tilde{\tau}_2'$ describing only the function proportional to $\boldsymbol{v}^{(2)}$, whereas the one proportional to the difference $(\boldsymbol{v}^{(2)} - \boldsymbol{\nu}^{(2)})$ and the more complicated functions of $\boldsymbol{p}$ is described by the time $\tilde{\tau}_2$. The difference $\tilde{\tau}_2' - \tilde{\tau}_2$ enters (7) to (9) since $f_{1i}^{(2)}$ are reduced to a form not involving the difference $(\boldsymbol{v}^{(2)} - \boldsymbol{\nu}^{(2)})$. In p-germanium $\tilde{\tau}_2' < \tilde{\tau}_2$ since $\sum\limits_{\boldsymbol{p}'} \boldsymbol{p}' W_{\boldsymbol{p}'2,\boldsymbol{p}2} < 0$ (this result for the isotropic approximation can be found in [14]). We note that the relaxation times $\tilde{\tau}_1$, $\tilde{\tau}_2$, $\tilde{\tau}_2'$ calculated here practically do not differ from the corresponding times obtained in [14] in the isotropic approximation, namely: $1/\tau_1 = 1/\tau_L^0 \times 6.13$, $1/\tau_2 = 1/\tau_L^0 \times 4.65$, $1/\tau_2' = 1/\tau_L^0 \times 5.6$, where $1/\tau_L^0 = (m_2 k_0 T)^{3/2}/(\sqrt{2}\,\pi \varrho h^4 C_L^2)$.

Substituting the functions (5) and (7) to (9) into (2) and integrating with a computer, we show that up to the terms $\sim m_1/m_2 = (|A| - \sqrt{B^2 + C^2/5})/(|A| + \sqrt{B^2 + C^2/5}) \ll 1$ these functions are the solution of (2) which supports the validity of the approach used.

From (3), (5), and (7) to (9) it is easy to obtain expressions for the galvanomagnetic coefficients, the anisotropy of the dispersion law is fully taken into account and enters only in the velocities. Note that a similar situation holds in n-Ge, where the anisotropy of the relaxation times is much smaller than that of the velocity (i.e. the effective masses) and can well approximated by the isotropic relaxation time [15].

The expression for the current density is of the form

$$\boldsymbol{j} = \sum_{i=1}^{2} [\sigma^{(i)} \boldsymbol{E} + \eta^{(i)} [\boldsymbol{E}\boldsymbol{H}] + \alpha^{(i)} H^2 \boldsymbol{E} + \beta^{(i)} [\boldsymbol{E}\boldsymbol{H}]\, \boldsymbol{H} + \gamma^{(i)} \boldsymbol{F}]\,, \tag{10}$$

where $\boldsymbol{F}$ is the vector with the components $E_1 H_1^2$, $E_2 H_2^2$, $E_3 H_3^2$ along the principal {100} axes of the crystal; the kinetic coefficients have the same symbols as in [16] (p. 360), for the acoustic scattering being

$$\left.\begin{aligned}
&\sigma^{(i)} = e \mathscr{P}_i \mu_i \bar{\sigma}^{(i)}\,; \qquad && \mathscr{P}_i = \frac{m_i^{3/2}}{m_1^{3/2} + m_2^{3/2}}\, \bar{\mathscr{P}}_i\,, \qquad && \mu_i = \frac{4 e \tau_i'}{3 \sqrt{\pi}\, m_i}\,,\\
&\eta^{(i)} = \frac{e}{c} \mathscr{P}_i \frac{3\pi}{8} \mu_i^2 \bar{\eta}^{(i)}\,, && \alpha^{(i)} = -\frac{e}{c^2} \mathscr{P}_i \frac{9\pi}{16} \mu_i^3 \bar{\alpha}^{(i)}\,, && \beta^{(i)} = \frac{e}{c^2} \mathscr{P}_i \frac{9\pi}{16} \mu_i^3 \bar{\beta}^{(i)}\,,\\
&\gamma^{(i)} = \frac{e}{c^2} \mathscr{P}_i \frac{9\pi}{16} \mu_i^3 \bar{\gamma}^{(i)}\,; && \bar{\gamma}^{(i)} = \bar{\alpha}^{(i)} - \bar{\beta}^{(i)} + 2 \bar{\delta}^{(i)}\,, && \tau_1' = \tau_1\,.
\end{aligned}\right\} \tag{11}$$

$\mathscr{P}_{1,2}$ is the hole concentration in bands 1, 2.

The expressions for all the factors with a bar are given in the Appendix. They are chosen so that in the isotropic approximation all the galvanomagnetic coefficients are of the general form (see [1, 16]), since in this case $\bar{\mathscr{P}}_i$, $\bar{\sigma}^{(i)}$, $\bar{\eta}^{(i)}$, $\bar{\alpha}^{(i)}$, $\bar{\beta}^{(i)}$ are equal to unity and $\bar{\delta}^{(i)} = 0$.

The calculation of these integrals for p-Ge yields $\bar{\mathscr{P}}_1 = 0.94$, $\bar{\mathscr{P}}_2 = 1$, $\bar{\sigma}^{(1)} = 1$, $\bar{\sigma}^{(2)} = 1.2$, $\bar{\eta}^{(1)} = 1$, $\bar{\eta}^{(2)} = 0.88$, $\bar{\alpha}^{(1)} = 1$, $\bar{\alpha}^{(2)} = 4$, $\bar{\beta}^{(1)} = 1$, $\bar{\beta}^{(2)} = 0.4$, $\bar{\delta}^{(1)} = 7 \times 10^{-4}$, $\bar{\delta}^{(2)} = -5.5 \times 10^{-2}$, $\bar{\gamma}^{(1)} = 0.038$, $\bar{\gamma}^{(2)} = 3.7$; i.e. the hole distribution between the bands 1 and 2 is here nearly the same as in the isotropic approximation in [14]: $\mathscr{P}_1/\mathscr{P}_2 = 0.043$ (0.046). Here and below the values for the isotropic approximation are given in brackets.

The contribution from the light holes to the conductivity and the Hall effect, too, is almost the same as in the isotropic approximation: $\sigma^{(1)}/\sigma^{(2)} = 0.26$ (0.33), $\eta^{(1)}/\eta^{(2)} = 2.3$ (2.34). Besides in the terms proportional to H^2, taking account of the dispersion law anisotropy gives no essential corrections only for the light holes, whereas for the heavy ones it is essential and can change the results considerably. A relative contribution from light holes to the corresponding coefficients is given by the ratios $\alpha^{(1)}/\alpha^{(2)} = 4$ (17), $\beta^{(1)}/\beta^{(2)} = 40.2$ (17), $\gamma^{(1)}/\gamma^{(2)} = 0.16$ ($\gamma^{(1)} = 0$, $\gamma^{(2)} = 0$). From (10) it is seen that the transverse MR $\Delta\varrho/\varrho_0 = (\varrho_H - \varrho_0)/\varrho_0$ is anisotropic, the MR is determined by the contribution of the light holes ($\alpha^{(1)} > \alpha^{(2)}$) and the anisotropy of the MR is entirely determined by the heavy holes ($\gamma^{(2)} \gg \gamma^{(1)}$, $\gamma^{(2)} \sim \alpha^{(2)}$).

Substituting (11) into (10) we have for the transverse MR

$$\left(\frac{\Delta\varrho}{\varrho_0}\right)_\perp = \frac{9\pi}{16}\left(\frac{\mu_2 H}{c}\right)^2 9.5\left[1 - \frac{0.29}{j^2}\sum_{\alpha=1}^{3} j_\alpha^2 \mathcal{H}_\alpha^2\right]. \tag{12}$$

With the current along $\langle 100 \rangle$ the expression in the square brackets is unity, the MR has a maximum value and does not depend on the orientation of $\boldsymbol{H}$ with respect to the crystallographic axes. The MR has a minimum, if $\boldsymbol{j} \parallel [110]$, $\boldsymbol{H} \parallel [1\bar{1}0]$. Note, that calculated in the isotropic approximation, the MR is isotropic as is to be expected and equals

$$\left(\frac{\Delta\varrho}{\varrho_0}\right) = \frac{9\pi}{16}\left(\frac{\mu_2 H}{c}\right)^2 8.4\,. \tag{13}$$

It is interesting that the difference between (12) and (13) is not large, though $\alpha^{(2)}$ and $\beta^{(2)}$ changed drastically with the account of the anisotropy. The longitudinal MR is

$$\left(\frac{\Delta\varrho}{\varrho_0}\right)_{\parallel} = \frac{9\pi}{16}\left(\frac{\mu_2 H}{c}\right)^2 2.8\left[1 - \frac{1}{j^4}\sum_{\alpha=1}^{3} j_\alpha^4\right]. \tag{14}$$

It has a minimum value, if $\boldsymbol{j} \parallel [100]$, and a maximum, if $\boldsymbol{j} \parallel [110]$, its anisotropy being much larger than that of the transverse MR. Provided $\boldsymbol{j} \parallel [110]$, the ratio of the transverse MR for $\boldsymbol{H} \parallel [1\bar{1}0]$ to the longitudinal MR is 6.9 which is near the experimental value 6.2 obtained at 300 K in [2].

From (10) it is easy to derive also the expression for the even Hall effect

$$\frac{E_H}{E_x} = \frac{9\pi}{16}\left(\frac{\mu_2 H}{c}\right)^2 2.8\,\frac{1}{j^2}\sum_{\alpha=1}^{3}\sum_{\beta,\gamma=1}^{3} \delta_{\alpha\beta\gamma} j_\alpha j_\beta \mathcal{H}_\alpha \mathcal{H}_\gamma\,, \tag{15}$$

where E_H is the field measured along the Hall direction. The even Hall effect is almost entirely due to the heavy holes since $E_H/E_x \sim \gamma^{(1)} + \gamma^{(2)}$, and $\gamma^{(1)}/\gamma^{(2)} = 0.16$. The orientational dependence of the even Hall effect on the directions of $\boldsymbol{E}$ and $\boldsymbol{H}$ coincides with the experimental [3, 4] one. It should be noted that in Fig. 4 of [4] the direction of the electric field at which $E_H/E_x = 0$ is not correct. This value is 54.6°. The even Hall effect as well as the longitudinal MR are zero in the isotropic approximation. The calculation of the current (10) for the case when $\varepsilon_i(\boldsymbol{k})$ in (1) with $\varkappa = 1/5$ is expanded in powers of the anisotropy, yields the same expressions as in the isotropic approximation, if the terms of first order in the anisotropy are retained. The values $(\Delta\varrho/\varrho_0)_\perp$, $(\Delta\varrho/\varrho_0)_\parallel$, and E_H/E_x for p-germanium taking into account the real dispersion law appear to be obtained for the first time.

4. Longitudinal and Transverse Anisotropy of Conductivity

Retaining only acoustic scattering the rate equation (2) can be solved for arbitrary electric fields with anisotropic $\varepsilon_i(\boldsymbol{k})$ (1) as was done in [17] for $\boldsymbol{H} = 0$ with the expansion in powers of $C^2(\sum_{\alpha>\beta} k_\alpha^2 k_\beta^2/k^4 - \varkappa)$. Then the symmetric part of the distribution function is the same in both bands and equal to the usual Druyvestyn's function [18],

$$\mathcal{F}_0^{(1,2)} = \mathcal{F}_0(x) = C_1(x + \varepsilon_E^2)^{\varepsilon_E^2}\, e^{-x}\,, \tag{16}$$

where

$$x = \frac{\varepsilon}{k_0 T}, \qquad \varepsilon_E^2 = \frac{2}{3} \frac{e^2 E^2 \tau_2' \tau_\varepsilon}{k_0 T\, m_2} \left(\bar{\sigma}^{(2)} + \frac{\tau_1}{\tau_2} \sqrt{\frac{m_1}{m_2}} \frac{\mathscr{P}_1}{\mathscr{P}_2} \bar{\sigma}^{(1)} \right), \tag{17}$$

C_1 is the normalization constant calculated from (3), $\tilde{\tau}_\varepsilon = \tau_\varepsilon \sqrt{k_0 T/\varepsilon_2}$ is the relaxation time of the energy in band 2, $1/\tau_\varepsilon = \left(a^2 m^{5/2} r_2' \sqrt{2 k_0 T}\right)/\pi \hbar^4 \varrho$ where r_2' is slightly different from r_2 in (7) of [17] for the isotropic approximation, the same is valid for the constant C_1, too, therefore we can use the function $\mathscr{F}_0(\varepsilon)$ of the isotropic approximation with high accuracy (the numerical value of r_2 for p-Ge is 0.962).

When calculating the functions $f_{10}^{(i)}, f_{20}^{(i)}, f_{30}^{(i)}$ we first made the same assumptions about the relaxation times as in Section 3. However, because of a slight difference between the times τ_2 and τ_2' in p-germanium, the numerical values of the resultant coefficients practically coincide with the case when a unique relaxation time $\tau_2' = \tau_2$ is introduced also for the heavy holes. Below only the results for $\tau_2' = \tau_2$ are given in order not to write out more cumbersome expressions with $\tau_2' \neq \tau_2$.

The conductivity $\sigma^{(i)}$ is isotropic and its dependence on the electric field is determined by the expression for $\sigma^{(i)}$ from (11) multiplied by

$$\frac{\sqrt{\pi}}{2} \int_0^\infty \mathscr{F}_0(x)\, dx \Big/ \int_0^\infty \sqrt{x}\, \mathscr{F}_0(x)\, dx\,. \tag{18}$$

Along with the second-rank diagonal tensor $\boldsymbol{\sigma}^{(i)}$ also the components of the fourth-rank tensor are non-zero, i.e. in (10) the term $\sum_{i=1}^{2} \sigma_{\alpha\beta\gamma\delta}^{(i)} E_\beta E_\gamma E_\delta$ must be added. Just this term is responsible for the longitudinal and transverse anisotropy of the conductivity which is characterized by the parameters $\gamma_{\|} = [j_x(E_x \| [100]) - j_x(E_x)]/j_x(E_x \| [100])$ and $\gamma_\perp = E_y/E_x$ where E_y is the electric field transverse to the current. Since $E_y \ll E_x$ we consider only the components of the conductivity tensor $\sigma_{xxxx} = \sum_{i=1}^{2} \sigma_{xxxx}^{(i)}$ and $\sigma_{yxxx} = \sum_{i=1}^{2} \sigma_{yxxx}^{(i)}$ for which we obtain

$$\sigma_{xxxx} = UW\,, \qquad \sigma_{yxxx} = VW\,, \tag{19}$$

where U and V are factors dependent on the orientations of the axes x and y in the crystal equal to

$$U = \sum_{\alpha=1}^{3} \cos^2 (\widehat{x,\alpha}) \sin^2 (\widehat{x,\alpha})\,, \qquad V = \sum_{\alpha=1}^{3} \cos^3 (\widehat{x,\alpha}) \cos (\widehat{y,\alpha})\,, \tag{20}$$

In particular, when the plane xy coincides with $\{110\}$ the sums over α in (20) are $\frac{1}{2} \sin^2 \varphi (3 \cos^2 \varphi + 1)$ and $-\frac{1}{4} \sin 2\varphi (3 \cos^2 \varphi - 1)$, respectively, where φ is the angle between the axis $\langle 001 \rangle$ and the current direction. W in (19) has the dimension of conductivity divided by the squared electric field and is given by the expression

$$W = \mathscr{P} \frac{e^4}{m_0^2} \times$$

$$\times \frac{\sum_{i=1}^{2} \left[S_1^{(i)} \int_0^\infty \tilde{\tau}_i^3 x^{1/2} \chi\, dx + 2 S_2^{(i)} \int_0^\infty x^{3/2} \left(\tilde{\tau}_i^2 \chi \frac{d\tilde{\tau}_i}{dx} + \tilde{\tau}_i^2 \frac{d(\tau_i, \chi)}{dx} + 1.2 x \tilde{\tau}_i \frac{d\tilde{\tau}_i}{dx} \frac{d(\tilde{\tau}_i \chi)}{dx} \right) dx \right]}{k_0 T \int_0^\infty x^{1/2} \mathscr{F}_0(x)\, dx (N^{(1)} + N^{(2)})}\,. \tag{21}$$

Here $\chi = -\mathrm{d}\mathcal{F}_0/\mathrm{d}x$ and the expressions for $S_1^{(i)}$, $S_2^{(i)}$, $N^{(i)}$ are presented in the Appendix.

If $\tilde{\tau}_i = \tau_i/\sqrt{x}$ as in the case of acoustic scattering, the expression for W essentially simplifies and for $\gamma_{\|}$ and $\gamma_{\perp}$ we have

$$\gamma_{\|} = \tfrac{7}{2}\, U \gamma_{\mathrm{sat}} \Gamma\,, \qquad \gamma_{\perp} = -\tfrac{7}{2}\, V\, \gamma_{\mathrm{sat}} \Gamma\,, \tag{22}$$

where

$$\Gamma = \varepsilon_E^2 \frac{\int\limits_0^\infty \frac{\mathcal{F}_0(x)}{x + \varepsilon_E^2}\, \mathrm{d}x}{\int\limits_0^\infty \mathcal{F}_0(x)\, \mathrm{d}x}\,, \qquad \gamma_{\mathrm{sat}} = \frac{9}{56\pi} \frac{\tau_2}{\tau_\varepsilon} \sqrt{\frac{m_2}{m_0}}\, \frac{\sum\limits_{i=1}^{2} \left(\frac{\tau_i}{\tau_2}\right)^3 (S_1^{(i)} - 1.4 S_2^{(i)})}{\left(\bar{\sigma}^{(2)} + \frac{\tau_1}{\tau_2} \sqrt{\frac{m_1}{m_2}} \frac{\mathcal{P}_1}{\mathcal{P}_2}\, \bar{\sigma}^{(1)}\right)^2}\,. \tag{23}$$

In [20] the integrals $S_k^{(i)}$ were calculated by expansion in powers of the anisotropy parameter. The following results were obtained: $S_1^{(i)} = 0$, $S_2^{(i)} = (-1)^{i+1}\,(4\pi/9) \times \times C^2/\sqrt{B^2 + C^2/5}$ and in p-Ge: $S_2^{(1)} = 4.31$, $S_2^{(2)} = -12.06$. Our calculations give $S_1^{(1)} = -0.689$, $S_1^{(2)} = -9.34$, $S_2^{(1)} = 4.4$, $S_2^{(2)} = -13.14$, i.e. for $S_2^{(i)}$ corrections are small and $S_1^{(2)}$ does not only become zero, but reaches a value of the order $S_2^{(2)}$. Because the coefficient $S_1^{(2)} - 1.4 S_2^{(2)}$ becomes almost half as large, not only the relative contributions from the heavy holes to $\gamma_{\|}$ and $\gamma_{\perp}$ decrease (they give the main contribution both here and in [17], but here their contribution is $-1.33(\tau_2/\tau_1)^3$ times that of the light holes, whereas in [17] it is $-2.8(\tau_2/\tau_1)^3$ times), but also $\gamma_{\|}$ and $\gamma_{\perp}$ themselves decrease more than twice (if $\tau_2/\tau_1 = 1$, $\gamma_{\|}$ and $\gamma_{\perp}$ decrease by a factor of 5 as compared with those calculated by expansion in powers of the anisotropy).

5. Linear Magnetoresistance

When holes are heated, there arises a magnetoresistance odd in the magnetic field. In the theory linear in H this can be readily calculated by the above scheme. The symmetric part of the distribution function can be calculated as before with neglection of the magnetic field. When the acoustic scattering dominates, it is of the form (16). By solving (4) for the functions $f_{rm}^{(i)}$ we obtain the following expression for the current density:

$$j_k^{(i)} = \sigma^{(i)} E_k + \eta^{(i)} E_l H_m \delta_{klm} + \sigma_{klmn}^{(i)} E_l E_m E_n + \sigma_{klmnr}^{(i)} E_l E_m E_n H_r\,. \tag{24}$$

All the coefficients entering in (24) depend on the electric field, for the acoustic scattering the dependence $\sigma^{(i)}(E)$ and $\sigma_{klmn}^{(i)}(E)$ are given in Section 4 and $\eta^{(i)}$ for the same case is obtained after multiplying $\eta^{(i)}$ from (11) by $\left(\int\limits_0^\infty \mathcal{F}_0(x)\, x^{-1/2}\, \mathrm{d}x\right)/\left(2 \int\limits_0^\infty \mathcal{F}_0(x)\, x^{1/2}\, \mathrm{d}x\right)$. Only the tensor $\sigma_{klmnr}^{(i)}$ is new here. When the magnetic field perpendicular to the current is along the axis z we are interested only in the $\sigma_{xxxxz} = \sum\limits_{i=1}^{2} \sigma_{xxxxz}^{(i)}$ component of this tensor. As in the preceding case, we obtain

$$\sigma^{(i)}_{xxxxz} = -\frac{1}{2} V \frac{e^5}{c m_0^3} \mathcal{P}[k_0 T \int_0^\infty x^{1/2} \mathcal{F}_0(x)\, \mathrm{d}x (N^{(1)} + N^{(2)})]^{-1} \times$$

$$\times \sum_{i=1}^{2} \left[10 \left(H^{(i)} - \frac{8}{3} F^{(i)} \right) \int_0^\infty \tilde{\tau}_i^3 \chi \frac{\mathrm{d}\tilde{\tau}_i}{\mathrm{d}x} x^{3/2}\, \mathrm{d}x + (G^{(i)} + 4.5 H^{(i)} - 15 F^{(i)}) \times \right.$$

$$\left. \times \int_0^\infty \tilde{\tau}_i^4 \chi x^{1/2}\, \mathrm{d}x + F^{(i)} \int_0^\infty x^{3/2} \tilde{\tau}_i \left(2 \frac{\mathrm{d}\tilde{\tau}_i}{\mathrm{d}x} \frac{\mathrm{d}(\tilde{\tau}_i^2 \chi)}{\mathrm{d}x} x - 5 \tilde{\tau}_i^2 \frac{\mathrm{d}(\tilde{\tau}_i \chi)}{\mathrm{d}x} \right) \right]. \tag{25}$$

The expressions for the integrals over the angles $H^{(i)}$, $G^{(i)}$, $F^{(i)}$ are given in the Appendix. Expanding in powers of the anisotropy the author of [13] obtained $G^{(i)} = -\frac{3}{4} H^{(i)}$, $H^{(i)} = (-1)^i (32\pi/9) \sqrt{m_0/m_i}\, C^2/\sqrt{B^2 + C^2/5}$, $F^{(i)} = 0$ (i.e. for p-Ge $H^{(1)} = -811.8$, $H^{(2)} = 290.5$). The present calculation yields $F^{(i)} = 0$, $G^{(i)} = -\frac{3}{4} H^{(i)}$, $H^{(1)} = -832.3$, $H^{(2)} = 270.1$, i.e. the difference between σ_{xxxxz} and the value calculated on account of the approximate theory is not large. The ratio $H^{(1)}/H^{(2)} = -3.05$ in the absolute value is larger than unity, therefore, for $\tau_1 = \tau_2$ we obtain that the main contribution to σ_{xxxxz} comes from the light holes which also determine the sign of this value.

For the MR $\Delta\varrho/\varrho_0$ we have

$$\left(\frac{\Delta\varrho}{\varrho_0} \right) = -E_x^2 H (\sigma^{(1)} + \sigma^{(2)})^{-1} \left[\sigma_{xxxxz} + 2 \sigma_{yxxx} \frac{\eta^{(1)} + \eta^{(2)}}{\sigma^{(1)} + \sigma^{(2)}} \right]. \tag{26}$$

The expressions for all the coefficients entering (26) are given above (the last term in the square brackets of (26) can be rewritten in the form $2\gamma_\perp (\eta^{(1)} + \eta^{(2)})/E_x^2$. The whole difference of the MR from that calculated originally by the expansion in powers of the anisotropy in [13] is involved in the last term in the square brackets (26), discussed in the foregoing section. In weak magnetic fields the linear MR is determined largely by the first term in the square brackets and that is the reason, why it is determined predominantly by the light holes.

6. Conclusion

From the above results it is seen that the calculation of the galvanomagnetic coefficients by the expansion in powers of the dispersion law anisotropy (1) with $\varkappa = 1/5$ yields a fair agreement with the numerical computations using an exact dispersion law, if these coefficients are non-zero in the zeroth- or first-order approximation in the anisotropy parameter. The coefficients, which become zero in the above approximations, are found to be non-zero in the exact calculation and can be rather essential for the corresponding effects. In the above calculations the coefficients concerned fully determined the even Hall effect, the longitudinal magnetoresistance, and the anisotropy of the transverse magnetoresistance, they essentially decrease the longitudinal and the transverse anisotropy of conductivity and the contribution to the linear magnetoresistance due to this anisotropy.

Appendix

The factors in (11) with the bar are of the following form, where it is needed to take into account that for $i = 1$, $\tau_1' = \tau_1$,

$$\bar{\mathcal{P}}_i = \frac{N^{(i)}}{m_i^{3/2}} \frac{m_1^{3/2} + m_2^{3/2}}{N^{(1)} + N^{(2)}}; \qquad N^{(i)} = \langle \Phi_i \rangle\,, \tag{A1}$$

with

$$\langle \Psi_i \rangle = \frac{1}{4\pi} \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \, \Psi_i / \Phi_i^{5/2} , \tag{A2}$$

$$\bar{\sigma}^{(i)} = \frac{3\omega_i}{N^{(i)}} [t_i I_1^{(i)} - (t_i - 1) I_2^{(i)}] . \tag{A3}$$

Here

$$\left.\begin{aligned} & I_1^{(i)} = \langle (R_\alpha^{(i)})^2 \rangle , \quad I_2^{(i)} = \langle R_\alpha^{(i)} r_\alpha^{(i)} \rangle , \quad \omega_i = \frac{m_i}{m_0}, \quad t_i = \frac{\tau_i}{\tau_i'}, \\ & R_\alpha^{(i)} = e_\alpha [|A| - (-1)^i M^{-1} (B^2 + 0.5C^2 \sum_{\beta \neq \alpha} e_\beta^2)] , \qquad r_\alpha = e_\alpha \omega_i^{-1} , \\ & e_\alpha = p_\alpha / p , \\ & \bar{\eta}^{(i)} = \frac{3\omega_i^2}{N^{(i)}} [t_i^2 K^{(i)} - (t_i - 1) \omega_i^{-1} (t_i I_1^{(i)} + I_2^{(i)})] , \end{aligned}\right\} \tag{A4}$$

where

$$\left.\begin{aligned} & K^{(i)} = \langle (R_\alpha^{(i)})^2 \chi_{\beta\beta}^{(i)} - R_\alpha^{(i)} R_\beta^{(i)} \chi_{\alpha\beta}^{(i)} \rangle ; \quad \alpha \neq \beta, \quad \alpha, \beta = 1, 2, 3 , \\ & \chi_{\alpha\beta}^{(i)} = \delta_{\alpha\beta} R_\alpha^{(i)} e_\alpha^{-1} + (-1)^i e_\alpha e_\beta \chi_{\alpha\beta}^0 , \\ & \chi_{\alpha\beta}^0 = C^2 M^{-1} [1 + \delta_{\alpha\beta} - e_\alpha^2 - e_\beta^2 - 2\xi + 0.5 C^2 M^{-2} \times \\ & \qquad \times (1 - e_\alpha^2 - 2\xi)(1 - e_\beta^2 - 2\xi)] , \qquad \xi = \sum_{\alpha > \beta = 1}^{3} e_\alpha^2 e_\beta^2 , \\ & \bar{\delta}^{(i)} = \frac{3\omega_i^3}{N^{(i)}} t_i^3 (L_1^{(i)} - L_2^{(i)}) , \qquad \bar{\alpha}^{(i)} = \frac{3\omega_i^3}{N^{(i)}} (t_i^3 L_3^{(i)} - \Delta L^{(i)}) , \end{aligned}\right\} \tag{A5}$$

where

$$\left.\begin{aligned} & L_1^{(i)} = \langle R_\alpha^{(i)} R_\gamma^{(i)} \chi_{\alpha\beta}^{(i)} \chi_{\beta\gamma}^{(i)} \rangle , \qquad L_2^{(i)} = \langle (R_\alpha^{(i)} \chi_{\beta\gamma}^{(i)}) \rangle ; \qquad \alpha \neq \beta \neq \gamma , \\ & L_3^{(i)} = \langle (R_\alpha^{(i)} \chi_{\beta\beta}^{(i)}) + (R_\alpha^{(i)} \chi_{\alpha\beta}^{(i)})^2 - 2 R_\alpha^{(i)} R_\beta^{(i)} \chi_{\alpha\beta}^{(i)} \chi_{\alpha\alpha}^{(i)} \rangle , \\ & \Delta L^{(i)} = \omega_i^{-1} (t_i - 1) [t_i^2 K^{(i)} + \omega_i^{-1} (t_i I_1^{(i)} + I_2^{(i)})] . \\ & \bar{\beta}^{(i)} = \frac{3\omega_i^3}{N^{(i)}} [t_i^3 (L_2^{(i)} - L_4^{(i)}) - \Delta L^{(i)}] , \end{aligned}\right\} \tag{A6}$$

where

$$L_4^{(i)} = \langle (R_\alpha^{(i)})^2 \chi_{\beta\beta}^{(i)} \chi_{\gamma\gamma}^{(i)} - 2 R_\alpha^{(i)} R_\beta^{(i)} \chi_{\alpha\beta}^{(i)} \chi_{\gamma\gamma}^{(i)} \rangle .$$

The integrals over the angles entering (21), (25) are given by the expressions

$$S_1^{(i)} = \langle \Phi_i [\chi_{\alpha\alpha}^{(i)} \chi_{\beta\beta}^{(i)} + 2 (\chi_{\alpha\beta}^{(i)})^2 - (\chi_{\alpha\alpha}^{(i)})^2] \rangle , \tag{A7}$$

$$S_2^{(i)} = \langle (R_\alpha^{(i)})^2 \chi_{\beta\beta}^{(i)} + 2 R_\alpha^{(i)} R_\beta^{(i)} \chi_{\alpha\beta}^{(i)} - (R_\alpha^{(i)})^2 \chi_{\alpha\alpha}^{(i)} \rangle , \tag{A8}$$

$$H^{(i)} = (-1)^i \, 8 \langle \varphi_{\alpha\alpha\alpha} R_\alpha^{(i)} (R_\beta^{(i)})^2 - \varphi_{\alpha\alpha\beta} (R_\beta^{(i)})^3 \rangle , \tag{A9}$$

$$\begin{aligned} G^{(i)} = (-1)^i \langle \Phi_i [& \varphi_{\alpha\alpha\beta} (14 \chi_{\alpha\alpha}^{(i)} R_\beta^{(i)} - 14 \chi_{\alpha\beta}^{(i)} R_\alpha^{(i)} - \\ & - 8 \chi_{\beta\beta}^{(i)} R_\beta^{(i)}) + \varphi_{\alpha\alpha\alpha} (10 \chi_{\alpha\beta}^{(i)} R_\beta^{(i)} - 2 \chi_{\beta\beta}^{(i)} R_\alpha^{(i)})] \rangle , \end{aligned} \tag{A10}$$

$$F^{(i)} = 4 \langle \Phi_i^{-1} [3 \chi_{\alpha\alpha}^{(i)} (R_\alpha^{(i)} R_\beta^{(i)})^2 - 2 \chi_{\alpha\beta}^{(i)} (R_\alpha^{(i)})^3 R_\beta^{(i)} - \chi_{\alpha\alpha}^{(i)} (R_\beta^{(i)})^4] \rangle . \tag{A11}$$

Here

$$\begin{aligned}\varphi_{\alpha\alpha\beta} = e_\alpha\{\chi^0_{\alpha\beta} + \chi^0_{\alpha\alpha}[2\delta_{\alpha\beta} - e_\alpha^2(2 + C^2M^{-2}(1 - e_\beta^2 - 2\xi)] + \\ + 2e_\alpha^2C^2M^{-1}[2(e_\alpha^2 + e_\beta^2 - 1 - \delta_{\alpha\beta} + 2\xi) + \\ + C^2M^{-2}(1 - e_\alpha^2 - 2\xi)\,(e_\alpha^2 + 2e_\beta^2 - 2 - \delta_{\alpha\beta} + 4\xi) - \\ - 0.5C^4M^{-4}(1 - e_\alpha^2 - 2\xi)^2\,(1 - e_\beta^2 - 2\xi)]\}\,.\end{aligned}$$

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