

## Nonlocality of Carrier Multiplication in Semiconductor Depletion Layers

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Analytical and numerical calculations of multiplication coefficients in the Schottky junctions are performed with inhomogeneity of the electric field taken into account. It is shown that in the thin depletion layers distribution functions of electrons and holes and their impact ionization (II) coefficients is nonlocal, i.e. they depend not only upon the electric field strength at a certain point but also on the fields in the vicinity of the point. In such nonlocal cases the introduction of the II coefficients is justified only for small multiplication coefficients, for the large ones they make no sense. If one assumes that the carriers — holes and electrons — differ from each other by the sign of the charge only, the nonlocality effect does not change just the values of the II coefficients obtained in the local approximation, but provides as well the difference between the electron and hole multiplication coefficients. This is due to the electric field asymmetry in the depletion layer.

Выполнены аналитический и численный расчеты коэффициентов умножения в слоях Шоттки, с учетом неоднородности электрического поля. Показано, что в узких истощенных слоях функция распределения электронов и дырок, а также их коэффициенты ударной ионизации (УИ) нелокальны, т.е. зависят не только от электрического поля в данной точке, но и от полей в её окрестности. В нелокальном случае понятие коэффициентов УИ может быть введено только для малых коэффициентов размножения, а при больших они теряют смысл. Если все параметры, характеризующие кинетику электронов и дырок, принять одинаковыми, то эффект нелокальности сводится не только к сильному исправлению значений коэффициентов умножения, рассчитанных в локальном приближении, но и к появлению различия электронного и дырочного коэффициентов умножения, связанного с асимметричным ходом электрического поля в истощенном слое.

### 1. Introduction

The time independent carrier multiplication process caused by impact ionisation (II) in flat depletion layers of semiconductors is, as a rule, described by the continuity equations

$$\frac{dj_p}{dx} = \frac{dj_n}{dx} = \alpha_p(x) j_p(x) - \alpha_n(x) j_n(x), \quad (1)$$

where  $j_{p,n}(x)$  are the drift current densities of holes and electrons:

$$j_p(x) = v_p(x) p(x), \quad j_n(x) = v_n(x) n(x), \quad (2)$$

$p$  and  $n$  are the concentrations of holes and electrons,  $v_{p,n}(x)$  are their drift velocities and  $\alpha_{p,n}(x)$  are their II coefficients.

The local approach to the II problem is based upon the assumption that (1) and (2) are valid and  $\alpha_{p,n}(x)$  and  $v_{p,n}(x)$  are functions of  $E(x)$  only:

$$\alpha_{p,n} = \alpha_{p,n}(E), \quad v_{p,n} = v_{p,n}(E), \quad E = E(x). \quad (3)$$

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Advantages of such an approach are that for a given semiconductor  $\alpha_{p,n}(E)$  and  $v_{p,n}(E)$  appear to be universal functions of temperature and field strength, and of its orientation relative to the crystallographic axis; these functions are completely independent of doping parameters (at least in the region of lattice scattering). Numerous attempts were made to obtain the field dependence of  $\alpha_{p,n}$  from the analysis of carrier multiplication in depletion layers of p-n or Schottky junctions in Ge, Si, GaAs, InAs, and other materials. Now it has become clear that such universal functions  $\alpha_{p,n}(E)$  may not be introduced due to the significant divergence of values obtained by various authors from the experimentally measured multiplication coefficients  $M_{n,p}(U)$ ,  $U$  being the bias voltage across the depletion layer. This divergency cannot be explained by either experimental or calculation errors.

The absence of universal empirical functions  $\alpha_{p,n}(E)$  is connected with two reasons:

- (i) ionization processes via deep impurity levels in addition to the main band-to-band process [1, 2];
- (ii) nonlocality of the II process that makes (3) incorrect.

Here we study just the second reason, which is essential in thin depletion layers, where high pre- and breakdown fields are obtained with low bias voltages.

## 2. Influence of the Inhomogeneity of the Electric Field on Small Multiplication Coefficients

Nonlocality of II was in some form taken into account in [3, 4], where just one type of nonlocality was considered — that of “dark space”. It is connected with a significant difference between the ionization threshold energy  $\varepsilon_{n,p}$  and the postionization energy  $\varepsilon'_{n,p}$  observed in most of the semiconductors [5, 6]. As a rule  $\varepsilon_{n,p} \gg \varepsilon'_{n,p}$ , hence both the primary and secondary particles can cause II again only after having run a path not less than  $d_{n,p} = \varepsilon_{n,p}/e\bar{E}$ ,  $\bar{E}$  being the mean electric field strength over the path. The same is true for carriers entering the depletion layer from the metal contact or from the neutral areas of a semiconductor where there is no electric field. Obviously the “dark space” effect may be neglected if

$$\alpha_{n,p}d_{n,p} \ll 1, \quad d_{n,p} \ll w, \quad (4)$$

where  $w$  is the depletion layer width. In this case the quantities  $d_{n,p}^{-1}$  are upper limits of  $\alpha_{n,p}$  ( $\alpha_{n,p} \leq d_{n,p}^{-1}$ ), which are reached near the ionization threshold when the II is the only way of carrier energy relaxation (in this case all the energy received from the electric field is spent to produce the electron-hole pair).

If the conditions (4) are not fulfilled it is clear that the II processes affect not only the normalization and asymptotics ( $\varepsilon \geq \varepsilon_{n,p}$ ) of the carrier distribution functions but determine all their energy and spatial structure. Therefore an essential spatial inhomogeneity of the ionization rate leads to an essential inhomogeneity of the distribution functions themselves and not only of their normalizing factors. Thus when the conditions (4) are not fulfilled the phenomenological attempts to take account of nonlocality effects without calculating the distribution functions, see e.g. [4], may not be considered as quite correct.

Here we shall point out another source of nonlocality which may be considered phenomenologically if the multiplication is low enough, this is the electric field inhomogeneity along the length  $d_{n,p}$ .

Let us assume that the distribution function is highly anisotropic in most part of the energy interval from the mean energy  $\bar{\varepsilon}_{n,p}$  up to the threshold  $\varepsilon_{n,p}$ . Furtheron let its energy dependence be determined by just the pre-collision drift in the electric

field [7 to 9]. Then in case of a homogeneous field the asymptotic form of the distribution function is given by

$$f_n(\varepsilon) \approx n \exp \left\{ - \int_0^\varepsilon \frac{d\varepsilon'}{e |E| l_n(\varepsilon')} \right\}. \quad (5)$$

Here  $l_n(\varepsilon)$  is the mean free path of the electrons drifting with the energy  $\varepsilon$  along the field. In the case of spatial inhomogeneity analogous asymptotics may be obtained from the equation

$$v_{p,x} \frac{\partial f}{\partial x} - eE(x) \frac{\partial f}{\partial p_x} + \frac{f}{\tau_p} \approx 0, \quad f = f(p, x). \quad (6)$$

Here  $p$  and  $p_x$  are the momentum and its projection upon the field direction,  $v_p$  and  $\varepsilon_p$  are velocity and energy of electron with momentum  $p$ ,  $\tau_p$  is the scattering time from this state. In (6) there is no term describing electrons coming to the state  $p$  since it may be neglected when high energy asymptotics are considered. Such an approach corresponds to the drift asymptotics in [7, 8]. An approximate solution of (6) is given by

$$f_n(\varepsilon, x) \approx n \left( \varphi(x) - \frac{\varepsilon}{e} \right) \exp \left\{ - \frac{1}{e} \int_0^\varepsilon \frac{d\varepsilon'}{\left| E \left( \varphi(x) - \frac{\varepsilon - \varepsilon'}{e} \right) \right| l_n(\varepsilon')} \right\}, \quad (7)$$

where  $n(\varphi)$  is the carrier concentration at the point with the potential  $\varphi$ ,  $E(\varphi)$  is the field strength at this point. The difference between (7) and (5) consists in the fact that in (7) the carrier concentration  $n(\varphi(x))$  at the point  $x$  is replaced by, generally speaking, another one  $n[\varphi(x) - \varepsilon/e]$ , and instead of the field  $E(\varphi(x))$  all the fields are present between  $E(\varphi(x))$  and  $E(\varphi(x) - \varepsilon/e)$ . Equations (5) and (7) are valid if  $\varepsilon \gg \bar{\varepsilon}_n$ , i.e. if the exponents are large enough. In this case relations (4) are fulfilled, so that

$$n \left( \varphi(x) - \frac{\varepsilon_n}{e} \right) \approx n(\varphi(x))$$

and also

$$p \left( \varphi(x) + \frac{\varepsilon_p}{e} \right) \approx p(\varphi(x)).$$

The continuity equations (1) keep their form unchanged, but in these equations one should put

$$\left. \begin{aligned} \alpha_n(x) &\approx \alpha_{n\infty} \exp \left\{ - \frac{1}{e} \int_0^{\varepsilon_n} \frac{d\varepsilon'}{\left| E \left( \varphi(x) - \frac{\varepsilon_n - \varepsilon'}{e} \right) \right| l_n(\varepsilon')} \right\}, \\ \alpha_p(x) &\approx \alpha_{p\infty} \exp \left\{ - \frac{1}{e} \int_0^{\varepsilon_p} \frac{d\varepsilon'}{\left| E \left( \varphi(x) + \frac{\varepsilon_p - \varepsilon'}{e} \right) \right| l_p(\varepsilon')} \right\} \end{aligned} \right\} \quad (8)$$

instead of

$$\alpha_{n,p}(x) \approx \alpha_{n,p\infty} \exp \left\{ - \frac{1}{e |E(x)|} \int_0^{\varepsilon_{n,p}} \frac{d\varepsilon'}{l_{n,p}(\varepsilon')} \right\}; \quad (9)$$

i.e. expressions (9) being used in a strictly local approach. Correction terms connected with the use of (8) instead of (9) are essential if the field  $E(x)$  varies significantly in the vicinity of its maximum value over a length of the order of  $d_{n,p}$ . The latter condition means that in intervals the order of  $d_{n,p}$  (with voltages  $\approx \varepsilon_{n,p}/e$  across them) practically all the II in the depletion layer must take place for voltages not too low as compared with the breakdown one. The latter takes place only in a rather thin depletion layer with low breakdown voltage (for example, exceeding  $\varepsilon_{n,p}/e$  by less than by an order of magnitude). Especially interesting in this sense are the so-called Read diodes with narrow highly doped multiplication layers [10].

If somewhere over the interval  $(\varphi(x) - \varepsilon_n/e, \varphi(x))$  for electrons or  $(\varphi(x), \varphi(x) + \varepsilon_p/e)$  for holes the field is equal to zero, the same happens to the coefficients  $\alpha_n(x)$  and  $\alpha_p(x)$  so that the "dark space" effect near the edges of depletion layers is automatically taken account of by (8).

In the case of smooth electric field

$$d_{n,p} \left| \frac{dE}{dx} \right| \ll |E| \quad (10)$$

(8) gives

$$\alpha_{n,p}(x) \approx \alpha_{n,p}^{(0)}(E(x)) \Psi_{n,p}(x), \quad (11)$$

where  $\alpha_{n,p}^{(0)}(E)$  is the local multiplication coefficients and  $\Psi_{n,p}(x)$  are the correction factors

$$\Psi_{n,p}(x) = \exp \left\{ \pm \left| \frac{4\pi N(x)}{e\kappa_d E^3(x)} \right| \int_0^{\varepsilon_{n,p}} \frac{(\varepsilon_{n,p} - \varepsilon') d\varepsilon'}{l_{n,p}(\varepsilon')} \right\}. \quad (12)$$

In deriving (12) account was taken of the fact that  $E(x)$  is determined by the Poisson equation

$$\frac{dE}{dx} = \frac{4\pi e}{\kappa_d} N(x),$$

where  $\kappa_d$  is the dielectric permeability and  $N$  is the difference between concentrations of ionized donors and acceptors:  $N = N_D - N_A$ . Correction factors  $\Psi_{n,p}(x)$  are essential if, due to field inhomogeneity, the quantities  $d_{n,p}(E)$  vary over the same distances not less than over the mean free path  $\bar{l}_{n,p}$ , i.e. if

$$\left| \frac{d_{n,p}}{E} \frac{dE}{dx} \right| \gtrsim \frac{\bar{l}_{n,p}}{d_{n,p}}. \quad (13)$$

It is easily seen that in sufficiently narrow depletion layers the condition (13) is fulfilled.

### 3. An Example: Schottky Junctions

Let us consider Schottky junctions in homogeneously doped n-type material with donor concentration  $N$ . Then

$$E = E_M \left( 1 - \frac{x}{w} \right), \quad \frac{E_M}{w} = - \frac{4\pi e N}{\kappa_d}, \quad (14)$$

where  $E_M < 0$ ,  $0 < x < w$ ,  $w$  is the thickness of the depletion layer,

$$\varphi = \varphi_0 \left[ 1 - \left( 1 - \frac{x}{w} \right)^2 \right], \quad (15)$$

$\varphi_0 = -E_M w/2 > 0$  being the bias voltage. Then using (14) and (15) and assuming  $l_{n,p} = \text{const}$  in (8) we obtain

$$\alpha_n(x) = \alpha_{n\infty} \exp \left\{ -\frac{2\varphi_0}{l_n |E_M|} \left[ \left( 1 - \frac{\varphi}{\varphi_0} + \frac{\varepsilon_n}{e\varphi_0} \right)^{1/2} - \left( 1 - \frac{\varphi}{\varphi_0} \right)^{1/2} \right] \right\} \quad (16)$$

for  $\varepsilon_n/e < \varphi(x) < \varphi_0$ ,

$$\alpha_p(x) = \alpha_{p\infty} \exp \left\{ -\frac{2\varphi_0}{l_p |E_M|} \left[ \left( 1 - \frac{\varphi}{\varphi_0} \right)^{1/2} - \left( 1 - \frac{\varphi}{\varphi_0} - \frac{\varepsilon_p}{e\varphi_0} \right)^{1/2} \right] \right\} \quad (17)$$

for  $0 < \varphi(x) < \varphi_0 - \varepsilon_p/e$ ; outside the  $\varphi(x)$  intervals just indicated  $\alpha_{n,p}(x) = 0$ .

Let us calculate the multiplication coefficients  $M_{n,p}$  under the low multiplication conditions

$$M_{n,p} - 1 \approx \int_0^w \alpha_{n,p}(x) dx \ll 1. \quad (18)$$

In this approach when the exponents in (16) and (17) are assumed to be large enough one obtains

$$M_n - 1 \approx \frac{2\varphi_0 \alpha_{n\infty}}{|E_M|} \int_0^\infty d\zeta \exp \left\{ -\frac{2\varphi_0}{l_n |E_M|} \left[ \left( 1 + \zeta^2 - 2\zeta \sqrt{1 - \frac{\varepsilon_n}{e\varphi_0}} \right)^{1/2} - \left( 1 - \frac{\varepsilon_n}{e\varphi_0} \right)^{1/2} + \zeta \right] \right\}, \quad (16')$$

$$M_p - 1 \approx \frac{2\varphi_0 \alpha_{p\infty}}{|E_M|} \int_0^\infty d\zeta \exp \left\{ -\frac{2\varphi_0}{l_p |E_M|} \left[ 1 - \zeta - \sqrt{(1 - \zeta)^2 - \frac{\varepsilon_p}{e\varphi_0}} \right] \right\}. \quad (17')$$

Two consequences of the nonlocality are evident from (16') and (17'):

(i) The quantities  $M_{n,p} - 1$  decrease compared to those calculated within the local approach. At  $\varepsilon_{n,p} \ll e\varphi_0$  we have

$$M_{n,p} - 1 \approx (M_{n,p}^{(0)} - 1) \exp \left\{ -\frac{\varepsilon_{n,p}^2}{4e^2 \varphi_0 l_{n,p} |E_M|} \right\}. \quad (19)$$

This decrease becomes large when the exponent in (19) rises exceeding 1. The decrease of  $M_p$  is due to the factor  $\Psi_p$  being less than 1 in (11) and (12). However,  $M_n$  decreases due to the "dark space" which exists near the point  $x = 0$ , affecting it stronger than the factor  $\Psi_n > 1$ . As a result the correction term in (19) appears to have the same structure for holes and electrons.

(ii) In case of complete symmetry of the parameters ( $\varepsilon_n = \varepsilon_p$ ,  $l_n = l_p$ ,  $\alpha_{n\infty} = \alpha_{p\infty}$ ) the approximate equation (19) gives  $M_n = M_p$ . However, the more accurate equations (16') and (17') maintain the inequality between  $M_n$  and  $M_p$  due to the asymmetry of  $E(x)$  within the depletion layer which is essential for the nonlocal effects. Under the assumptions used above this inequality is rather small and is not given correctly since the field dependence of  $\alpha_{n\infty}$ ,  $\alpha_{p\infty}$  is lost here. Such a dependence is also nonlocal and it may be as important as that taken account of above.

As far as (8) is obtained with the conditions (4) assumed to be valid, these equations become unsuitable when  $M_{n,p}$  are large within the layers where the effect of the field inhomogeneity is most significant, i.e. such may be the situation in the pre-breakdown region. In these layers the main contribution to the multiplication comes from spatial intervals of the order of  $d_{n,p}$  lying close to the point where the field strength reaches its maximum. Thus the breakdown condition here takes the form  $\alpha_{n,p} d_{n,p} \approx 1$ .

#### 4. Calculation of $M_{n,p}$ by the Monte-Carlo Procedure

To describe the II when the conditions (4) are not fulfilled it is necessary to find the spatially inhomogeneous distribution functions  $f_{n,p}$  within the depletion layers. Here we present the results of numerical calculations for a greatly simplified model of a semiconductor. It is assumed that the carriers — holes and electrons — differ from each other only by the sign of charge. Such an identity of the carriers not only simplifies the calculation but enables us as well to obtain the same local coefficients  $\alpha_n(E)$  and  $\alpha_p(E)$ . (Experiments on GaAs [11, 12] give  $\alpha_n(E) \approx \alpha_p(E)$ .)

The carriers are scattered with equal angular probabilities emitting or absorbing dispersionless optical phonons via the deformation potential interaction. Moreover the emission probability strongly exceeds that of absorption since we put  $\hbar\omega_0/kT = 10$  ( $\omega_0$  is the phonon frequency and  $T$  is the lattice temperature). The II threshold energy is chosen to be  $\varepsilon_n = \varepsilon_p = 10\hbar\omega_0 = 100kT$ ; a particle appearing over the threshold immediately generates an electron-hole pair with zero energy and falls to the bottom of the band. An isotropic parabolic dispersion law is assumed with an effective mass  $m = 0.35m_0$  where  $m_0$  is the free electron mass. (This comes into a certain contradiction with the assumption just made concerning the particle energy after II. That assumption would have taken place if a "third body" were involved in the II. However, such processes have a low probability.) Within the layer  $0 \leq x \leq w$  the particles move in a given electric field  $E(x)$ . Two cases of multiplication were considered:

(i) Initial electrons with energy  $\varepsilon = kT$ , distributed isotropically over the hemisphere  $v_x > 0$ , start from the point  $x = 0$  and  $M_p$  is calculated.

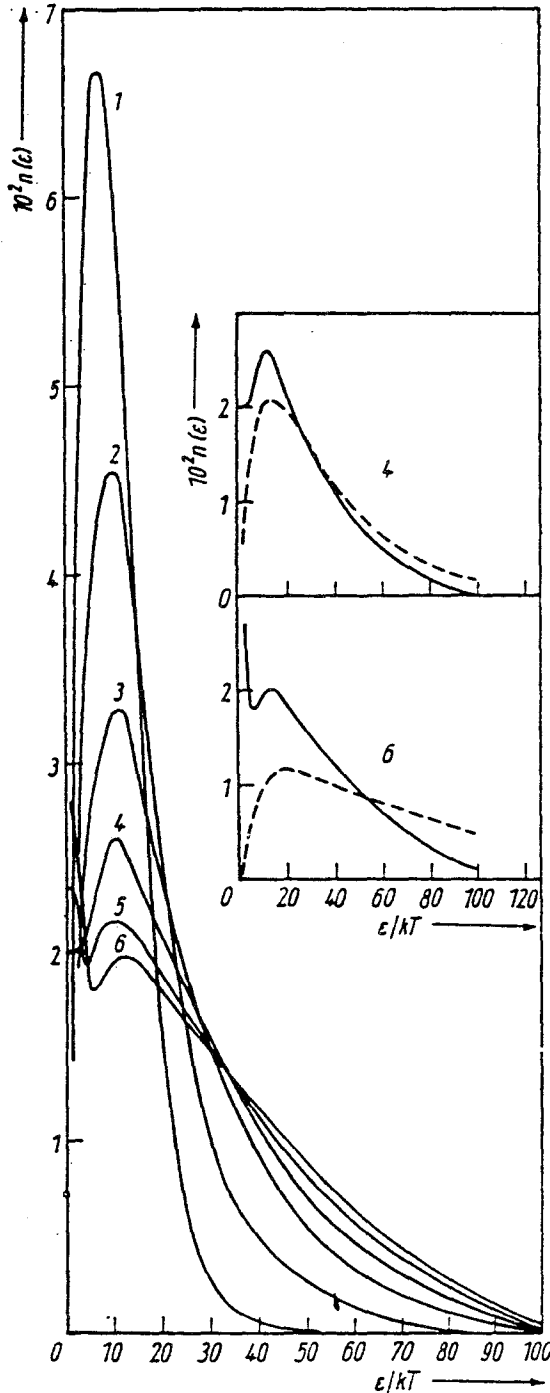


Fig. 1. Dependence of the electron concentration  $n(\varepsilon, x)$  upon  $\varepsilon$  at the point  $x = 2d(E)$  for various values of the dimensionless field strength  $E'$ . (1)  $E' = 0.25$ , (2) 0.5, (3) 0.75, (4) 1.0, (5) 1.25, (6) 1.5; — calculation with and --- without the II being taken into account. Here  $\int_0^\infty n(\varepsilon) d\varepsilon = 1$

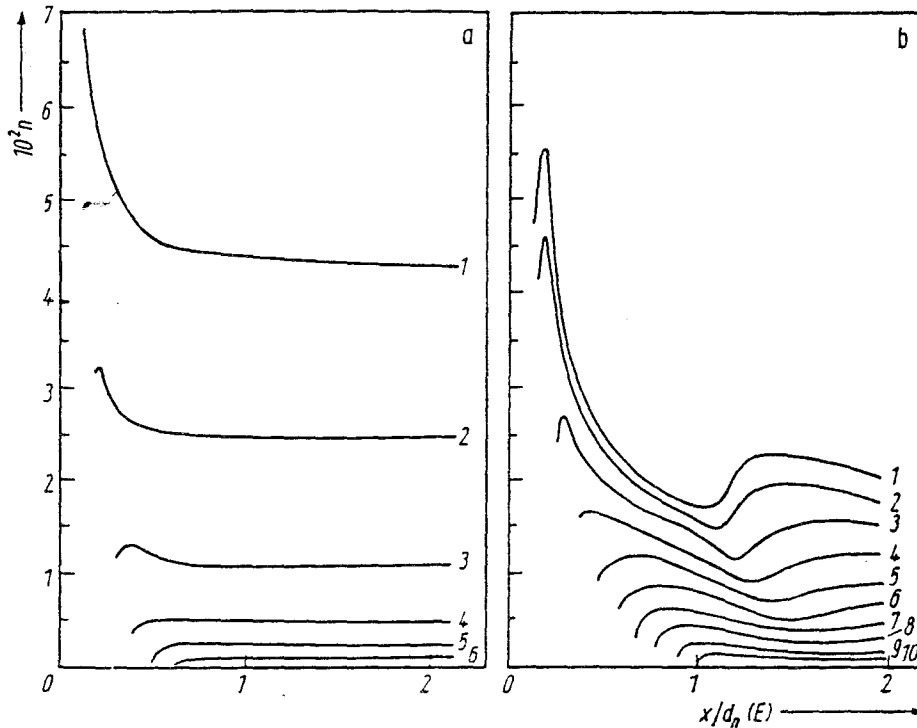


Fig. 2. Dependence of the electron concentration  $n(\epsilon, x)$  upon  $x$  at a)  $E' = 0.5$  for various values of  $\epsilon/kT$ :  $\epsilon/kT = (1) 10, (2) 20, (3) 30, (4) 40, (5) 50, (6) 60$ , b)  $E' = 1.5$ ;  $\epsilon/kT = (1) 12, (2) 20, (3) 30, (4) 40, (5) 50, (6) 60, (7) 70, (8) 80, (9) 90, (10) 100$

(ii) Initial holes with energy  $\epsilon = kT$  distributed isotropically over the hemisphere  $v_x < 0$ , start from the point  $x = w$  and  $M_p$  is calculated.

The many-particle Monte-Carlo method was used to calculate the motion and multiplication of the particles [13 to 18].

The motion of the particles was considered in fields of three different types:

A)  $E = \text{const}$  at  $0 \leq x \leq w = 2.5d_n(E)$ , where  $d_n(E) = \epsilon_n/eE$ ;

B)  $E = E_M(1 - 0.9x/w)$ ,  $0 \leq x \leq w(E_M)$ ;

C)  $E = E_M(1 - ax)$  at  $0 \leq x < x_0$ ,

$E = E_m = E_M(1 - ax_0)$  at  $x_0 \leq x \leq w = d_n(E_M)$ .

$E_M, a, x_0$  were interdependent so as to obtain the bias voltage  $U(x_0)$  of the form

$$U(x_0) = E_M x_0 - \frac{ax_0^2}{2} = 0.95 \frac{\epsilon_n}{e}.$$

Fig. 1 shows the energy dependence of the electron concentration  $n(\epsilon) = f_0(\epsilon) g(\epsilon)$  normalised to one electron for the A-type field at the point  $x = 2d_n(E)$  ( $f_0(\epsilon)$  is the isotropic part of the distribution function and  $g(\epsilon)$  is the density of states). At such a distance from the cathode the electron distribution function (with no ionization taken into consideration) is approximately saturated and is close to a homogeneous one. The calculations were performed for six values of the dimensionless field strength

$$E' = 5 \sqrt{2m^*} \frac{eEl(\epsilon_n)}{\epsilon_n},$$

here  $m^* = m/m_0$ ,  $l(\epsilon_n) = v(\epsilon_n) \tau_n(\epsilon_n)$  is the mean free path of the electron with energy  $\epsilon_n$ ,  $v$  is the velocity, and  $\tau$  is the scattering time. It is seen that at  $E' \approx 1$  the II process

alters the distribution function for all energies: if II is not considered, the mean energy rises quickly on raising the field strength, but taking account of the II tends to give saturation in the  $n(\varepsilon)$  dependence at fields  $E' \approx 1$ . According to simple estimates the distribution function has the form

$$f_0(\varepsilon) = \int_{\varepsilon}^{\varepsilon_n} \frac{d\varepsilon'}{g(\varepsilon') v^2(\varepsilon') \tau(\varepsilon')} \quad (20)$$

It saturates due to diffusion heating of the electrons up to the energies  $\varepsilon_n$ , so that at  $\varepsilon \gg \hbar\omega_0$  one obtains  $n(\varepsilon) \approx 1 - \sqrt{\varepsilon/\varepsilon_n}$ . The function (20) makes sense for fields

$$\sqrt{\varepsilon_n \hbar\omega_0} < e E l(\varepsilon_n) < \varepsilon_n, \quad (21)$$

i.e.  $1.3 < E' < 4.2$  so that we observe it just tending to set up. At  $eEl(\varepsilon_n) > \varepsilon_n$  (i.e. at  $E' > 4.2$ ) the function  $f_0(\varepsilon)$  must acquire a collisionless character (i.e. it must become constant) over the interval  $0, \varepsilon_n$ . However, the condition

$$eEl(\varepsilon_n) > \sqrt{\varepsilon_n \hbar\omega_0}$$

is already sufficient for the II coefficient  $\alpha_n(E)$  to reach its maximum value  $d_n^{-1}(E)$ .

The dependence  $n(\varepsilon, x)$  upon  $x$  is shown in Fig. 2 a and b for some fixed energies  $\varepsilon$  at two values of field strength  $E' = 0.5$  (Fig. 2a), where the II is still insignificant and almost does not disturb  $n(\varepsilon)$  and  $E' = 1.5$  (Fig. 2b), when the disturbing effect of the II is high enough. Attention is attracted in the figures by the energy dependent "dark space"  $d(\varepsilon, E) = \varepsilon/eE$ , and also by the significant decrease of  $n(\varepsilon, x)$  on increasing  $x$  at small  $\varepsilon$  due to electrons acquiring higher energies. Comparison of Fig. 2 a and b demonstrates the transformation of the distribution functions due to II. This transformation is displayed especially brightly by the presence of the second maxima with  $x$  at small  $\varepsilon$  connected with electrons appearing after the II (Fig. 2b).

In Table 1 the multiplication coefficients  $M_n$  and  $M_p$  in the field of type B are given for two effective dopings N. The doping is determined by the parameter  $\gamma = kTm^*/(2\varepsilon_n) (l(\varepsilon_g)/l_D)^2$  where  $l_D^2 = \kappa_d kT/4\pi e^2 N$ . The depletion width  $w$ , depending upon  $E$  and N is given by

$$w = 0.09 \frac{E_M}{\gamma} \sqrt{\frac{m^*}{2}} l(\varepsilon_n).$$

The II nonlocality is displayed not only by the strong decrease of  $M_{n,p}$  but as well by their inequality caused by the field asymmetry. This effect is seen especially

Table 1

$\gamma$	$E'_M$	$M_n$	$M_p$	$(1 - M_p)/(1 - M_n)$
0.002	0.8	1.0122	1.0175	1.43
	1.0	1.1134	1.1423	1.25
	1.2	1.5280	1.5720	1.08
0.05	3.18	1.1486	1.235	1.58
	3.2	1.159	1.231	1.45
	3.7	1.854	2.080	1.26
	4.2	3.085	3.457	1.17



strongly in the case of extremely narrow layers ( $\gamma = 0.05$ ), where the II occurs at  $w$  values slightly exceeding  $d_n(E)$ . Much higher asymmetry  $((1-M_p)/(1-M_n) \approx 5$  to 10) is obtained in fields of type C, where the multiplication is concentrated within the narrow layer  $0 \leq x \leq x_0$  with the properly selected thickness. The electrons entering this layer are perfectly cold and the holes are slightly heated in the region of low field  $x_0 < x \leq w$ , where the II cannot occur at all.

The computer experiment described for a completely artificial model of a semiconductor has the only aim to make some qualitative trends clear. The numerical results were obtained for relatively high  $M_{n,p}$ , that is why we cannot compare them with the estimates of Section 3 which in the case of this model are valid for very low values of  $M_{n,p} - 1$ .

### References

- [1] D. J. ROBBINS, *phys. stat. sol. (b)* **98**, 11 (1980).
- [2] V. A. KUZMIN, N. N. KRYUKOVA, and A. S. KYUREGYAN, *Fiz. Tekh. Poluprov.* **9**, 729 (1975).
- [3] R. VAN OVERSTRAETEN and H. DE MAN, *Solid State Electronics* **13**, 583 (1970).
- [4] Y. OKUTO and C. R. CROWELL, *Phys. Rev. B* **10**, 4284 (1974).
- [5] C. L. ANDERSON and C. R. CROWELL, *Phys. Rev. B* **5**, 2267 (1972).
- [6] R. A. BALLINGER, K. A. MAJOR, and J. R. MALLINSON, *J. Phys. C* **6**, 2573 (1973).
- [7] Q. A. BARAFF, *Phys. Rev.* **123**, 2507 (1962).
- [8] L. V. KELDysh, *Zh. eksper. teor. Fiz.* **48**, 1692 (1965).
- [9] Z. S. GRIBNIKOV, *Zh. eksper. teor. Fiz.* **74**, 2112 (1978).
- [10] D. KERROL, *SVC-generatory na goryachikh elektronakh*, Mir, Moskva 1972 (p. 253).
- [11] Q. E. STILLMAN, C. M. WOLFF, J. A. ROSSI, and A. Q. FOYT, *Appl. Phys. Letters* **24**, 471 (1974).
- [12] I. P. PEARSALL, R. E. NAHORY, and J. R. CHELIKOWSKY, *Phys. Rev. Letters* **39**, 295 (1977).
- [13] P. A. LEBWOHL and P. J. PRICE, *Solid State Commun.* **9**, 1721 (1971).
- [14] J. Q. RUCH, *IEEE Trans. Electron Devices* **ED-19**, 652 (1972).
- [15] T. KUROSAWA, *Proc. VIII. Internat. Conf. Phys. Semicond., Kyoto 1966*; *J. Phys. Soc. Japan, Suppl.* **21**, 424 (1966).
- [16] J. K. POŽELA, *Plazma i elektricheskie neustoichivosti v poluprovodnikakh*, Nauka, Moskva 1977 (p. 227).
- [17] P. J. PRICE, *Solid State Electronics* **21**, 9 (1978).
- [18] A. MATULIONIS, J. POŽELA, and A. REKLAITIS, *Sbornik mnogodolinnye poluprovodniki*, Izd. Mokslas, Vilnius 1979 (p. 7).

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