

Determination of the deformation potential constants of semiconductors similar to *p*-type germanium from the temperature dependence of the piezoresistance

N. T. Gorbachuk, V. V. Mitin, Yu. A. Tkhorik, and Yu. M. Shvarts

Institute of Semiconductors, Academy of Sciences of the Ukrainian SSR, Kiev

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The temperature dependence of the piezoresistance was studied for *p*-type germanium with resistivities $\rho_{300} = 40, 16, \text{ and } 0.02 \Omega \cdot \text{cm}$ in the temperature range 77-300°K. The measurements were carried out for the current *i* and uniaxial compression *X* parallel to the $\langle 100 \rangle$ and $\langle 111 \rangle$ axes of the crystal. The anisotropy of the dispersion of light and heavy holes was taken into account in the interpretation of the experimental data and in the calculation of the deformation potential constant *d*. The resulting values of the deformation potential constant are $d = 3.6-4.1$, whereas the isotropic approximation based on the same experimental results would yield $d \approx 6.0$ eV. The anisotropy of the dispersion law not only affects the value of *d* but it also explains the observed difference in the signs of the piezoresistance corresponding to $i \parallel X \parallel \langle 111 \rangle$ and $i \parallel X \parallel \langle 100 \rangle$ in lightly doped samples where an increase in the pressure leads to a monotonic decrease of the resistivity ρ for $i \parallel X \parallel \langle 111 \rangle$. For $i \parallel X \parallel \langle 100 \rangle$, an initial increase is followed by a decrease at greater pressures. Such a nonmonotonic dependence $\rho(X)$ for $i \parallel X \parallel \langle 100 \rangle$ does not occur in heavily doped samples and the piezoresistance has the same sign for both directions. It is also noted that a reliable value of the deformation potential constant *b* cannot be obtained from the temperature dependence of the piezoresistance.

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1. The deformation potential constants are important parameters of the semiconductor band structure and, therefore, many attempts have been made to determine them with increasing accuracy.¹ Since the piezoresistance coefficients π_{ij} include the deformation potential constants as factors, it should be possible to determine these constants by measuring the variation of the semiconductor resistance under strain. It is usual to measure the temperature dependence of the longitudinal piezoresistance for a uniaxial compression or dilation *X* along the $\langle 100 \rangle$ and $\langle 111 \rangle$ axes since the slope of such a dependence is proportional to π_{11} and π_{44} , respectively (we shall further use the notation $\pi_{\langle 100 \rangle}$ and $\pi_{\langle 111 \rangle}$).

2. According to Refs. 2 and 3, the temperature-dependent components of the piezoresistance of semiconductors similar to *p*-type Ge are given by

$$\pi_{\langle hkl \rangle}^T = \frac{0.3\pi_{\langle hkl \rangle}}{\bar{E}(1+\lambda)k_0T} (\xi_2 \xi_2, \langle hkl \rangle - \lambda \xi_1 \xi_1, \langle hkl \rangle), \quad (1)$$

where

$$\xi_i = \frac{\langle \tau_i \rangle}{\langle \epsilon \tau_i \rangle}, \quad \langle \epsilon^k \tau_i \rangle = \frac{\int_0^\infty \epsilon^{1/2+k} \tau_i(\epsilon) e^{-\epsilon} d\epsilon}{\int_0^\infty \epsilon^{1/2} e^{-\epsilon} d\epsilon}; \quad (2)$$

$\epsilon = E/k_0T$ is a dimensionless energy; k_0 is the Boltzmann constant; *T* is the temperature; the indices 1 and 2 refer, respectively, to light and heavy holes; $\xi_{i,\langle hkl \rangle}$ are coefficients depending on the anisotropy of the dispersion law and on the direction $\langle hkl \rangle$, $\bar{E} = [(2B^2 + D^2)/5]^{1/2}$ (*B* and *D* are the band structure parameters); $\lambda = \sigma_1/\sigma_2$; σ_1 is the conductivity due to holes of type *i*; $\eta_{\langle 100 \rangle} = 2Bb/(c_{11} - c_{12})$,

$\eta_{\langle 111 \rangle} = Dd/3c_{44}$, c_{ik} are the elastic moduli; *b* and *d* are deformation potential constants. Analytic expressions for the

coefficients $\xi_{i,\langle hkl \rangle}$, which appear in Eq. (1) are very cumbersome and will be given in the Appendix. The anisotropy of the dispersion law is neglected in many papers^{4,5} and, as will be seen in the Appendix, this corresponds to the approximation $\xi_{i,\langle hkl \rangle} = 1$. However, the values of $\xi_{i,\langle hkl \rangle}$ calculated with an allowance for the actual anisotropic dispersion law may differ significantly from unity [for example, for *p*-type Si and $X \parallel \langle 100 \rangle$, by up to 70% (see Ref. 3) and by up to 10% for *p*-type GaAs and $X \parallel \langle 111 \rangle$ (see Ref. 6)]. There is large scatter in the values of the band structure constants *A*, *B*, and *C* for the same material quoted in literature⁷⁻¹² and, therefore, we have calculated the parameters $\xi_{i,\langle hkl \rangle}$ for various sets of these constants; the results of such calculations are quoted in Table 1. For *p*-type Ge, all the parameters $\xi_{i,\langle hkl \rangle}$ with the exception of $\xi_2, \langle 100 \rangle$ are almost identical for all the sets of values of the band structure constants considered. If the anisotropy of the dispersion law is taken into account, the contribution of heavy holes to $\pi_{\langle 111 \rangle}$ increases compared with the isotropic approximation, whereas the contribution to $\pi_{\langle 100 \rangle}$.

3. Figure 1 shows the results of our measurements of the temperature dependences of $\pi_{\langle 100 \rangle}$ and $\pi_{\langle 111 \rangle}$ for samples of *p*-type germanium with resistivities $\rho_{300^\circ K} = 40, 16, \text{ and } 0.02 \Omega \cdot \text{cm}$. It follows that the piezoresistances of lightly doped semiconductors corresponding to $i \parallel X \parallel \langle 100 \rangle$ and $i \parallel X \parallel \langle 111 \rangle$ have opposite signs. In fact, $\lambda < 1$ for all the scattering mechanisms and $\xi_2 \approx \xi_1$ and, therefore, it follows from Eq. (1) and Table 1 that the contribution of heavy holes to $\pi_{\langle 111 \rangle}$ is dominant and the slopes of curves 2 and 4 are positive. On the other hand, setting $\xi_2 = \xi_1$, we find that $\pi_{\langle 100 \rangle} < 0$ provided

$$\lambda > \lambda_c. \quad (3)$$

Here, $\lambda_c = 0.29$ for the first choice of values of the constants in Table 1 and $\lambda_c = 0.06$ for the other three possibilities.

TABLE 1

Material	No.	Band structure constants			Ref- erence	ξ_{11} (100)	ξ_{12} (100)	ξ_{13} (111)	ξ_{14} (111)	d, eV
		A	B	C						
Ge	1	-13.0	-8.9	10.3	[7]	1.14	0.34	0.92	1.3	-4.1
	2	-13.1	-8.3	12.5	[8]	1.2	0.07	0.89	1.4	-3.7
	3	-13.27	-8.63	12.04	[9]	1.2	0.067	0.89	1.4	-3.6
	4	-12.35	-8.26	12.07	[10]	1.2	0.068	0.89	1.44	-3.8
Si	5	-4.1	-1.6	3.3	[7]	1.3	0.53	0.86	1.2	—
	6	-4.0	-1.1	4.1	[8]	1.6	0.19	0.79	1.3	—
	7	-4.38	-1.0	4.80	[10]	1.8	0.031	0.76	1.3	—
	8	-4.28	-0.75	5.25	[11]	2.1	0.22	0.72	1.3	—
	9	-4.27	-0.63	5.03	[12]	2.2	0.17	0.71	1.3	—
GaAs	10	-7.39	-4.93	5.06	[9]	1.1	0.52	0.94	1.26	—

ties. It is usually assumed that $\lambda = 0.34$ for pure p-type germanium where the scattering by acoustic lattice vibrations is dominant. Consequently, the condition (3) is satisfied for all the sets of values of the constants included in Table 1, which explains the fact that curves 1 and 3 in Fig. 1 have negative slopes at high temperatures. In heavily doped semiconductors (curve 5) and also at low temperatures in pure p-type Ge (curves 1 and 3), the slopes of the curves $\pi_{<100>}$ are positive, which should be related to a change of sign in the inequality (3). It should be noted that, if the scattering by ionized impurities predominates, then $\lambda \approx 0.1$ and the inequality (3) could change sign only for the first set of values of the constants.

Using the experimental results in the range where the temperature dependence of $\pi_{<111>}$ is linear ($4 < 10^3/T < 9$) and setting $\lambda = 0.34$, we can apply Eq. (1) to curve 2 for a pure sample and determine the value of d; these values are listed in Table 1. It should be noted that the slope of curve 4 in the interval $10^3/T \approx 4-6$ is close to the slope of curve 2, which is due to the fact that the impurity scattering is unimportant in both samples in this temperature range. For a heavily doped semiconductor (curve 6), it is impossible to determine a range of temperatures where one of the scattering mechanisms predominates and, therefore, we have not used curve 6 to determine the value of d.

If the anisotropy of the dispersion law is neglected

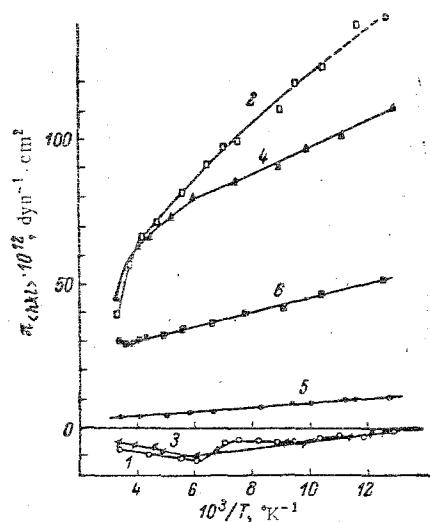


FIG. 1. Temperature dependences of $\pi_{<100>}$ and $\pi_{<111>}$ for p-type Ge. 1), 3), 5) $<100>$; 2), 4), 6) $<111>$; $\rho_{300}^{\circ}\text{K}$ ($\Omega \cdot \text{cm}$): 1), 2) 40; 3), 4) 16; 5), 6) 0.02.

($\xi_{11}, \langle hkl \rangle = 1$), practically identical values of d are obtained for all the sets of band structure constants included in Table 1 ($d \approx 6.0$ eV). This value is rather different from the value of d obtained with an allowance for the anisotropy of the dispersion law. Consequently, the anisotropy of the dispersion law not only affects the value of d but it has to be taken into account to explain the complex dependences of the piezoresistance of p-type Ge on the doping and temperature. The first set of values of the constants A, B, and C is preferable for p-type Ge since it explains the change of the sign of the slope of $\pi_{<100>}(1/T)$. Moreover, the corresponding value of d is in better agreement with other available data than the values obtained for other sets of band structure constants. However, it should be noted that the determination of the deformation potential constants from the temperature dependence of π_{ij} is not sufficiently reliable since the band structure constants A, B, and C and also the values of λ and ξ_j are not known with the required accuracy. The contributions of light and heavy holes to $\pi_{<100>}$ almost compensate one another and, therefore, $\pi_{<100>}$ is particularly sensitive to the values of λ and ξ_1 .

Figure 2 shows the dependence of the longitudinal piezoresistance of weakly doped p-type Ge on the pressure at 4°K. For $\parallel X \parallel <100>$ and small strain, the principal contribution to the piezoresistance is due to light holes [Eq. (1) and curves 1 and 3 in Fig. 1] and, therefore, the resistance is initially an increasing function of pressure. When the pressure is further increased, the density of light holes is reduced and the resistance decreases (see Ref. 2 for a more detailed discussion indicating that the effective mass of light holes in the direction of the strain increases as a function of the pressure whereas the effective mass of heavy holes decreases). For the $\parallel X \parallel <111>$ direction, the piezoresistance is governed by heavy holes and, therefore, the resistance decreases as a function of the pressure.

5. Figure 3 shows the temperature dependence of the Hall mobility for different values of the load X. When X is increased, the slopes of the curves $\mu(T)$ for a pure sample increase. The temperature at which the slope of $\mu(T)$ changes is equal to the temperature corresponding to the change of slope of $\pi_{<111>}$ on curve 4 (Fig. 1), which is due to the effect of an additional scattering mechanism (by ionized impurities) as the temperature is lowered. For a heavily doped sample, the slope of $\mu(T)$ increases as a function of X for temperatures higher than that corresponding to the maximum of $\mu(T)$ and it decreases at lower temperatures because of the different relative contribution of the scattering by ionized impurities. We have noted in Ref. 1.

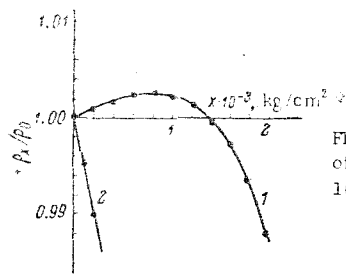


FIG. 2. Longitudinal piezoresistance of p-type Ge at 300°K. Here, $\rho_{300^\circ K} = 16 \Omega \cdot \text{cm}$; 1) $[100]$; 2) $[111]$.

that it is impossible to determine the energy dependence of the relaxation time from the slope of $\mu(T)$ in strained films of p-type Ge. This fact is illustrated by Fig. 3. The change in the slope of the temperature dependence of the mobility is greater for the drift mobility than for the Hall mobility.

APPENDIX

The parameters $\xi_{i, \langle hkl \rangle}$ which appear in Eq. (1) are given by

$$\xi_{i, \langle 100 \rangle} = 1.25 I_1^{(i)} / I_2^{(i)}, \quad \xi_{i, \langle 111 \rangle} = 5 I_3^{(i)} / I_2^{(i)}, \quad (\text{A.1})$$

where

$$I_1^{(i)} = \int d\Omega z^2 R_i^{(i)} \bar{B} \Phi_i^{-3/2} M^{-1} \left\{ \frac{4}{3} \Phi_i [(1-3z^2) R_3 M^{-1} + 2] + (1-3z^2) R_i^{(i)} \right\}, \quad (\text{A.2})$$

$$I_2^{(i)} = \int d\Omega (z R_i^{(i)})^2 \Phi_i^{-3/2}, \quad (\text{A.3})$$

$$I_3^{(i)} = \int d\Omega \bar{B} \Phi_i^{-3/2} M^{-1} \left\{ \frac{4}{3} \Phi_i [z^2 R_i^{(i)} (1-2x^2 R_2 M^{-1}) + x^2 R_i^{(i)} (1-2z^2 R_3 M^{-1})] - R_i^{(i)} R_x^{(i)} x z R_5 \right\},$$

$$\int d\Omega = \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi, \quad x = \cos \varphi \sin \theta, \quad y = \sin \varphi \sin \theta, \quad z = \cos \theta, \quad (\text{A.4})$$

$$M = (B^2 + C^2 R_4)^{1/2}, \quad \Phi_i, z = A \pm M, \quad C^2 = D^2 - 3B^2,$$

$$R_i^{(1,2)} = A \pm R_3, \quad R_x^{(1,2)} = A \pm R_1,$$

$$R_1 = \left[B^2 + \frac{1}{2} C^2 (z^2 + y^2) \right] M^{-1}, \quad R_2 = \left[B^2 + \frac{1}{2} C^2 (z^2 + x^2) \right] M^{-1},$$

$$R_3 = \left[B^2 + \frac{1}{2} C^2 (x^2 + y^2) \right] M^{-1}, \quad R_5 = xy + xz + yz,$$

$$R_4 = x^2 z^2 + x^2 y^2 + y^2 z^2.$$

Neglecting the anisotropy of the dispersion law leads to averaging with $C = 0$ and B and $D/\sqrt{3}$ replaced by \bar{B} . It can then be easily seen that $I_1^{(i)} = \frac{4}{15} I_0^{(i)}$, $I_2^{(i)} = \frac{1}{3} I_0^{(i)}$, and

$$I_3^{(i)} = \frac{1}{15} I_0^{(i)} \quad \text{where} \quad I_0^{(1,2)} = 4\pi/(A \pm B)^{3/2} \quad \text{and Eq. (A.1) yields} \\ \xi_{i, \langle hkl \rangle} = 1.$$

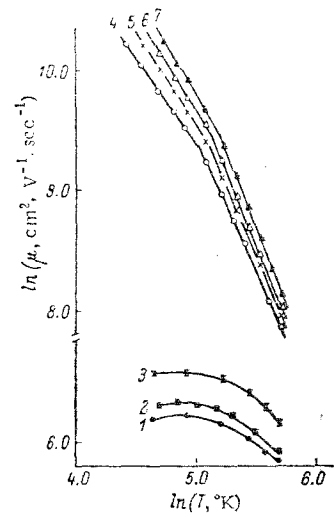


FIG. 3. Temperature dependences of the Hall mobility in p-type Ge for different values of the load. Here, $\rho_{300^\circ K} (\Omega \cdot \text{cm})$: 1)-3) 0.02; 4)-7) 16. X (kg/cm^2): 1), 4) 0; 2), 5) $3 \cdot 10^3$; 3) 10^4 ; 6) $5 \cdot 10^3$; 7) $7 \cdot 10^3$.

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