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Piezo Hall Effect in p-Germanium

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The piezo Hall effect is investigated on p-Ge samples of low ($\varrho_{300\,\mathrm{K}} = 0.02\,\Omega\mathrm{cm}$) and high ($\varrho_{300\,\mathrm{K}} = 16\,\Omega\mathrm{cm}$) resistivity in the temperature range 77 to 300 K for the case of current $\boldsymbol{i}$ and uniaxial stress $\boldsymbol{x}$ being parallel to the $\langle 100\rangle$ and $\langle 111\rangle$ crystal axes. Taking into account the dispersion law anisotropy for light and heavy holes allows to explain the experimental temperature dependence of the piezoresistivity coefficients for samples of low and high resistivity and orientational dependence of this effect. The deformation potential constants b and d are obtained which vary in the range $b = -(2.2$ to $2.6)$ eV, $d = -(4.4$ to $5.1)$ eV depending on the values of the band parameters A, B, and C given in literature. These values of b, d are considerably different from those obtained by means of the same experimental data, but without taking into account the dispersion law anisotropy.

В интервале температур 77 до 300 K образцах p-Ge с малым ($\varrho_{300\,\mathrm{K}} = 0{,}02\,\Omega\mathrm{cm}$) и большим ($\varrho_{300\,\mathrm{K}} = 16\,\Omega\mathrm{cm}$) сопротивлением исследован пьезо-Холл-эффект для случая, когда ток $\boldsymbol{i}$ и одноосное давление $\boldsymbol{x}$ параллельны осям $\langle 100\rangle$ и $\langle 111\rangle$ кристалла. При трактовке экспериментальных результатов учтена анизотропия закона дисперсии легких и тяжелых дырок, что позволило объяснить различие температурных зависимостей коэффициентов пьезосопротивления для образцов с малым и большим ϱ, а также ориентационную зависимость эффекта. Получены значения констант деформационного потенциала b и d, которые в зависимости от известных в литературе значений зонных констант A, B и C изменяются в пределах $b = -(2{,}2$ до $2{,}6)$ eV, $d = -(4{,}4$ до $5{,}1)$ eV, что существенно отличается от b и d, полученных по тем же экспериментальным данным без учета анизотропии закона дисперсии.

1. Introduction and Formulae for Calculation

The piezo Hall effect in p-Ge has been investigated in [1, 2]. However, the dispersion law anisotropy (DLA) was not taken into account in the calculations and thus in the interpretation of the experimental results. Taking account of the DLA influences sufficiently the results of the piezoresistivity calculation (see, for example, [3]). Therefore, it is of interest to consider this influence on the piezo Hall effect, too.

The dependence of the Hall coefficient upon the value of the uniaxial stress X is determined by the expression

$$R(x) = R_0[1 + (\delta_{\langle hkl\rangle} + \tfrac{1}{2}\pi_{\langle hkl\rangle})\,X]\,, \tag{1}$$

where $R_0(\mu_1\sigma_1 + \mu_2\sigma_2)/[c(\sigma_1 + \sigma_2)]$; μ_i; σ_i are the Hall mobility and conductivity for the light ($i = 1$) and heavy ($i = 2$) holes, respectively, $\delta_{\langle hkl\rangle} + \frac{1}{2}\pi_{\langle hkl\rangle}$ is the piezo Hall effect coefficient, $\pi_{\langle hkl\rangle}$ is the longitudinal piezoresistance coefficient, $\delta_{\langle hkl\rangle} = \delta^0_{\langle hkl\rangle} + \delta^T_{\langle hkl\rangle}$, $\delta^0_{\langle hkl\rangle}$ are temperature independent and $\delta^T_{\langle hkl\rangle}$ are temperature dependent components of $\delta_{\langle hkl\rangle}$.

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For the interpretation of the experimental results only $\delta^T_{\langle hkl\rangle}$ are used. The expression for $\delta^T_{\langle hkl\rangle}$ may be easily obtained according to [4]

$$\delta^T_{\langle hkl\rangle} = \frac{0.3\,\eta_{\langle hkl\rangle}}{\bar{B}(1+\lambda)\,k_0T}\,(\lambda\zeta_1\xi_{1,\langle hkl\rangle} - \zeta_2\xi_{2,\langle hkl\rangle})\,. \tag{2}$$

Here

$$\eta_{\langle 100\rangle} = \frac{2Bb}{C_{11}-C_{12}}\,,\qquad \eta_{\langle 111\rangle} = \frac{Dd}{3C_{44}}\,,\qquad \lambda = \frac{\sigma_1\mu_1}{\sigma_2\mu_2}\,, \tag{3}$$

$$\zeta_i = \frac{\langle\tau_i^2\rangle}{2\langle\varepsilon\tau_i^2\rangle}\,,\qquad \langle\varepsilon^k\tau_i^l\rangle = \frac{\int\limits_0^\infty \varepsilon^{1/2+k}\tau_i^2(\varepsilon)\,\mathrm{e}^{-\varepsilon}\,\mathrm{d}\varepsilon}{\int\limits_0^\infty \varepsilon^{1/2}\,\mathrm{e}^{-\varepsilon}\,\mathrm{d}\varepsilon}\,. \tag{4}$$

ε ist the hole energy in k_0T units, $\bar{B} = \sqrt{B^2 + C^2/5}$, B, D, C are the valence band constants [4]. In (2) only the parameters $\xi_{i,\langle hkl\rangle}$ depend on DLA. Their analytical expression is written in the Appendix because it is cumbersome. It follows from the Appendix that the neglect of DLA gives $\xi_{i,\langle hkl\rangle} = 1$, while taking into this circumstance leads to the deviation of $\xi_{i,\langle hkl\rangle}$ from unity.

We calculated $\xi_{i,\langle hkl\rangle}$ using the formulae of the Appendix and different sets of constants A, B, and C which are known [5 to 10] for p-Ge, p-Si, and p-GaAs. The results are listed in Table 1. It is seen from the table that the parameters $\xi_{i,\langle hkl\rangle}$ for Ge slightly differ from one set to another due to a slight distinction of the different sets of constants A, B, and C. Quite another situation takes place in Si, for which the scatter of values A, B, and C is large.

2. Experimental Results and Comparison with Calculation Data

Samples of p-Ge of resistivity $\varrho_{300\,\mathrm{K}} = 16\ \Omega\mathrm{cm}$ and $\varrho_{300\,\mathrm{K}} = 0.02\ \Omega\mathrm{cm}$ and sizes of $0.7\times 0.7\times 8\ \mathrm{mm}^3$ were investigated. The long dimension of samples coincided with $\langle 100\rangle$ or $\langle 111\rangle$ directions. The dependence of R and ϱ on X was measured under small loads (in the region of linear dependence of these values on the stress) in the temperature range from 77 to 300 K for $\boldsymbol{i}\parallel\boldsymbol{x}\parallel\langle 100\rangle$ and $\boldsymbol{i}\parallel\boldsymbol{x}\parallel\langle 111\rangle$ directions ($\boldsymbol{i}$ is the current vector). $\pi_{\langle hkl\rangle} = (\varrho_x - \varrho_0)/\varrho_0X$ and $\delta_{\langle hkl\rangle} + \frac{1}{2}\pi_{\langle hkl\rangle} = (R_x - R_0)/R_0X$ (i.e. experimental dependence of $\delta_{\langle hkl\rangle}$ on temperature) were determined (see Fig. 1).

It is seen from Fig. 1 that in the relatively pure semiconductor (curves 1 and 2)

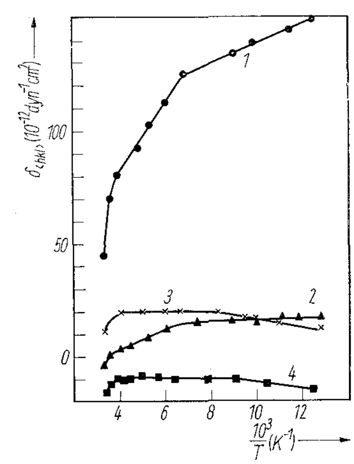

Fig. 1. Temperature dependence of $\delta_{\langle 100\rangle}$ ((1) and (3)) and $\delta_{\langle 111\rangle}$ ((2) and (4)) in p-Ge. (1) and (2) $\varrho_{300\,\mathrm{K}} = 16\ \Omega\mathrm{cm}$, (3) and (4) $\varrho_{300\,\mathrm{K}} = 0.02\ \Omega\mathrm{cm}$

Table 1

semiconductor	No.	band constants A	B	$\|C\|$	ref.	$\xi_{1,\langle 100\rangle}$	$\xi_{2,\langle 100\rangle}$	$\xi_{1,\langle 111\rangle}$	$\xi_{2,\langle 111\rangle}$	b (eV)	d (eV)
germanium	1	−13.0	−8.9	10.3	[5]	1.2	0.32	0.91	1.38	−2.6	−4.4
	2	−13.1	−8.3	12.5	[6]	1.31	0.17	0.87	1.44	−2.3	−5.1
	3	−13.27	−8.63	12.4	[7]	1.29	0.18	0.87	1.44	−2.3	−5.1
	4	−12.35	−8.26	12.07	[8]	1.30	0.096	0.87	1.49	−2.2	−5.4
silicon	5	− 4.1	−1.6	3.3	[5]	1.45	0.46	0.84	1.25		
	6	− 4.0	−1.1	4.1	[6]	2.14	1.01	0.72	1.27		
	7	− 4.38	−1.0	4.80	[8]	2.59	0.72	0.65	1.24		
	8	− 4.27	−0.63	5.03	[9]	3.29	1.14	0.60	1.14		
	9	− 4.28	−0.25	5.25	[10]	3.12	1.54	0.59	1.17		
gallium arsenide	10	− 7.39	−4.93	5.06	[7]	1.14	0.48	0.92	1.28		

$\delta_{\langle 100\rangle}$ and $\delta_{\langle 111\rangle}$ are positive and increase with rising $1/T$. This indicates the predominating contribution of light holes to $\delta_{\langle hkl\rangle}$ — see (2). Supposing $\lambda = 2.8$, $\zeta_i = 1$ (this is correct, if the scattering on acoustic phonons is predominant), $C_{44} = 6.67 \times 10^{11}$ dyn per cm^2, $C_{11} - C_{12} = 8.02 \times 10^{11}$ dyn/cm^2 [11]. After that we obtain the values of the deformation potential constants (see Table 1) from a comparison of theoretical and experimental slopes $\delta_{\langle hkl\rangle}(1/T)$ for the region where $\delta_{\langle hkl\rangle}$ depends linearly on $1/T$.

By neglecting DLA ($\zeta_{i,\langle hkl\rangle} = 1$) b and d are quite equal for the all sets of band parameters listed in Table 1 ($b \approx -4.4$ eV, $d \approx -2.8$ eV). These values are considerably different from b and d obtained by us with taking into account of DLA. Moreover this result ($|b| > |d|$) is in contradiction to literature data [12]. Therefore, it is necessary to take into account DLA in the interpretation of the experimental results. Especially it must be noted that in contrast to the case $\boldsymbol{i} \parallel \boldsymbol{x} \parallel \langle 100\rangle$ for which $\lambda\xi_{1,\langle 100\rangle} \gg \xi_{2,\langle 100\rangle}$ and the contribution of the light holes is principal, the value $\xi_{2,\langle 111\rangle}$ is close to $\lambda\xi_{1,\langle 111\rangle}$ for $\boldsymbol{i} \parallel \boldsymbol{x} \parallel \langle 111\rangle$. That is why the difference $\lambda\xi_{1,\langle 111\rangle} - \xi_{2,\langle 111\rangle}$ is very sensitive to the choice of the λ value. In particular, if λ is smaller than 1.52; 1.66; 1.71, and 1.66 for the first, second, third, and fourth sets of constants, respectively, this difference has always another sign.

The slope of $\delta_{\langle hkl\rangle}(1/T)$ for a heavily doped semiconductor is practically zero at temperatures larger than 120 K (curves 3, 4). This is caused by decreasing of ζ_i and λ (for scattering on ionized impurities $\zeta_i = 1/9$, $\lambda \lesssim 0.36$). Although the slopes of curves 3 and 4 are small, it should be noted that they have different signs. This may be explained by predominant contributions of heavy holes to $\delta_{\langle 111\rangle}$ and light holes to $\delta_{\langle 100\rangle}$ for $\lambda \lesssim 1$ (see (2)). The slope of the $\delta_{\langle hkl\rangle}(1/T)$ curves changes in the sample with $\varrho = 0.02\ \Omega$cm, if $T < 120$ K. This is caused by the transition to another conductivity mechanism. In this temperature range R has a maximum and the reduction of R with decreasing temperature indicates a contribution of hopping conductivity [13].

3. Conclusion

Comparing the values of b and d obtained with taking into account DLA and without it, we come to the following conclusion.

It is necessary to take into account CLA in the interpretation of the piezo Hall effect in p-Ge (in particular, in the experimental determination of the deformation potential constants). This circumstance allows to account for the experimentally observed features of the piezo Hall effect in p-Ge and to obtain reasonable estimations for the deformation potential constants b and d.

The accuracy of determination of the deformation potential constants is limited by the precision with which the band parameters A, B, and C are known. In Si, where the scatter of values A, B, and C is large, the corresponding scatter of values $\xi_{i,\langle hkl\rangle}$ is large, too. This creates difficulties for the determination of b and d.

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Appendix

The parameters $\xi_{i,\langle hkl\rangle}$ in (2) are defined by the expressions

$$\xi_{i,\langle 100\rangle} = \frac{5}{3}\frac{I_1^{(i)}}{I_2^{(i)}}, \qquad \xi_{i,\langle 111\rangle} = \frac{10}{3}\frac{I_3^{(i)}}{I_2^{(i)}},$$

where

$$I_1^{(i)} = \int \mathrm{d}\Omega\, \bar{B}\Phi_i^{-5/2} M^{-1} \{\tfrac{3}{2}(1 - 3z^2)(x^2 R_x^{(i)2} R_{zz}^{(i)} - yz R_y^{(i)} R_z^{(i)} R_{yz}^{(i)}) -$$
$$- \Phi_i[x^2 R_x^{(i)}(R_x^{(i)}\,\Delta R_{zz} + 2\Delta R_x R_{zz}^{(i)}) -$$
$$- yz(R_y^{(i)} R_z^{(i)}\,\Delta R_{yz} + R_{yz}^{(i)}[R_y\,\Delta R_z + R_z^{(i)}\,\Delta R_y])]\} ,$$

$$I_2^{(i)} = \int \mathrm{d}\Omega\, \Phi_i^{-5/2}(x^2 R_x^{(i)2} R_{zz}^{(i)} - yz R_y^{(i)} R_z^{(i)} R_{yz}^{(i)}) ,$$

$$I_3^{(i)} = \int \mathrm{d}\Omega\, \bar{B}\Phi_i^{-5/2} M^{-1} \{\tfrac{3}{2}(R_5 R_{zz}^{(i)} - \tfrac{1}{3}\Phi_i\,\Delta\bar{R}_{zz})(yz R_y^{(i)} R_z^{(i)} - x^2 R_x^{(i)2}) +$$
$$+ \tfrac{3}{2}(R_5 R_{yz}^{(i)} - \tfrac{1}{3}\Phi_i\,\Delta\bar{R}_{yz})(yz R_y^{(i)} R_z^{(i)} + xz R_x^{(i)} R_x^{(i)} - z^2 R_z^{(i)2} - x^2 R_x^{(i)2}) +$$
$$+ \tfrac{1}{2}\Phi_i\,[R_{zz}^{(i)}(2x R_x^{(i)}\,\Delta\bar{R}_x - z R_z^{(i)}\,\Delta\bar{R}_y - y R_y^{(i)}\,\Delta\bar{R}_z) +$$
$$+ R_{zy}^{(i)}(2z R_z^{(i)}\,\Delta\bar{R}_z + 2x R_x^{(i)}\,\Delta\bar{R}_x - y R_y^{(i)}\,\Delta\bar{R}_z - z R_z^{(i)}(\Delta\bar{R}_y + \Delta\bar{R}_x) -$$
$$- x R_x^{(i)}\,\Delta\bar{R}_z)]\} .$$

Here

$$\int \mathrm{d}\Omega = \int_0^\pi \sin\theta\,\mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\varphi , \quad x = \cos\varphi\sin\theta , \quad y = \sin\varphi\sin\theta , \quad z = \cos\theta ,$$

$$M = (B^2 + C^2 R_4)^{1/2} , \qquad \Phi_{1,2} = A \pm M , \qquad C^2 = D^2 - 3B^2 ,$$

$$R_z^{(1,2)} = A \pm R_3 , \qquad R_x^{(1,2)} = A \pm R_1 , \qquad R_y^{(1,2)} = A \pm R_2 ,$$

$$R_1 = [B^2 + \tfrac{1}{2} C^2(z^2 + y^2)]\, M^{-1} , \qquad R_2 = [B^2 + \tfrac{1}{2} C^2(z^2 + x^2)]\, M^{-1} ,$$

$$R_3 = [B^2 + \tfrac{1}{2} C^2(x^2 + y^2)]\, M^{-1} , \qquad R_4 = x^2z^2 + x^2y^2 + y^2z^2 ,$$

$$R_5 = xy + xz + yz ,$$

$$\Delta R_{zz} = -2 - \frac{R_3}{M}\left\{1 - 3z^2 + z^2[2 - 3(1 - 3z^2)] - 2z^2(1 - 3z^2)\,\frac{B^2}{M^2}\right\},$$

$$\Delta R_{yz} = -yz\,\frac{1}{M}\left\{2R_3\left(1 - 3\,\frac{1 - 3z^2}{M}\,R_2\right) - 4R_2 + (1 - 3z^2)\,\frac{2B^2 + C^2}{M}\right\},$$

$$\Delta R_{xy} = 1 - \frac{R_{1,2}}{M}(1 - 3z^2) , \qquad \Delta R_z = -2 - \frac{R_3}{M}(1 - 3z^2) ,$$

$$R_{zz}^{(1,2)} = R_2^{(1,2)} \mp \frac{C^2}{2M}\,z^2\left\{4(x^2 + y^2 - R_4) + \frac{C^3}{M^2}(x^2 + y^2 - 2R_4)^2\right\},$$

$$R_{yz}^{(1,2)} = \pm\frac{C^2}{M}\,yz\left\{-x^2 + 2R_4 - \frac{C^2}{2M^2}(x^2 + y^2 - 2R_4)(x^2 + z^2 - 2R_4)\right\},$$

$$\overline{\Delta R}_{zz} = -\frac{2R_3}{M}\left[2xz + yz + \left(1 - \frac{6R_3}{M}\,z^2\right)R_5\right] - \frac{4B^2}{M^2}\,z^2 R_5 ,$$

$$\overline{\Delta R}_{zy} = 1 - \frac{2R_3}{M}\left[xz + z^2 - yz\,\frac{6R_2}{M}\,R_5\right] - \frac{2R_2}{M}(y^2 + yx) -$$
$$- 2yz\,\frac{2B^2 + C^2}{M^2}\,R_5 ,$$

$$\overline{\Delta R}_{x,y,z} = x + y + z - (x, y, z)(1 - 2R_5 R_{1,2,3} M^{-1}) .$$

Neglecting the anisotropy of the dispersion law corresponds to replacement of B and $D/\sqrt{3}$ by the mean value $\bar{B}$ and $C = 0$. It is easy to see that in this approximation

$$I_1^{(i)} = \tfrac{1}{5}\,\bar{I}_0^{(i)}\,, \qquad I_2^{(i)} = \tfrac{1}{3}\,\bar{I}_0^{(i)}\,, \qquad I_3^{(i)} = \tfrac{1}{10}\,\bar{I}_0^{(i)}\,,$$

where

$$\bar{I}_0^{(1,2)} = 4\pi(A \pm B)^{1/2} \quad \text{and} \quad \xi_{i,\langle hkl\rangle} = 1\,.$$

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