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Piezoresistance of Uniaxially Deformed n-Si

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Piezoresistance of nondegenerate n-Si is calculated for the case when the directions of the current and compression axis coincide with the ⟨100⟩ axis. Low temperatures are considered ($T<100$ K), when electrons are scattered by acoustic phonons, ionized impurities, and electrons of its own and other valleys. The piezoresistance is shown to depend not only on the impurity concentration but also on the scattering intensity of the electrons of one valley by those of the other valleys. The latter may be determined by comparing the experimental and theoretical deformation resistance dependences under the conditions when the acoustic scattering is inessential due to low temperatures or high doping levels.

Рассчитано пьезосопротивление невырожденного n-Si для случая, когда ток и ось сжатия совпадают с осью ⟨100⟩. Рассмотрены низкие ($T < 100$ K) температуры, когда рассеяние электронов осуществляется на акустических колебаниях решетки, ионизированных примесях, электронах своей долины и электронах других долин. Показано, что пьезосопротивление зависит не только от концентрации примесей, но и от интенсивности рассеяния электронов одной долины на электронах другой долины. Эта интенсивность может быть определена из сравнения экспериментальных зависимостей сопротивления от давления с теоретическими для случая, когда рассеяние на акустических колебаниях решетки не существенно вследствие низких температур или высоких уровней легирования.

1. Introduction

It is known that the consideration of transport phenomena in heavily doped semiconductors and nonheating electric fields implies the account of momentum relaxation due to both ionized impurity scattering and to electron–electron (e–e) scattering [1 to 3]. In this case in multi-valley semiconductors one should distinguish between electron scattering by electrons from its own valley and that from a foreign valley (intravalley and intervalley e–e scattering, respectively, in terms of [4]).

In this paper the piezoresistance of uniaxially deformed non-degenerate n-Si is calculated for the case when the current and the axis of compression coincide with the ⟨100⟩ axis (x axis). The aim is to clarify the role of the intervalley e–e scattering. The low-temperature case ($T < 100$ K) is considered when scattering by optical f- and g-phonons does not contribute to the momentum relaxation (see e.g. [5, 6]). Therefore in pure semiconductors only the acoustic scattering is significant and the deformation just leads to a relative change of the electron concentration in various valleys but not to a mobility variation.

On the other hand, at still lower temperatures or at high doping levels, when the Coulomb contribution to the momentum relaxations is essential, the situation becomes markedly different. The electron redistribution among the valleys at deformation P causes a change of the relative role of the intra- and intervalley e–e scattering, i.e. to a dependence of mobility upon P.

2. Results

To calculate the above-mentioned pressure dependence of the mobility, the rate equations for each valley must be solved. Let us write the distribution function for each of the valleys in the following form:

$$f^{(\alpha)}(\boldsymbol{p}) = f_0^{(\alpha)}(\varepsilon)\,[1 + F^{(\alpha)}(\boldsymbol{p})]; \qquad |F^{(\alpha)}(\boldsymbol{p})| \ll 1\,, \tag{1}$$

$$f_0^{(\alpha)}(\varepsilon) = \frac{n_\alpha}{n_0} f_0(\varepsilon); \qquad F^{(\alpha)}(\boldsymbol{p}) = \frac{eE_x}{B_{\text{imp}} \cdot 3n_0} \sqrt{\frac{\pi kT}{m_\perp}} \left(\boldsymbol{p} \cdot \boldsymbol{C}^{(\alpha)} + \boldsymbol{p} \cdot \boldsymbol{A}^{(\alpha)} \frac{\varepsilon}{kT}\right). \tag{2}$$

Here $f_0(\varepsilon)$ is the equilibrium Boltzman distribution function, ε is the electron energy, $\boldsymbol{p}$ is the crystal momentum, E_x is the external electric field, $B_{\text{imp}} = 2\pi e^4 \ln (h/b_0)$ is the coupling constant of electrons with ionized impurities [1, 4, 7], h is the screening radius, $b_0 = e^2/3kT$; $e = e_0/\sqrt{\varkappa}$, $\varkappa$ is the dielectric susceptibility, $m_\perp$ and $m_\|$ stand for the longitudinal and transverse effective masses, respectively, n_0 is the equilibrium electron concentration in one valley, $\boldsymbol{C}^{(\alpha)}$ and $\boldsymbol{A}^{(\alpha)}$ are unknown parameters. Following [4] one obtains for them the following set of equations:

$$\left.\begin{aligned}
&C_x^{(\alpha)}(T_{xx}^{(\alpha)} + R_{xx}^{(\alpha)}) + A_x^{(\alpha)}(T_{xx}^{(\alpha)} + 3R_{xx}^{(\alpha)}) + \sum_{\beta \neq \alpha} \frac{n_\beta}{n_0} M_{xx}^{(\alpha\beta)}(C_x^{(\alpha)} - C_x^{(\beta)}) + \\
&+ \sum_{\beta \neq \alpha} \frac{n_\beta}{n_0} (A_x^{(\alpha)} N_{xx}^{(\alpha\beta)} - A_x^{(\beta)} K_{xx}^{(\alpha\beta)}) + 1 = 0\,, \\
&\frac{2}{5} C_x^{(\alpha)} (T_{xx}^{(\alpha)} + 3R_{xx}^{(\alpha)}) + \frac{4}{5} A_x^{(\alpha)}(T_{xx}^{(\alpha)} + 6R_{xx}^{(\alpha)}) + A_x^{(\alpha)} L_{xx}^{(\alpha)} \frac{n_\alpha}{n_0} + \\
&+ \sum_{\beta \neq \alpha} \frac{n_\beta}{n_0} (C_x^{(\alpha)} - C_x^{(\beta)})\, \varphi_{xx}^{(\alpha,\beta)} + \sum_{\beta \neq \alpha} (A_x^{(\alpha)} \Psi_{xx}^{(\alpha,\beta)} - A_x^{(\beta)} \Phi_{xx}^{(\alpha,\beta)}) \frac{n_\beta}{n_0} + 1 = 0\,, \\
&C_x^{(2)} = C_x^{(3)}; \quad A_x^{(2)} = A_x^{(3)}; \quad C_y^{(\alpha)} = C_z^{(\alpha)} = A_y^{(\alpha)} = A_z^{(\alpha)} = 0\,.
\end{aligned}\right\} \tag{3}$$

The valleys are labelled so that the valleys[1]) 1, 2, 3 lie on the x, y, z axes, respectively. All the coefficients of (3), except of $R_{xx}^{(\alpha)}$, are given in the Appendix of [4]. The new coefficient $R_{kk}^{(\alpha)}$ characterizes the scattering by acoustic phonons; in the relaxation time approximation $\left(\tau_{kk}^{(\alpha)} \equiv \tau_{kk}^{0,(\alpha)} \sqrt{kT/\varepsilon}\ [8]\right)$ it has the form

$$R_{kk}^{(\alpha)} = \frac{8}{3} \frac{(kT)^{3/2}}{B_{\text{imp}} \cdot 3n_0 \sqrt{m_\perp}} \frac{m_k}{\tau_{kk}^{0,(\alpha)}}\,. \tag{4}$$

The same holds for $T_{kk}^{(\alpha)}$ [4], the coefficients $R_{kk}^{(\alpha)}$ are related by $R_{xx}^{(1)} = R_{yy}^{(2)} = R_{zz}^{(3)} = R_2$; $R_{yy}^{(1)} = R_{zz}^{(1)} = R_{xx}^{(2)} = R_{zz}^{(2)} = R_{xx}^{(3)} = R_{yy}^{(3)} = R_1$ and since $\tau_\perp^0/\tau_\|^0 = 3/2$ [8], $R_2 = 7.02\,R_1$. Further the relations (3) differ from similar ones in [4] by the multipliers n_α/n_0 and n_β/n_0, taking account of the change of the intra- and intervalley e–e scattering at deformation, here

$$n_1 = n_0 \frac{3\gamma}{\gamma + 2}, \quad n_2 = n_3 = n_0 \frac{3}{\gamma + 2}, \quad \gamma \equiv \frac{n_1}{n_2} = \exp\left[-\frac{\Xi_{\text{u}} P}{(C_{11} - C_{12})\, kT}\right]. \tag{5}$$

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[1]) Here as well as in [4] the two valleys placed on the ⟨100⟩ axis are considered as one valley.

Ξ_u is the deformation potential constant, C_{11} and C_{12} are the elastic constants, k is the Boltzmann constant, and P is the pressure.

Using the expression for the current

$$\boldsymbol{j} = \sum_\alpha \boldsymbol{j}^{(\alpha)} = -e \sum_{\alpha, \boldsymbol{p}} \frac{\partial \varepsilon}{\partial \boldsymbol{p}} f^{(\alpha)}(\boldsymbol{p}) \equiv \sigma(P)\,\boldsymbol{E}$$

and (1), (2), we obtain for the conductivity

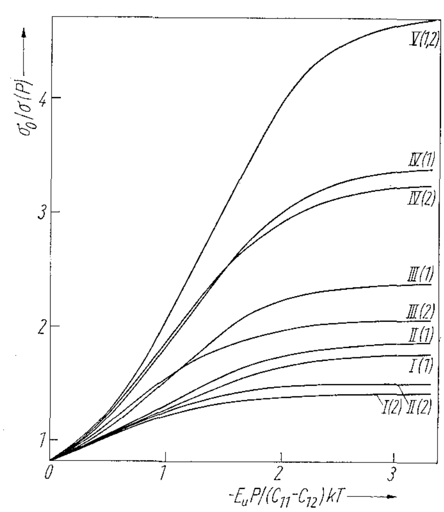
$$\sigma(P) = e n_0 \frac{3}{\gamma + 2} (\gamma \mu_{\|} + 2\mu_{\perp}) \,, \tag{6}$$

where

$$\left.\begin{aligned} \mu_{\|} = \mu_{\|}^{(1)} &= -e \sqrt{\frac{\pi}{m_\perp}} \, \frac{(kT)^{3/2}}{B_{\mathrm{imp}} \cdot 3n_0} \left(C_x^{(1)} + \frac{5}{2} A_x^{(1)}\right), \\ \mu_{\perp} = \mu_{\perp}^{(2)} &= -e \sqrt{\frac{\pi}{m_\perp}} \, \frac{(kT)^{3/2}}{B_{\mathrm{imp}} \cdot 3n_0} \left(C_x^{(2)} + \frac{5}{2} A_x^{(2)}\right) \end{aligned}\right\} \tag{7}$$

are the mobilities. Substituting into (7) the solutions of (3) for C_x and A_x we obtain the pressure dependence of the conductivity. An uncompensated semiconductor is considered, the impurity concentration being equal to the total electron concentration $3n_0$. Since in n-Si the static and high-frequency dielectric susceptibilities are the same, the coupling constants of electrons with impurities B_{imp} coincide with those of their own valley $B_{\alpha\alpha}$ [9], the parameter $b = B_{\alpha\alpha}/B_{\mathrm{imp}}$ entering [4] into the coefficients of (3) is put equal to unity. The piezoresistance, i.e. the P dependence of $\sigma_0/\sigma(P)$, is shown in Fig. 1. The parameter

$$\alpha = \frac{R_2}{T_2} = \frac{22.2t}{\ln(264t)}\,; \qquad t = \left(\frac{T}{100}\right)^3 \frac{10^{15}}{n}\,; \qquad (n = 3n_0) \tag{8}$$



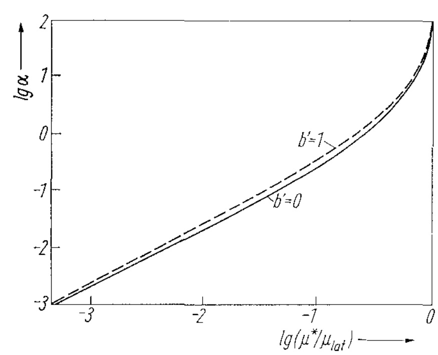


Fig. 2. The parameter α versus the ratio of effective mobility μ^* to the lattice mobility μ_{lat} for $b' = 0$ and $b' = 1$

Fig. 1. The piezoresistance $\sigma_0/\sigma(P)$ versus pressure P for different values of the parameters α and b'. I. $\alpha = 0$; II. $\alpha = 0.1$; III. $\alpha = 1$; IV. $\alpha = 10$; V. $\alpha = 1000$; (1) $b' = 0$, (2) $b' = 1$

characterizes the relative role of acoustic phonon scattering which is small when $\alpha \ll 1$ and dominates when $\alpha \gg 1$; while $b' = B_{\alpha\beta}/B_{\alpha\alpha}$ ($B_{\alpha\beta}$ being the coupling constant of α valley electrons with those of the $\beta \neq \alpha$ valley[2])) characterizes the relative role of the intervalley e–e scattering. From Fig. 1 it is seen that the piezoresistance decreases on increasing both the contribution of the Coulomb mechanism to the momentum relaxation and the intensity of the intervalley e–e scattering (increase of b'). To determine b' from Fig. 1 one must know the value of α for the semiconductor in question. This can be obtained from (8) or from Fig. 2 where α is given as a function of the ratio of the effective mobility $\mu^* = \sigma_0/3n_0e$ to the lattice mobility μ_{lat} ($\mu_{\text{lat}} = 2 \times 10^7 \times (T^0k)^{-3/2}$ [8]).

In the high pressure case ($\gamma \gg 1$) and in absence of deformation ($\gamma = 1$) we obtain respectively from (6)

$$\sigma_{(-\infty)} = 3n_0e\mu_{\parallel}^{(-\infty)}\,, \qquad \sigma_0 = en_0\,(\mu_{\parallel}^0 + 2\mu_{\perp}^0)\,. \tag{9}$$

Here the indexes "0" and "$-\infty$" remind the value of P.

From (9) and (6) one obtains easily

$$\gamma = \frac{3\sigma_0 - \sigma_{(-\infty)} - 2\sigma + 3n_0e\,[2(\mu_{\perp} - \mu_{\perp}^0) + (\mu_{\parallel}^{(-\infty)} - \mu_{\parallel}^0)]}{\sigma - \sigma_{(-\infty)} + 3n_0e\,[\mu_{\parallel}^{(-\infty)} - \mu_{\parallel}^0]}\,, \tag{10}$$

assuming $\mu_{\parallel}$ and $\mu_{\perp}$ to be independent of pressure leads to vanishing of the square bracketed expression in (10), thus

$$\gamma_1 = \frac{3\sigma_0 - \sigma_{(-\infty)} - 2\sigma}{\sigma - \sigma_{(-\infty)}}\,. \tag{11}$$

Fig. 3 shows the $\ln \gamma_1$ dependence upon P usually employed when analysing the experimental data (see, for example, [6, 10]). When the acoustic phonon scattering is dominating $\mu_{\parallel}$ and $\mu_{\perp}$ are independent of P and $\gamma_1 = \gamma$ so that $\ln \gamma_1$ (curves IV, V on Fig. 3) is a linear function of P.[3]) At small α when $\mu_{\parallel}$ and $\mu_{\perp}$ depend on P, $\gamma_1 \neq \gamma$.[4]) At $b' = 0$ the $\ln \gamma_1$ curve occurs to be lower than the $\ln \gamma(P)$ straight line. At $b' = 0$ the intervalley scattering is absent and the increase of

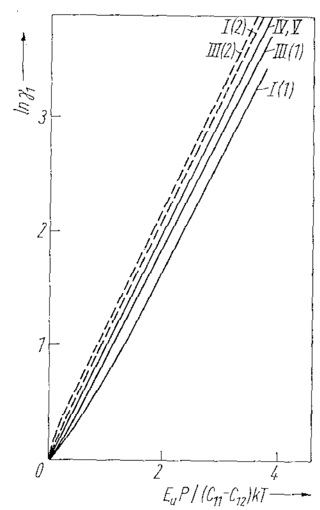


Fig. 3. $\ln \gamma_1$ versus pressure P for different values of the parameters α and b'. The labelling of the curves is the same as in Fig. 1

[2]) As it was pointed out in [4] $B_{\alpha\alpha}$ and $B_{\alpha\beta}$ may differ only due to their being obtained by averaging the Coulomb interaction potential over the Bloch functions of a given valley ($B_{\alpha\alpha}$) or of different valleys ($B_{\alpha\beta}$).

[3]) The linear dependence of $\ln \gamma_1$ on P was employed in [6, 10] to determine the deformation potential constants in n-type Ge.

[4]) Note that a deviation from the linear dependences was observed in [5] at high temperatures ($T > 100$ K), when the relative role of scattering by f- and g-phonons varied with pressure. This leads to a pressure dependence of the mobility.

pressure has almost no influence on the conductivity in the valley 1 ($\mu_{\parallel} \approx$ const), where the carrier concentration increases no more than by a factor of three. At the same time it does influence the mobility $\mu_{\perp}$ as the relevant carrier concentration decreases exponentially and the e–e scattering becomes unimportant ($\mu_{\perp} > \mu^0_{\perp}$) in the valleys 2 and 3. Then as it is seen by comparing (10) and (11), $\ln \gamma > \ln \gamma_1$. At $b' = 1$ the intervalley scattering tends to an equalisation of $\mu_{\parallel}$ and $\mu_{\perp}$. Therefore $\mu_{\parallel}$ will decrease with growing pressure because of the reduction of carriers in the valleys 2 and 3, and $\ln \gamma_1 > \ln \gamma$. Known experimental piezoresistance data of n-Si [6] concern the case of comparatively high temperatures and low concentrations ($\alpha \gg 1$), so $\ln \gamma_1$ occurs to be approximately a linear function of P.

3. Conclusion

The dependences of α upon μ (Fig. 2) and of the piezoresistance upon pressure (Fig. 1) make it possible to determine the parameter b' characterizing the intensity of the intervalley e–e scattering. Especially convenient in our opinion is the analysis using (11) and Fig. 3 due to a strong influence of b' upon the value and the sign of the difference ($\ln \gamma_1/\ln \gamma - 1$).

To conclude the knowledge of b' is necessary not so much for the correct interpretation of the piezoresistance experiments as for the proper choice of approximations for studying the hot electron problem in multivalley semiconductors. At $b' \to 0$ the individual valley electron temperature approximation seems appropriate, whereas at $b' \to 1$ it is more likely that the electron temperature is the same in all the valleys.

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