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Linear Magnetoresistance in p-Ge for Weakly Heating Electric Fields

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In p-Ge crystals light and heavy holes satisfy the anisotropic dispersion laws (reference (1)) which in the linear anisotropy approximation are of the form

$$\varepsilon_i(\vec{p}) = \frac{p^2}{2m_i}\left[1 + A_i\left(1 - 5\,\frac{p_{x'}^2 p_{y'}^2 + p_{x'}^2 p_{z'}^2 + p_{y'}^2 p_{z'}^2}{p^4}\right)\right], \tag{1}$$

where $m_{1,2} = \frac{m_o}{A \pm B'}$, $A_{1,2} = \mp \frac{C^2}{10\,B'(A \pm B')}$, $B' = \sqrt{B^2 + C^2/5}$, A, B, and C are band parameters (reference (1)), m_o is the mass of the free electron; x', y', z' coincide with the $\{100\}$ axes of the crystal. Inspite of the small anisotropy of light holes ($-A_1/A_2 = m_1/m_2 = 0.13 \ll 1$) and their number ($p_1/p_2 = (m_1/m_2)^{3/2} \ll 1$, where p_1 and p_2 are light and heavy hole concentrations), the contribution from light holes to the effects caused by the anisotropy of the dispersion law can be much in excess of $-A_1 p_1/A_2 p_2 = (m_1/m_2)^{5/2} \ll 1$ and be comparable with the contribution from heavy holes. For instance, in reference (2) it was shown that the contribution γ_i from each type of holes to the longitudinal and transverse anisotropy of conductivity at weakly heating fields is proportional to the product of mobility $\mu_i \sim 1/m_i$ and heating parameter $e^2E_x^2\mu_i\tau_i/kT$ and $A_i p_i$. For equal relaxation times ($\tau_1 = \tau_2$) the contribution from light holes $\gamma_1/\gamma_2 = -\sqrt{m_1/m_2}$ can amount to 36% of that from heavy holes and apparently cannot be neglected, as it was done in reference (3).

Below the transverse linear magnetoresistance (MR)[1)] is calculated for the case of weak heating when the contribution from each type of holes δ_i is determined by the product of γ_i by $\omega_i\tau_i = eH\tau_i/cm_i$ and $-\delta_1/\delta_2 = \sqrt{m_2/m_1} > 1$ (here $\tau_1 = \tau_2$), i.e. light holes can make the main contribution to the effect causing a sign opposite

1) The linear MR here, as for n-Ge crystals (4) is due to carrier fluxes and the anisotropic field E_a present in the direction of the Hall field E_H also for $H = 0$ (2, 3). Therefore, the total field $\vec{E}_\perp = \vec{E}_H + \vec{E}_a$ and the semiconductor state on the whole are different under magnetic field inversion (4).

to that obtained by neglecting light holes.

The linear MR is calculated in the approximation linear in the anisotropy A_i. The rate equations are solved separately in each of the bands since the "cross terms", due to which the equations of different bands are coupled, give a small contribution to the effect like in (2, 5). The hole distribution function $f^{(i)}$ in each of the bands is written as the sum of the symmetric $f_s^{(i)}$ and asymmetric $f_a^{(i)}$ (2, 3). Since the inter-band hole distribution and deviation of $f_s^{(i)}$ from the equilibrium Boltzmann distribution function in our case of weak heating do not contribute to the effect (this can be easily shown, like in (2, 3)), only the equation for $f_a^{(i)}$ must be solved. Representing $f_a^{(i)}$ in series of the electric and magnetic field powers

$$f_a^{(i)} = \sum_{l=1}^{3} \sum_{m=0}^{1} E_x^l H^m f_{a,lm}^{(i)} \tag{2}$$

(here E_x is the external electric field, H is normal to the current magnetic field), substituting (2) into the rate equation for $f_a^{(i)}$ and equating the terms with the same powers of E_x and H, we obtain solutions for $f_{a,lm}^{(i)}$. Then in the relaxation time approximation ($\tau_i = \tau_i(\varepsilon)$) which, according to (3), is valid in p-Ge for scattering by lattice vibrations, the expression for the MR $\Delta\sigma/\sigma_o = (\sigma_o - \sigma_H)/\sigma_o$ will be

$$\frac{\Delta\sigma}{\sigma_o} = A_2 \frac{eH\tau_{20}}{c\, m_2} \frac{e^2 E_x^2 \tau_{20}^2}{m_2 kT} \frac{\sin 2\alpha (3\cos^2\alpha - 1)}{\left[1 + \sqrt{\frac{m_1}{m_2}} \frac{\tau_{10}}{\tau_{20}}\right] \int_0^\infty e^{-x} x^{3/2} \psi\, dx} K , \tag{3}$$

$$K = \frac{\left[1 + \sqrt{\frac{m_2}{m_1}} \left(\frac{\tau_{10}}{\tau_{20}}\right)^2\right]\left[1 - \sqrt{\frac{m_1}{m_2}} \left(\frac{\tau_{10}}{\tau_{20}}\right)^3\right]}{\left[1 + \sqrt{\frac{m_1}{m_2}} \left(\frac{\tau_{10}}{\tau_{20}}\right)\right]} M + \left[\sqrt{\frac{m_2}{m_1}} \left(\frac{\tau_{10}}{\tau_{20}}\right)^4 - 1\right] L , \tag{4}$$

where

$$M = \frac{5}{2} \frac{\int_0^\infty e^{-x} x^{3/2} \psi^2 dx}{\int_0^\infty e^{-x} x^{3/2} \psi dx} \int_0^\infty e^{-x} x^{1/2} \psi \left[\psi^2 + \frac{2}{3} x \psi \frac{\partial\psi}{\partial x} + \frac{4}{5} x^2 \psi \frac{\partial\psi}{\partial x} - \frac{4}{5} x^2 \left(\frac{\partial\psi}{\partial x}\right)^2\right] dx , \tag{5}$$

$$L = \frac{25}{12} \int_0^\infty e^{-x} x^{1/2} \psi^3 (3\psi + 8x \frac{\partial \psi}{\partial x}) \, dx \qquad (6)$$

$\tau_{i0} = \tau_i(\varepsilon = kT)$; $\psi(x) = \tau_i/\tau_{i0}$; $x = \varepsilon/kT$; α is the angle between the $[001]$ axis and the current in the $(\bar{1}10)$ plane, $\alpha > 0$ when the current is deviated towards the $[110]$ axis, $H > 0$ if $\vec{H} \parallel [\bar{1}10]$, and $H < 0$ if $\vec{H} \parallel [1\bar{1}0]$.

In the expressions (3, 4) the terms summed to unity define the relative contribution from light holes to each multiple in the square brackets (it is taken into account that $p_1/p_2 = (m_1/m_2)^{3/2}$ and $A_1/A_2 = - m_1/m_2$). If in (4) the contribution from light or heavy holes is neglected, we obtain, respectively, $K_2 = M - L$ and $K_1 = -\sqrt{m_2/m_1}(\tau_{10}/\tau_{20})^4 K_2$, i.e. the MR due to each hole type has reverse sign with $|K_1| > |K_2|$ for $\tau_{10} \approx \tau_{20}$.

For scattering on lattice vibrations $\tau_{10} \approx \tau_{20}$ (2, 3, 5); from (5, 6) at 300, 200, 150, and 100 $^{\circ}$K 12 M is 0.9, 1.7, 6.6, and 12.5 and 12 L is 2, 4, 14.6, and 31.5, respectively, and $K > 0$. If the contribution from light holes is neglected, $K < 0$ because $L > M$, i.e. light holes make the major contribution to the MR and determine its sign.[2)]

The above calculation holds for $eH\tau_{1,2}/cm_{1,2} \ll 1$ and $e^2E_x^2\tau_{1,20}\tau_{1,2}^{so}/kT\,m_{1,2} \ll 1$, where $\tau_i^{so} = \tau_i^s$ $(x = 1)$ is the energy relaxation time.

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References

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2) As the magnetic field H increases above $\omega_1\tau_1 \approx 1$ so that for the heavy holes it is still close to a weak one because of $\omega_1\tau_1 \gg \omega_2\tau_2$, the contribution from the light holes decreases $\sim 1/\omega_1\tau_1$ (they are magnetized in considerably lower H then the heavy holes do (5) and no longer contribute to K) and the sign of K is reversed.

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