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Anomalous Hall effect in a many-valley semiconductor under multivalued Sasaki effect conditions

Z. S. Gribnikov and V. V. Mitin

Institute of Semiconductors, Academy of Sciences of the Ukrainian SSR, Kiev
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When a many-valley n-type semiconductor is subjected to a heating electric field, which gives rise to the multivalued Sasaki effect, and to a weak transverse magnetic field, the latter displaces the domain wall separating regions with different directions of the transverse (anisotropic) electric Sasaki field. This displacement gives rise to an additional "anomalous" Hall emf, which is superimposed on the "normal" emf. The effect disappears when the domain wall is displaced to one of the surfaces of the sample. In addition to the displacement of a wall in the direction producing the anomalous emf of the correct (electron) sign, it is possible, in principle, to have a situation in which the wall displacement is opposite so that the anomalous emf (and the total Hall emf) has the incorrect (i.e., hole) sign.

We have already shown[1] that a layered domain structure is established in a plate ($-d \le y \le d$) made of a many-valley semiconductor subjected to a longitudinal electric field E_x which produces the multivalued Sasaki effect.[2-4] These domains have different values of the transverse electric field E_y and different electron densities in the valleys. In the simplest two-valley case, discussed in ref. 1 and below (this case was studied experimentally in ref. 5), the domain structure is very sensitive to a small deviation of the direction of the current (along the X axis) from the symmetry axis of the valleys. Therefore, we may expect a structure of this kind to be highly sensitive to a transverse magnetic field $H_z = H$. The change in the domain structure produced by this field alters the transverse potential (Sasaki emf) and this may be manifested by the anomalous Hall effect, with anomalies not only of the magnitude of the effect but also of its sign.

If we assume that the magnetic field is weak (in the sense that the normal Hall angle is small), we find that the quasineutrality approximation yields the following differential equation describing the dependence $\zeta = \zeta(y)$, where $\zeta = E_y/a^+E_x$:

$$\frac{d^2\zeta}{dy^2} - \gamma \frac{d}{dy}[1-(\zeta-\Omega)^2] = \alpha^2(\zeta)[\zeta - L(\zeta) - \Omega], \tag{1}$$

where

$$L(\zeta) = \frac{\tau_1(\zeta)-\tau_2(\zeta)}{\tau_1(\zeta)+\tau_2(\zeta)}, \quad \alpha^2(\zeta) = \frac{2}{D}[\tau_1^{-1}(\zeta)+\tau_2^{-1}(\zeta)],$$

$$\gamma = \frac{\mu E_x}{D}a^+, \quad \Omega = \frac{(\mu_{yx}^{(1)}+\mu_{yx}^{(2)})}{(\mu_{yx}^{(1)}-\mu_{yx}^{(2)})} \sim H,$$

$$\mu = \mu_{yy}^{(1)}+\mu_{yy}^{(2)}, \quad D = D_{yy}^{(1)}+D_{yy}^{(2)}, \quad a^+ = \frac{(\mu_{yx}^{(2)}-\mu_{yx}^{(1)})}{\mu},$$

$\mu_{ik}^{(1,2)}$ and $D_{ik}^{(1,2)}$ are the tensors of the mobilities and diffusion coefficients of electrons in the first and second valleys, respectively. The times $\tau_{1,2}(\zeta)$ are the transition times of electrons from the first to the second valley and conversely. It is assumed that

$$\left.\begin{aligned} &\tau_{1,2}(\zeta) = \tau(P_{1,2}), \\ &P_{1,2} = eE_x^2[\mu_{xx}^{(1,2)} + (\mu_{xy}^{(1,2)}+\mu_{yx}^{(1,2)})a^+\zeta + \mu_{yy}^{(1,2)}(a^+\zeta)^2]. \end{aligned}\right\} \tag{2}$$

Equation (1), subject to the condition (2), describes approximately the problem in question if the following conditions are satisfied (in the exact formulation, this problem is described by a spatially inhomogeneous Boltzmann equation).

1. The model with an independent energy balance of the valleys and with the diffusion approximation is obeyed.

2. The fields cause weak heating, so that $\mu_{yy}^{(1)} \approx \mu_{yy}^{(2)}$, $D_{yy}^{(1)} \approx D_{yy}^{(2)}$, and $\mu_{yx}^{(1)} \approx -\mu_{yx}^{(2)}$ (if the latter is true in the absence of a magnetic field, then $\Omega \propto H$).

3. The dependence $\zeta(y)$ obtained from Eqs. (1) and (2) can be represented by regions of smooth variation (domains and "thick" domain walls with a thickness of the order of the "extended" diffusion length) and intervals of rapid variation ("thin" domain walls of thickness of the order of the "contracted" diffusion length, where the diffusion in question occurs during an intervalley transition time).

The first of the above conditions, adopted also in ref. 4, is, in fact, the condition for the existence of the multisign Sasaki effect because a significant intervalley electron–electron energy exchange reduces greatly the range of existence of this effect or makes it impossible (see, for example, ref. 6). The second and third conditions are necessary for the reduction of the transport problem to the solution of an ordinary second-order differential equation (this is always typical of inhomogeneous heating problems, for example, ref. 7; the procedure in problems of this kind is considered in greater detail in ref. 8).

Equation (1) is solved subject to the effective boundary conditions at $y = \pm d$:

$$\left.\frac{d\zeta}{dy}\right|_{y=\pm d} - \gamma[1-(\zeta^{\pm}-\Omega)^2] = \pm\alpha_{\pm}^2(\zeta^{\pm})[\zeta^{\pm} - L_{\pm}(\zeta^{\pm}) - \Omega], \tag{3}$$

where $\zeta^{\pm} = \zeta(\pm d)$, $\alpha_{\pm}^2(\zeta)$ and $L_{\pm}(\zeta)$ depend on the surface rate intervalley transitions $S_1^{\pm}(\zeta)$ and $S_2^{\pm}(\zeta)$ in exactly the same way as $\alpha^2(\zeta)$ and $L(\zeta)$ depend on $\tau_1^{-1}(\zeta)$ and $\tau_2^{-1}(\zeta)$. It should also be pointed out that the right-hand side of Eq. (3) is fully justified only when a domain or a thick domain wall emerges on the effective surface, so that the quantities $P_{1,2}^{\pm}$ are the true arguments of the rates $S_{1,2}^{\pm}$. In those cases when a domain is separated from the surface by a thin wall, the boundary condition (3) is only qualitative.

Figure 1 shows the phase plane of Eq. (1) and the singularities, separatrices, and some other characteristic phase trajectories found from the equation

$$\frac{dp}{d\zeta} = \frac{\alpha^2(\zeta)[\zeta - L(\zeta) - \Omega] - 2\gamma p(\zeta-\Omega)}{p}, \quad p \equiv \frac{d\zeta}{dy}. \tag{1'}$$

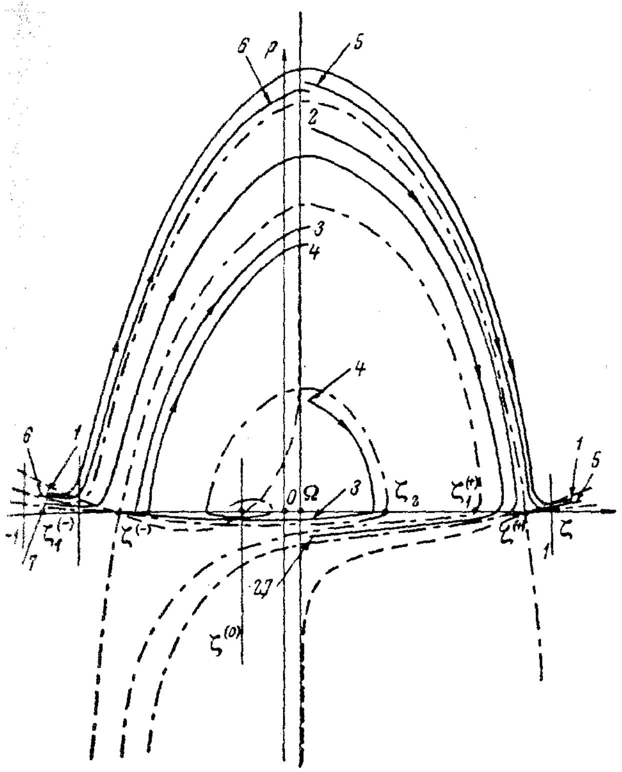

Fig. 1. Phase plane of Eq. (1'). The dashed curve is $p' = 0$, the chain curves are separatices, i.e., the trajectories passing through saddles, and the continuous curves are sections of the phase trajectories considered in text (the numbers identify the beginning and end of each section).

All three singularities of Eq. (1') lie on the $p = 0$ axis at $\zeta = \zeta^{(-)}, \zeta^{(0)}, \zeta^{(+)}$ [as in ref. 1, we shall consider the simplest case when the right-hand side of Eq. (1) has only three roots: $\zeta^{(-)} < \zeta^{(0)} < \zeta^{(+)}$]. The singularities $\zeta^{(-)}$ and $\zeta^{(+)}$ are saddle points (two phase trajectories – separatrices – pass through each of these points and, if $H = 0$, these trajectories merge pairwise):

$$\zeta^{(\pm)}(H) = \pm\zeta^{(+)}(0) + \frac{\Omega}{1 - L'(\zeta^{(+)})}, \tag{4}$$

where $L'(\zeta^{(+)}) = (dL/d\zeta)|_{\zeta=\zeta^{(+)}(0)}$, $L'(\zeta^{(+)}) < 1$. The point $\zeta^{(0)}$ for $H = 0$ is a center located at $\zeta^{(0)} = 0$, whereas for $H \neq 0$ it moves from the coordinate origin to

$$\zeta^{(0)}(H) = -\frac{\Omega}{L'(0) - 1}, \quad L'(0) > 1, \tag{5}$$

and as long as $\Omega^2 < \Omega_c^2 \equiv (\alpha^2/\gamma^2)[L'(0) - 1]^2/[L'(0)]^2$ this point is a focus (this is the situation shown in Fig. 1), whereas for $\Omega^2 > \Omega_c^2$ (in the opposite case, the concept of domains and their walls is invalid), we have $\alpha^2 \ll \gamma^2$ (in the opposite case, the concept of domains and their walls is invalid), we have $\Omega_c^2 \ll 1$.

If $H = 0$, the distributions $\zeta(y)$ have, depending on the conditions on the surfaces $\pm d$, either a two-domain structure in which the domains with $\zeta \approx \zeta^{(+)}(0)$ and $\zeta \approx \zeta^{(-)}(0) = -\zeta^{(+)}(0)$ are separated from one another by a thin or thick domain wall, respectively (these are known as the S^-S^+ structures and M^+M^- structures, respectively – see refs. 1 and 8 for details) or they have a one-domain structure with thin and thick domain walls near the surfaces (these are known as the S^-, S^+, M^-, or M^+ domains – refs. 1 and 8). The transverse emf across a sample is absent only in the case of symmetric two-domain structures; in all other cases, it can be calculated approximately from the equation

$$V(0) = \int_{-d}^{+d} E_y dy = a^+ E_x \int_{-d}^{+d} \zeta(y)\, dy \approx a^+ E_x \zeta^{(+)}(0)\,[d^+(0) - d^-(0)], \tag{6}$$

where $d^+(0)$ is the thickness of a "+" domain (i.e., an S^+ or M^+ domain) and $d^-(0)$ is the thickness of a "−" domain (an S^- or M^- domain), where $d^+ + d^- = 2d$. In weak magnetic fields, we have

$$V(H) \cong V(0) + 2da^+ \frac{\Omega E_x}{1 - L'(\zeta^{(+)})} - 2a^+\zeta^{(+)}(0)\, E_x \Delta(\Omega), \tag{7}$$

where $\Delta(\Omega)$ is the magnetic-field-induced displacement of a thin or thick domain wall which is regarded as positive if it causes expansion of a "+" domain and negative if it causes expansion of a "−" domain.

The second term on the right-hand side of Eq. (7) describes the "normal" Hall emf which, in this situation, is not completely trivial because it also includes the effect of a redistribution of electrons between the valleys by the heating action of the Hall field. The third term on the right-hand side of Eq. (7) represents the "anomalous" emf associated with the change in the domain structure under the influence of an external magnetic field.

We shall consider several limiting cases since we do not know the actual dependences $S^{\pm}_{1,2}(\zeta)$ in their general form.

A. $S^+_{1,2} = S^-_{1,2} = 0$, which clearly occurs if the electrons do not reach a real surface because of the repulsive Coulomb potential. In the domains near the surface, we then have

$$\zeta^{\pm} = \pm 1 + \Omega. \tag{8}$$

It follows from Fig. 1 that these boundary conditions correspond to a unique distribution $\zeta(y)$ found from a phase trajectory of type 1. For $H = 0$, this distribution represents a symmetric two-domain S^-S^+ structure with a thin domain wall at $y = 0$ and with $\zeta \simeq \zeta^{(+)}(0)$ for $y > 0$ and $\zeta \simeq \zeta^{(-)}(0) = -\zeta^{(+)}(0)$ for $y < 0$. If $\Omega > 0$, this wall is displaced in the direction of negative values of y and, in a contracting S^- domain, the highest value of $\zeta(y)$ does not exceed $\zeta_1^{(-)}$ (Fig. 1), whereas the condition of continuity of the fluxes in a thin domain wall yields

$$\zeta_1^{(-)}(H) \simeq -\zeta^{(+)}(H) + 2\Omega. \tag{9}$$

The domain thicknesses $d^+(H)$ and $d^-(H)$ may be found from the integrals of the phase characteristics

$$d^+(H) \simeq \int_{\zeta^{(+)}+z}^{\zeta^+} d\zeta/p(\zeta), \quad d^-(H) \cong \int_{\zeta^-}^{\zeta_1^{(-)}-z} d\zeta/p(\zeta), \tag{10}$$

where $\zeta^{(+)}(H) + z$ represents the value of $\zeta(y)$ in a "+" domain near a thin wall (minimum value in a "+" domain) and z is found from the condition $d^+ + d^- = 2d$. The functions $p(\zeta)$ in Eq. (10) can be expressed approximately in the form $P(\zeta) \simeq (dp/d\zeta)|_{\zeta=\zeta^{(\pm)}}(\zeta - \zeta^{(\pm)}) \cong \pm(dp/d\zeta)|_{\zeta=\zeta^{(+)}(0)}(\zeta - \zeta^{(\pm)})$, so that the value of z found with the aid of Eqs. (4), (8), and (9) is

$$z^2 + 2\Omega \frac{L'(\zeta^{(+)})}{1 - L'(\zeta^{(+)})} z - e^{-2p'd}\,[1 - \zeta^{(+)}(0)]^2 = 0, \tag{11}$$

where the quantity $p' \equiv (dp/d\zeta)|_{\zeta=\zeta^{(+)}(0)}$ is estimated approximately (ref. 8) as follows:

$$p' = \frac{1}{2\gamma\zeta}\frac{d}{d\zeta}\,[x^2(\zeta)\,(\zeta - L(\zeta))]\,|_{\zeta=\zeta^{(+)}(0)},$$

i.e., this quantity is of the order of the reciprocal of the extended diffusion length ($\sim\alpha^2/\gamma$). In sufficiently thick samples ($2p'd \gg 1$), even very weak magnetic fields cause strong displacements of domain walls. In fact, we have

$$\Delta = \frac{1}{2}\,(d^{(-)} - d^{(-)}) \cong \frac{1}{2p'}\ln\left(1 + \frac{z_1}{z}\right), \tag{12}$$

where $z_1 = 2\Omega L'(\zeta^{(+)})/[1 - L'(\zeta^{(+)})]$. The value of Δ is small as long as $z_1 \ll z \cong e^{-p'd}[1 - \zeta^{(+)}(0)]$; then, $\Delta \cong (1z_1/2p'z) \sim \Omega$ and the anomalous Hall emf may be many times larger than the normal emf:

$$\frac{\partial V_2}{\partial V_1} = \frac{e^{p'd}}{p'd} L'(\zeta^{(+)}) \frac{\zeta^{(+)}(0)}{1 - \zeta^{(+)}(0)}. \tag{13}$$

If $z_1 > z \cong e^{-2p'd}[1 - \zeta^{(+)}(0)]^2/z_1$, the value of Δ saturates, approaching d:

$$\Delta = d - \frac{1}{p'}\ln\left\{\frac{[1 - L'(\zeta^{(+)})](1 - \zeta^{(+)})}{2\Omega L'(\zeta^{(+)})}\right\}, \tag{14}$$

so that the dependence V(H) has the form shown in Fig. 2 (curve A). It should be noted that, in this case, the multisign Sasaki effect is, in fact, single-valued: There is only one unique distribution $\zeta(y)$.

B. $S^+_{1,2} = S^-_{1,2} = \infty$, which will occur in the case of a strong accumulation of electrons near the surface, for example, in the case of heavy doping of surface layers. We shall assume that $L_\pm(\zeta)$ is such that the equations

$$\zeta^\pm - L_\pm(\zeta^\pm) - \Omega = 0$$

have unique solutions $\zeta^\pm = \Omega/[1 - L'_\pm(0)]$, $L'_\pm(0) < 1$. For values of Ω close to zero, these values of $\zeta^\pm$ for a thick sample ($p'd \gg 1$) correspond to three solutions which do not contain unstable (in this situation) thin bulk domain walls:[3] an M^+ structure, corresponding to a section of the trajectory of type 2 in the phase plane of Fig. 1, an M^- structure (section of the trajectory of type 3), and an M^+M^- structure (section of the trajectory of type 4). If $\Omega = 0$, the last structure has a thick domain wall at y = 0 which separates an M^- domain located in the y < 0 region for which $\zeta \cong \zeta^{(+)}(0)$ and an M^+ domain in the y > 0 region for which $\zeta \cong \zeta^{(-)}(0)$. There are thin walls near both surfaces in this structure. As $\Omega > 0$ increases, the thick domain walls shift rapidly in the direction of negative values of y and the maximum value of ζ in a contracting M^- domain does not exceed ζ_2 (Fig. 1). This structure, like the M^- structure, is possible for $\Omega > 0$ as long as $\zeta_1 > \zeta^- > 0$. This condition ceases to be satisfied near $\Omega \approx \Omega_c$ for which the central singularity changes from a focus to a node. Thus, for $\Omega > \Omega_c$ (and for $\Omega < -\Omega_c$), the infinite rates of the intervalley transitions on both surfaces correspond to a one-domain M^+ structure (and a corresponding M^- structure) in which only the normal Hall emf may be manifested in the $-\Omega_c < \Omega < \Omega_c$ range: There are three values of the transverse emf and the intermediate value corresponding to an M^+M^- structure varies rapidly with the magnetic field, which is responsible for the strong anomalous emf of sign opposite to that of the normal emf, i.e., the displacement of a thick domain wall in the $\Omega > 0$ case corresponds to a negative sign of $\Delta(\Omega)$.

Figure 3 shows the dependences V(H) which are possible in this case.

C. $S^+_{1,2} = 0$, $S^-_{1,2} = \infty$. If $\Omega \geq 0$, this case corresponds to a unique one-domain S^+ structure with a thin domain wall "pinned" to the $y = -d$ surface (section of the phase trajectory 5 in Fig. 1). The anomalous Hall effect is not observed.

D. $S^-_{1,2} = 0$, $S^+_{1,2} = \infty$. If $\Omega \geq 0$, this case corresponds to a one-domain S^- structure with a thin wall near the y = d surface (section of the phase trajectory 6 in Fig. 1). However, this is not the only solution. We can easily show that the same boundary conditions are satisfied also by a solution corresponding to a section of the phase trajectory 7 in Fig. 1, which – following the terminology used in ref. 1 – should be called an S^-M^+ structure (with a thin domain wall in the bulk of the sample). If $\Omega = 0$, this solution is no longer obtained but, on approach to the limit $\Omega \to 0$, the thin domain wall tends to occupy the position in the middle of the sample (y = 0); when Ω is increased, this wall is displaced in approximately the same way as an S^-S^+ wall in case A.

If $\Omega < 0$, an S^- structure is the only solution of the problem in case D, but a solution of the M^-S^+ type appears in case C. The possible dependences V(H) in cases C and D are shown in Fig. 2 (curves C and C', D and D').

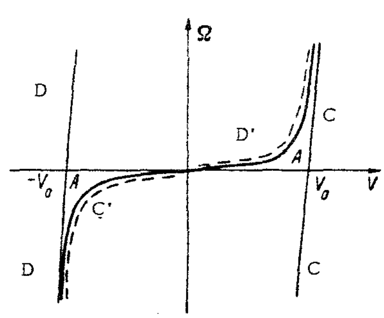

Fig. 2. Dependences of the transverse emf on the magnetic field plotted for different cases: $S^+_{1,2} = S^-_{1,2} = 0$ (A), $S^+_{1,2} = \infty$ (C for $\Omega > 0$ and C and C' for $\Omega < 0$), $S^+_{1,2} = \infty$, $S^-_{1,2} = 0$ (D and D' for $\Omega > 0$ and D for $\Omega < 0$).

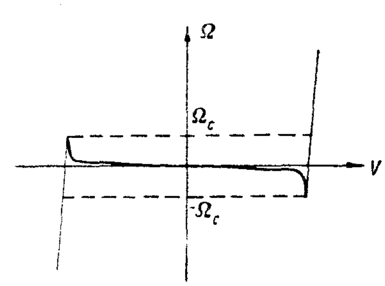

Fig. 3. Dependences of the transverse emf on the magnetic field in the $S^+_{1,2} = S^-_{1,2} = \infty$ case.

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