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Institute of Semiconductors, Academy of Sciences of the Ukrainian SSR, Kiev

Domain Structure and Transverse Conductivity of a Many-Valley Semiconductor in the Multivalued Sasaki Effect

By

Z. S. GRIBNIKOV and V. V. MITIN

The transverse conductivity (TC) of a semiconductor is studied in the range of the multivalued Sasaki effect in a heating electrical field. The negative TC due to the Erlbach effect does not arise in this field. The TC depends on the surface conditions of the sample and on the domain structure arising in these fields. The case of a small TC caused by a displacement of a domain wall under the influence of a transverse current is possible together with the case of a large positive differential TC. Such conductivity can be both positive and negative depending on the direction of the wall displacement. The case when the driving field and the transverse current are homogeneous has been studied.

Рассмотрена поперечная проводимость (ПП) полупроводника в греющем электрическом поле при многозначном эффекте Сасаки. В таком поле не возникает отрицательной ПП, соответствующей эффекту Эрлбаха. ПП зависит от условий на поверхностях образца и возникающей при таких полях доменной структуры. Наряду со случаем большой положительной дифференциальной ПП возможны случаи малой ПП, обусловленной смещением под действием поперечного тока доменной стенки. В зависимости от направления смещения проводимость может быть как положительной, так и отрицательной. Исследовался случай однородных тянущего поля и поперечного тока.

1. Introduction

The multivalued Sasaki effect in a many-valley semiconductor, considered in [1 to 3], and the negative TC (transverse conductivity) effect (the Erlbach effect — see [4] and [5 to 7]) are rather mutually exclusive than interrelated (see [7 to 9]) phenomena; a homogeneous semiconductor state, for which the Erlbach effect was predicted, is unstable, and the negative TC does not exist in stable homogeneous states. Hence the experimentally observed TC [10] should be treated by a spatially inhomogeneous (domain) semiconductor state, appearing in the multivalued Sasaki effect [3], rather than according to theories such as [4].

Below we consider (in more detail than in[3]) an inhomogeneous structure appearing in a two-valley semiconductor at a symmetrical current direction. The occurrence of the "absolute" negative TC is shown in principle to be possible in a domain model; for its emergence the boundaries between domains at a transverse current have to displace in a way that a domain with anisotropic field directed against the transverse "ohmic" field widens and a domain with field directed along the current gets narrow.

2. Physical Model and Basic Equation

The considered effects are typical for semiconductors with a high mobility, when the Boltzmann kinetic equation is valid, but it cannot be solved exactly. The system of current continuity equations

$$-\frac{1}{e}\nabla \boldsymbol{i}_\alpha = \sum_{\beta \neq \alpha}\left(\frac{n_\alpha}{\tau_\alpha} - \frac{n_\beta}{\tau_\beta}\right) \tag{1}$$

is solved to get approximate results. Here $\boldsymbol{i}_\alpha$ is the electron current of the α-valley (α-electrons further), n_α is their concentration, τ_α is the transition time of α-electrons to any other valley (in n-Ge $\alpha = 1$ to 4; in n-Si $\alpha = 1$ to 3, where the valley is a pair of equivalent valleys lying on the same axis). We shall consider in detail only the case of two valleys ($\alpha = 1, 2$) lying in the xy-plane. This case is realised in deformed n-Ge and was experimentally studied in [10].

Equation (1) is supplemented by the quasi-neutrality condition

$$\sum_\alpha n_\alpha = \text{const} \tag{2}$$

and by the condition of constancy of the transverse current

$$\sum_\alpha i_{\alpha y} = i_y \tag{3}$$

resulting from the homogeneity in x- and z-directions.

At inhomogeneous heating of electrons the currents have the form

$$i_{\alpha x} = e\frac{\partial}{\partial y}(D_{xy}^{(\alpha)} n_\alpha) + e n_\alpha(\mu_{xx}^{(\alpha)} E_x + \mu_{xy}^{(\alpha)} E_y)\,, \tag{4}$$

$$i_{\alpha y} = e\frac{\partial}{\partial y}(D_{yy}^{(\alpha)} n_\alpha) + e n_\alpha(\mu_{yx}^{(\alpha)} E_x + \mu_{yy}^{(\alpha)} E_y)\,. \tag{5}$$

Introducing $n_{1,2} = (1 \pm f)\, n_0$ according to (2) we obtain from (3)

$$\vartheta = \frac{E_y}{E_x} = \frac{i_y + e n_0 \mu E_x(a^- + a^+ f) - e n_0 \dfrac{\partial}{\partial y}[D(1 + \bar{a}_1 f)]}{e n_0 \mu E_x (1 + a_1 f)}\,, \tag{6}$$

where

$$\mu = \mu_{yy}^{(1)} + \mu_{yy}^{(2)}\,, \qquad D = D_{yy}^{(1)} + D_{yy}^{(2)}\,, \qquad a^\pm = -(\mu_{yx}^{(1)} \mp \mu_{yx}^{(2)})/\mu\,,$$

$$a_1 = (\mu_{yy}^{(1)} - \mu_{yy}^{(2)})/\mu\,, \qquad \bar{a}_1 = (D_{yy}^{(1)} - D_{yy}^{(2)})/D\,.$$

Equation (1) taking into account (3) and (6) becomes

$$\frac{\mathrm{d}}{\mathrm{d}y}\left\{\frac{\mathrm{d}}{\mathrm{d}y}[D(\bar{a}_1 + f)] - \frac{a_1 + f}{1 + a_1 f}\frac{\mathrm{d}}{\mathrm{d}y}[D(1 + a_1 f)]\right\} +$$

$$+ E_x \frac{\mathrm{d}}{\mathrm{d}y}\left[\mu\,\frac{(a_1 a^- - a^+)(1 - f^2)}{1 + a_1 f}\right] + \frac{i_y}{e n_0}\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{a_1 + f}{1 + a_1 f}\right) = 2(\tau_1^{-1} + \tau_2^{-1})(f - \beta)\,, \tag{7}$$

where

$$\beta = (\tau_1 - \tau_2)/(\tau_1 + \tau_2)\,.$$

In order to turn (7) into an ordinary differential equation determining the dependence $f(y)$, the "parameters" $\tau_{1,2}$, μ, D, $a^{\pm}$, a_1, $\bar{a}_1$ have to be expressed in terms of f and its derivatives. It is impossible to perform this exactly (because it is impossible to exactly replace the Boltzmann equation by an ordinary differential equation). Therefore, the necessity of introducing additional assumptions arises.

1. The low-field limit of the Sasaki effect (the field $E_c^{(1)}$ in [2]) lies practically in the range of weak heating fields. Therefore, if the field E_x is not very much higher than $E_c^{(1)}$, the energy dependences of the electron distribution functions in different valleys differ slightly and are close to their equilibrium values. Therefore, all "parameters" of equation (7), except the strongly distribution function-dependent times $\tau_{1,2}$, are close to their equilibrium values and are y-independent.

2. It is supposed beforehand that the dependence $f(y)$ obtained from (7) may be represented in the form of domains, where the electron concentration changes smoothly and the diffusion components of the current i_{xy} are small, and domain walls, where f changes sharply with y and the currents i_{xy} do not change essentially in the walls. In the domains the times $\tau_{1,2}$ are determined (as in the homogeneous case [2]) by the specific powers

$$P_{1,2} = \frac{\boldsymbol{i}_{1,2}\boldsymbol{E}}{n_{1,2}} = e\boldsymbol{E}\boldsymbol{\mu}_{1,2}\boldsymbol{E} = eE_x^2\,[\mu_{xx}^{(1,2)} + (\mu_{xy}^{(1,2)} + \mu_{yx}^{(1,2)})\,\vartheta + \mu_{yy}^{(1,2)}\vartheta^2]\,. \tag{8}$$

calculated (as seen from (8)) without taking into account the diffusion components. The exact knowledge of the form of the right-hand side of equation (7) is not necessary in the domain walls, because they are supposed to be narrow. Therefore, we consider everywhere that $\tau_{1,2}$ are functions of $P_{1,2}$, determined by (8), and calculating the relation between ϑ and f for $P_{1,2}$ (8), we neglect the derivative $\partial f/\partial y$ in (6).

3. Domain Structure for Symmetrical Directions of the Driving Field

Let the x-direction and the valley symmetry axis coincide. According to the assumption of Section 2 about the parameters, in this case $a_1 = \bar{a}_1 = a^- = 0$, so that

$$f = \zeta - \lambda\,, \tag{9}$$

where $\zeta = \vartheta/a^+$, $\lambda = i_y/(en_0a^+\mu E_x)$, and (7) becomes

$$\frac{\mathrm{d}^2\zeta}{\mathrm{d}y^2} - \gamma\left[\frac{\mathrm{d}}{\mathrm{d}y}\,(1-\zeta^2) + \lambda\,\frac{\mathrm{d}\zeta}{\mathrm{d}y}\right] = \alpha^2(\zeta)\,[\zeta - L(\zeta) - \lambda]\,, \tag{10}$$

where

$$\gamma = \mu E_x a^+/D\,, \qquad \alpha^2 = 2(\tau_1^{-1} + \tau_2^{-1})/D\,, \qquad \beta \equiv L(\zeta)\,.$$

The phase trajectories of (10) $p = p(\zeta)$ (where $p = \mathrm{d}\zeta/\mathrm{d}y$) are determined from the equation

$$p\left[\frac{\mathrm{d}p}{\mathrm{d}\zeta} + \gamma(2\zeta - \lambda)\right] = \alpha^2(\zeta)\,[\zeta - L(\zeta) - \lambda]\,. \tag{11}$$

At first we consider the rather simple case without transverse current ($\lambda = 0$). The case with current will be considered in Section 4.

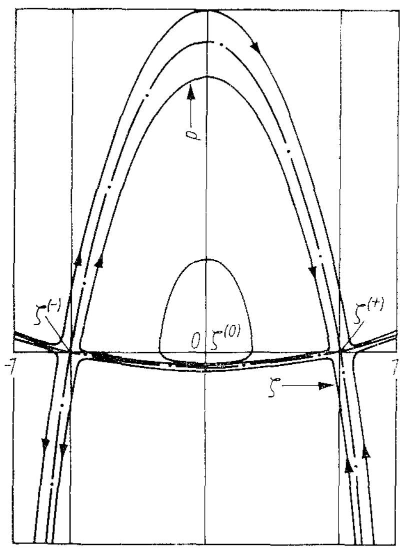


Fig. 1. The phase trajectories of (10) at $\lambda = 0$

We shall restrict ourselves to the case, when in the homogeneous state there are only three stationary values of ζ obtained from the equation

$$\zeta - L(\zeta) = 0 \tag{12}$$

and marked by $\zeta^{(+)}$, $\zeta^{(0)}$, and $\zeta^{(-)}$, at $\lambda = 0$: $1 > \zeta^{(+)} > 0$, $\zeta^{(0)} = 0$ and $\zeta^{(-)} = -\zeta^{(+)}$. There are three critical points on the phase plane of equation (10). They lie on the axis $p = 0$ (Fig. 1), the points $(\zeta^{(+)}, 0)$ and $(\zeta^{(-)}, 0)$ are saddle points and the point $(0, 0)$ is a vortex. The phase trajectories presented on Fig. 1 are separated passing through the saddle points by two separatrixes whose equations at

$$\gamma^2 \gg \alpha^2 \tag{13}$$

are approximately

$$1.\quad p \approx \gamma[(\zeta^{(+)})^2 - \zeta^2]\,, \qquad 2.\quad p \approx \frac{\alpha^2(\zeta)\,(\zeta - L(\zeta))}{2\gamma\zeta} \tag{14}$$

(the latter is the extremum line equation).

The concept of domains and domain walls that substantiates (10), is valid only provided (13) is fulfilled. At $\gamma^2 \lesseqgtr \alpha^2$, the separatrixes (and the phase trajectories near to these) have such a form that the neglect of the diffusion currents in the first case and of the intervalley transitions in the other are a bad approximation.

At $E_x \approx E_c^{(1)}$ the condition (13) can be approximated by

$$\tau_\varepsilon \ll \tau_\alpha\,, \tag{13'}$$

where τ_ε is the energy relaxation time of electrons. The condition (13′) is also essential for introducing the independent valley model and holds in cases when the multivalued Sasaki effect is possible.

For determining $\zeta(y)$ from equation (10) it is necessary to use the boundary conditions on the surfaces $y = \pm d$. They are formulated in terms of the surface intervalley transition velocity $s_{1,2}^{\pm}$ (see [11 to 13]) and have the form (at $\lambda = 0$)

$$\left[\frac{d\zeta}{dy} - \gamma(1 - \zeta^2)\right]\Bigg|_{y=\pm d} = \pm\, \alpha_\pm^2(\zeta)\,(\zeta - L^\pm(\zeta)|_{y=\pm d}\,, \tag{15}$$

where $\alpha_\pm^2(\zeta)$ and $L^\pm(\zeta)$ are expressed in terms of $s_{1,2}^{\pm}$ in such a way that $\alpha^2(\zeta)$ and $L(\zeta)$ are determined in terms of $\tau_{1,2}^{-1}$. The lack of information on the real values of $s_{1,2}^{\pm}(\zeta)$ prevents the use of the conditions (15). (In (15) one should use effective values of the velocities inside the space charge regions near the surfaces, because (10) was derived supposing quasi-neutrality.)

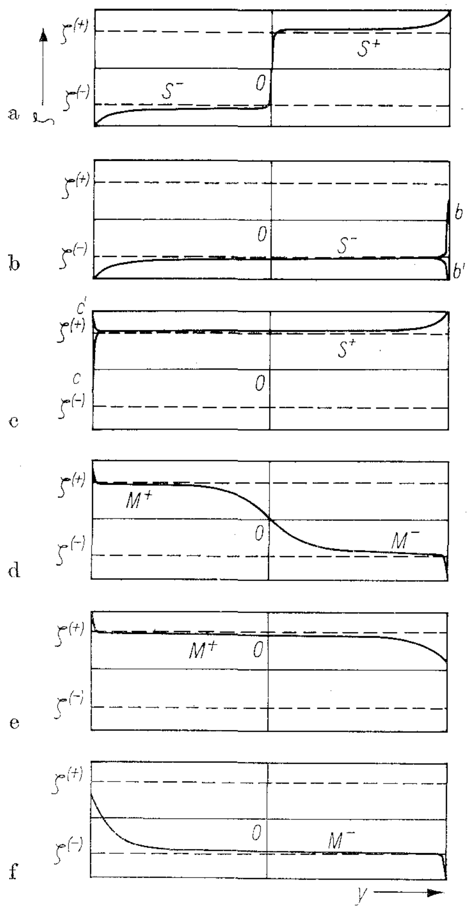


Fig. 2. a) S⁻S⁺-structure; b) S⁻-structure; c) S⁺-structure; d) M⁺M⁻-structure; e) M⁺-structure; f) M⁻-structure

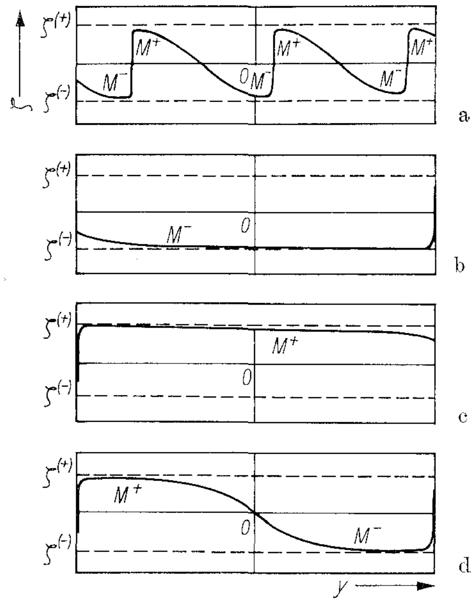


Fig. 3. a) Unstable structure with volume M⁻M⁺-walls; b) M⁻-structure; c) M⁺-structure; d) M⁺M⁻-structure

The situation is simplified in two cases:

Case 1: $\alpha_\pm^2 \ll \gamma$ and a domain goes out directly on the surface (a domain wall does not separate the one from the surface — see below). From (15) we have

$$\zeta^\pm = \pm 1 \,. \tag{15'}$$

Case 2: $\alpha_\pm^2 \gg \gamma$. It follows from this condition approximately

$$\zeta^\pm - L^\pm(\zeta^\pm) = 0 \,. \tag{16}$$

The equations (16) are analogous to the volume equation (12) and may also have several solutions at a sufficiently sharp rise of $s^\pm$ with P. We shall restrict ourselves to the case of the "trivial" surface solution[1])

$$\zeta^\pm = 0 \quad \text{at} \quad \alpha_\pm^2 \to \infty \,. \tag{16'}$$

The separatrix equations (14) estimate two characteristic lengths $l_1 \approx 1/\gamma$ and $l_2 \approx \gamma/\alpha^2$, according to (13) $l_2 \gg l_1$. Considering thick samples only

[1]) These considerations are valid only in the case when a domain goes out on the surface. If a domain wall lies near the surface, $\zeta^\pm$ do not characterize the carrier distribution function near the surface, and (16) becomes senseless.

($d \gg l_2$) we are interested only in the phase trajectories passing near one or both saddle points[2]).

If only one surface value of $\zeta^{\pm}$ lies outside the interval $(\zeta^{(-)}, \zeta^{(+)})$, then such sample is characterized by a segment of an unclosed phase trajectory passing outside the central region limited by the separatrixes (Fig. 1), and only the distribution $\zeta(y)$ (always stable) corresponds to the given $\zeta^{\pm}$. Six types of such solutions $\zeta(y)$ are possible. Their structures are shown on Fig. 2; the designations are listed in the caption (see also [15]).

If both surface values $\zeta^{\pm}$ lie within the interval $(\zeta^{(-)}, \zeta^{(+)})$, then the sample is characterized by the segment of a closed phase trajectory passing within the central region limited by the separatrixes (Fig. 1) and surrounding the vortex (0, 0). The given $\zeta^{\pm}$ correspond to an infinite set of possible distributions $\zeta(y)$ being an interchange of M-structures (M^+M^--structures) separated from each other by M^-M^+-type domain walls (Fig. 3a). This corresponds to a multiple circulation around the vortex. Only three simplest structures from this infinite set (Fig. 3b, c, d) are stable and the remaining ones are certainly unstable, because the volume M^-M^+-wall (as distinct from the S^-S^+-wall (Fig. 2a)) is unstable. Consider a fluctuation displacement δy_0 of the narrow domain wall from its equilibrium position at $y = y_0$ supposing that it takes place for unchanged carrier distribution in the rest of the sample. The equality of the currents $i_{1,2y}$ near the domain wall is broken $(i_{1,2y}(y_0 + \delta y_0 + 0) \neq i_{1,2y}(y_0 + \delta y_0 - 0))$ at this new position of the wall, this inequality is such that it attempts to return the S^-S^+-wall in the equilibrium position $y = y_0$ and to displace the M^-M^+-wall from this position.

In four special cases, when the boundary conditions (15′) or (16′) are realized on the surfaces of a sample, the following structures are possible: A) $\zeta^+ = 1$, $\zeta^- = -1$, symmetrical S^-S^+-structure (Fig. 2a); B) $\zeta^+ = 1$, $\zeta^- = 0$, S^+-structure (Fig. 2c); C) $\zeta^+ = 0$, $\zeta^- = -1$, S^--structure (Fig. 2b); D) $\zeta^- = \zeta^+ = 0$, in this case three structures depicted on Fig. 3b, c, d are possible.

4. Influence of a Transverse Current on Domain Structures

In the three-valued Sasaki effect the difference $\zeta - L(\zeta)$ has the form shown on Fig. 4. For small λ on the phase plane of (10) three critical points exist on the axis $p = 0$ and at $\lambda \ll \lambda_m$ (Fig. 4)

$$\zeta^{(\pm)}(\lambda) = \zeta^{(\pm)}(0) + k^{(+)}\lambda; \qquad \zeta^{(0)}(\lambda) = -k^{(0)}\lambda; \qquad k^{(0)}, k^{(+)} > 0\,. \tag{17}$$

As for $\lambda = 0$ the points $\zeta^{(\pm)}(\lambda)$ are saddle points through each a pair of separatrixes passes. However, as distinct from the case $\lambda = 0$, these separatrixes do not

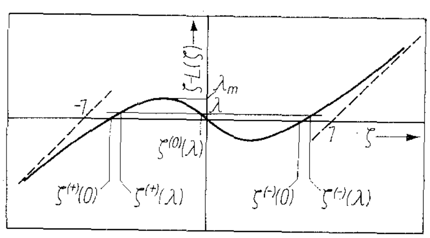


Fig. 4. The function $\zeta - L(\zeta)$ and the position of the critical points

[2]) The domain structure for nonheating field at $l_1 < d < l_2$ and at the distance $\approx l_2$ from the surface in thick samples with more than two valleys is considered in [14].

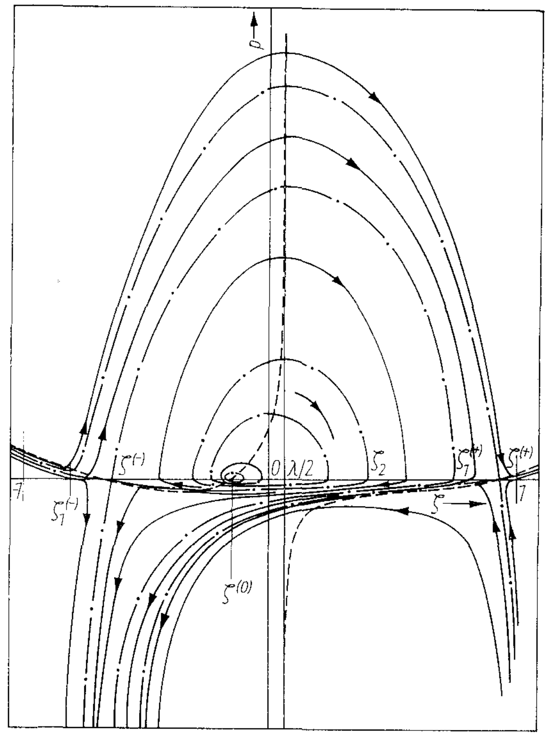


Fig. 5. The separatrixes (dashed lines) and the trajectories (solid lines) of (10) at $\lambda > 0$. The lines $dp/d\zeta = 0$ are dash-dotted

coincide, generally, with each other. The mutual arrangement of the separatrixes depends on that of the points $\zeta^{(\pm)}(\lambda)$ and $\zeta_1^{(\pm)}(\lambda)$. The quantities $\zeta_1^{(\pm)}(\lambda)$ are the values of ζ on one side of the narrow domain wall, if on the other side these values are $\zeta^{(\mp)}(\lambda)$. They are determined from the approximate conditions of current continuity through the narrow wall

$$\zeta_1^{(\pm)}(\lambda) = -\zeta^{(\pm)}(\lambda) + \lambda \tag{18}$$

or

$$\zeta_1^{(\pm)}(\lambda) = \zeta^{(\pm)}(0) + (1 - k^{(+)})\,\lambda\,. \tag{18'}$$

Since $k^{(+)} = [1 - (dL/d\zeta)|_{\zeta=\zeta^{(\pm)}(0)}]^{-1}$, then at $(dL/d\zeta) > 0$ the inequalities $\zeta^{(\pm)}(\lambda) > \zeta^{(\pm)}(0) > \zeta_1^{(\pm)}(\lambda)$ are fulfilled and (at $\lambda = 0$) the family of phase trajectories takes the form shown on Fig. 5. The third critical point is a focal point (as shown on Fig. 5) if the condition $0 < \lambda^2 < (\alpha^2/\gamma^2)\left(1/k^{(0)}(k^{(0)} + \frac{1}{2})^2\right) = \lambda_c^2$ is fulfilled and is a nodal point, if $\lambda^2 > \lambda_c^2$. In addition to the mentioned points we are interested in the point $(0, \zeta_2)$ shown on Fig. 5; at $\lambda \approx \lambda_c$ this point approaches the point $(0, \zeta^{(0)})$ and then gets into the halfplane $\zeta < 0$.

Consider the influence of a current on the boundary conditions at $y = \pm d$. Our analysis is based on the assumption of the homogeneity of the driving

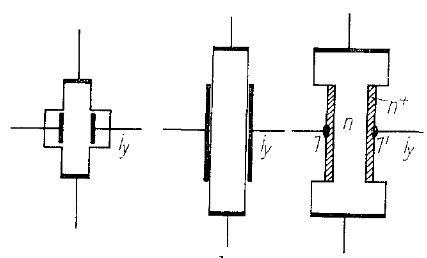


Fig. 6. Experimental scheme for studying the transverse conductivity: a) scheme with inhomogeneous E_x and i_y used in [10]; b), c) scheme with homogeneous E_x and i_y

field E_x and the transverse current i_y. In the experiments [10] these two assumptions were not fulfilled (see Fig. 6a). On Fig. 6b, c the diagrams of possible experimental schemes are shown in which the assumptions of the homogeneity are fulfilled. On Fig. 6b the transverse current is applied by capacitive electrodes which do not shunt the stationary driving field E_x; the current i_y must change with time slowly enough to keep the quasi-stationary situation. In this case we have on the surfaces

$$\left.\begin{aligned} -i_{1y}^{\pm}/e &= \pm\,(s_1^{\pm}n_1^{\pm} - s_2^{\pm}n_2^{\pm}) \pm \frac{\partial N_1^{\pm}}{\partial t}\,,\\ -i_{2y}^{\pm}/e &= \pm\,(s_2^{\pm}n_2^{\pm} - s_1^{\pm}n_1^{\pm}) \pm \frac{\partial N_2^{\pm}}{\partial t}\,, \end{aligned}\right\} \tag{19}$$

where $N_{1,2}^{\pm}$ are the surface electron concentrations. Supposing $\pm\dfrac{\partial N_{1,2}^{\pm}}{\partial t} = \pm\dfrac{n_{1,2}^{\pm}}{2n_0}\dfrac{\partial N^{\pm}}{\partial t} = -\dfrac{n_{1,2}^{\pm}}{2n_0}\dfrac{i_y}{e}$ (here $N^{\pm} = N_1^{\pm} + N_2^{\pm}$) we get

$$\frac{\mathrm{d}\zeta^{\pm}}{\mathrm{d}y} - \gamma\,[1 - (\zeta^{\pm} - \lambda)^2] = \pm\,\alpha_{\pm}^2(\zeta^{\pm})\,\big(\zeta^{\pm} - L^{\pm}(\zeta^{\pm}) - \lambda\big) \tag{20}$$

instead of (15). At very small and very great surface velocities of intervalley scattering the boundary conditions are simplified.

Case 1: ($\alpha_{\pm}^2 \ll \gamma$)

$$\zeta^{\pm} = \pm\,1 + \lambda\,. \tag{21}$$

Case 2: ($\alpha_{\pm}^2 \gg \gamma$) $\zeta^{\pm} - L^{\pm}(\zeta^{\pm}) - \lambda = 0$; for the "trivial" form of $L^{\pm}(\zeta)$ and small λ's we get

$$\zeta^{\pm}(\lambda) = k_0^{\pm}\lambda\,, \qquad k_0^{\pm} > 0\,. \tag{22}$$

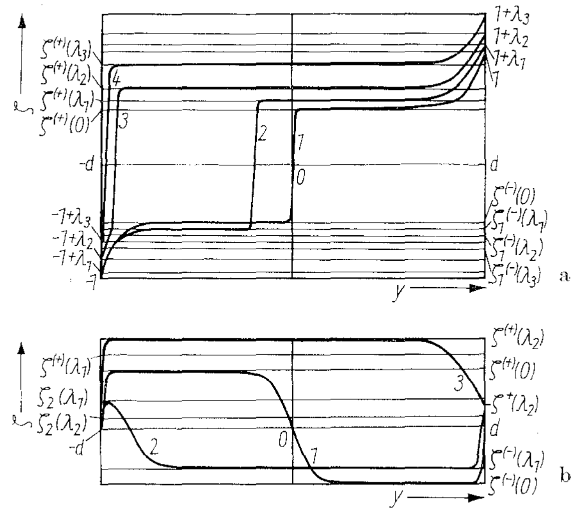
In another more cumbersome experimental scheme, depicted on Fig. 6c, both E_x and i_y may be stationary. The longitudinal conductivity of the sample is given by n$^+$-regions, situated on the borders of the samples whose doping is chosen so that: 1. the multivalued effect is absent there; 2. they is a small spreading resistance for the transverse current contacts 1,1′; 3. it is possible to obtain sufficiently great fields E_x. The current $i_{1,2y}$ and the ratio $n_{1,2}/n_0$ (i.e. the value f) are continuous on the boundary between n- and n$^+$- regions. If the electron concentration in the n$^+$-region is far in excess of $2n_0$, the transverse current field in this region can be neglected as well as the influence of the contact with the weakly doped n-region on the electron distribution in valleys. Then $f^{\pm} = 0$, so that

$$\zeta^{\pm}(\lambda) = \lambda\,. \tag{23}$$

Thus, the n$^+$n-contact behaves as a surface with infinite intervalley transition velocity, and $k_0^{\pm} = 1$ (compare with (22)).

We consider at $\lambda > 0$ four limiting cases, mentioned at the end of the previous section. In case A, when $\zeta^{\pm} = \pm\,1 + \lambda$, the inequalities $\zeta^- < \zeta^{(-)}$ and $\zeta^{(+)} < \zeta^+$ are fulfilled (because $k^{(+)} \to 1$ at $\zeta^{(+)} \to 1$), but beginning with the current $\lambda = \lambda_k \equiv \big(1 + \zeta^{(+)}(0)\big)/k^{(+)}$, $\zeta^-(\lambda) > \zeta_1^-(\lambda)$ is fulfilled. (The currents λ_k, in general, are not small; thus these estimations are valid only at $1 - \zeta^{(+)}(0) \ll k^{(+)}$.) For $\lambda < \lambda_k$ the solution has the form of the S$^-$S$^+$-structure with a

Fig. 7. The domain structures $\zeta(y)$ in the presence of transverse current ($\lambda > 0$). a) Case A. (1) $\lambda = 0$; (2) $\lambda = \lambda_1 < 2\lambda_s$; (3) $\lambda = \lambda_2 > 2\lambda_s$; (4) $\lambda = \lambda_3 > \lambda_k$; b) case D. (1) $\lambda = 0$. M^+M^--structure; (2) $\lambda = \lambda_1 < \lambda_c$. M^+M^--structure; (3) $\lambda = \lambda_2 > \lambda_c$. M^+-structure



narrow domain wall that displaces with increasing λ in negative direction and, as will be shown in the next section, at

$$\lambda > \frac{1 - \zeta^{(+)}(0)}{k^{(+)} - \frac{1}{2}} \, e^{-dp'} \equiv 2\lambda_s\,, \tag{24}$$

where $p' \equiv \left.\frac{dp}{d\zeta}\right|_{\zeta=\zeta^+(0)} \approx \frac{\alpha^2}{\gamma k^{(+)}\zeta^{(+)}(0)}$, the S^+-domain occupies almost the whole sample leaving for the S^--domain a layer with thickness $\approx 1/p' \approx l_2$ near the surface. This rudiment of the S^--domain vanishes at $\lambda > \lambda_k$. All stages of the structure change are shown on Fig. 7a.

In the asymmetrical cases B ($\zeta^+ = 1 + \lambda$, $\zeta^- = k_0^-\lambda$) and C ($\zeta^- = -1 + \lambda$, $\zeta^+ = k_0^+\lambda$) the same (as for $\lambda = 0$) S^+- and S^--structures are retained. In addition to this in the case C at $\lambda > 0$ the S^-M^+-structure, not found at $\lambda = 0$, is possible; the narrow S^-M^+-wall behaves as the S^-S^+-wall in the case A, that causes the same TC of this structure. The M^-S^+-structure, analogous to the latter one, is possible in the case B at $\lambda < 0$.

In the case D, when $\zeta^\pm = k_0^\pm\lambda$, at $\lambda = 0$ we have three solutions. This situation remains at $\lambda \ll \lambda_c$; at $\lambda > \lambda_c$ it follows $\zeta_2 < \zeta^+$, and solutions in the form of the M^--structure and the M^+M^--structure are impossible. The solution in the form of the M^+-structure is unambiguous. The current dependence of the M^+M^--structure is of interest. The M^+-domain presses soon to the surface $y = d$ with the rise of $\lambda > 0$ leaving the whole volume for the M^--domain, i.e. the wide M^+M^--wall moves in the direction of the S^-S^+-wall in the case A (see Fig. 7b).

5. Tranverse Conductivity of Samples with Various Structure of Solutions

Let us calculate the transverse voltage

$$V = \int_{-d}^{d} E_y \, dy = a^+ E_x \int_{-d}^{d} \zeta(y) \, dy\,. \tag{25}$$

At small currents, when the formulae (17) are valid,

$$V \approx V_0 + 2da^+k^{(+)}\lambda E_x + 2a^+\zeta^{(+)}E_x\Delta(\lambda)\,, \tag{26}$$

where $\Delta(\lambda)$, the displacement of the domain wall (narrow or wide) connected with the transverse current is positive, if the "+" domain (S^+- or $M^\pm$) widens at this displacement and is negative otherwise. The second term on the right-hand side of (26) characterizes the "ordinary" transverse ohmic resistance of the sample

$$\delta V_1 = i_y\,\frac{2dk^{(+)}}{e\mu n_0} \tag{27}$$

((27) takes the traditional form at $k^{(+)} = 1$ when the transverse current does not alter the valley distribution of carriers). The last term on the right-hand side of (26) determines the "extraordinary" transverse resistance connected with the current displacement of the domain wall. It can be both positive and negative at $\lambda > 0$.

In the case A at $\lambda > 0$ the domain wall displacement widens the S^+-domain and $\Delta(\lambda) > 0$. Let us write the thickness of the S^+- and S^--domains for the calculation of $\Delta(\lambda)$ as

$$d^+ \approx \int\limits_{\zeta^{(+)}+z}^{\zeta^+} \mathrm{d}\zeta/p(\zeta)\,, \qquad d^- \approx \int\limits_{\zeta^-}^{\zeta_1^{(-)}-z} \mathrm{d}\zeta/p(\zeta)\,, \tag{28}$$

where $\zeta^{(+)} + z$ is the value of $\zeta(y)$ in the S^+-domain near the narrow domain wall separating it from the S^--domain, and

$$2\Delta(\lambda) = d^+ - d^-\,, \qquad 2d = d^+ + d^-\,. \tag{29}$$

z is determined from the second equality (29) and is substituted into the first one. For wide samples the main contribution to the integrals $d^\pm$ make the values of ζ close to $\zeta^{(\pm)}$, so that it can be taken,

$$p(\zeta) = p'_\pm(\zeta - \zeta^{(\pm)})\,, \tag{30}$$

and $p'_\pm \approx \pm\, p'$, respectively, where $p'_\pm = (\mathrm{d}p/\mathrm{d}\zeta)|_{\zeta=\zeta^{(\pm)}}$.

The value of z is determined from the quadratic equation obtained by the integration of (28) taking into account the smallness of λ

$$z^2 + 2\lambda z(k^{(+)} - \tfrac{1}{2}) - \left(1 - \zeta^{(+)}(0)\right)^2 \mathrm{e}^{-2p'd} = 0\,. \tag{31}$$

If the strong inequality, opposite in sense to (24), is valid,

$$z = [1 - \zeta^{(+)}(0)]\,\mathrm{e}^{-p'd}\,, \qquad \Delta(\lambda) \approx \frac{1}{2p'}\,\frac{\lambda}{\lambda_s}. \tag{32}$$

In thick samples ($p'd \gg 1$) the "extraordinary" resistance is greater than the ordinary one:

$$\frac{\delta V_2}{\delta V_1} = \frac{k^{(+)} - \frac{1}{2}}{k^{(+)}}\,\frac{\zeta^{(+)}}{1 - \zeta^{(+)}}\,\frac{\mathrm{e}^{p'd}}{p'd}. \tag{33}$$

When the condition (24) is fulfilled

$$z \approx \frac{\left(1 - \zeta^{(+)}(0)\right)^2}{2\lambda(k^{(+)} - \frac{1}{2})}\,\mathrm{e}^{-2p'd}\,, \qquad \Delta(\lambda) \approx \frac{1}{p'}\ln\left(\frac{\lambda}{\lambda_s}\right), \tag{34}$$

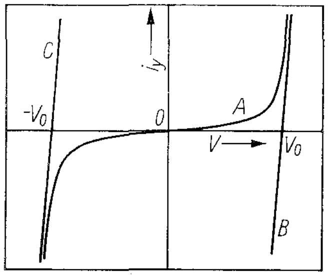


Fig. 8. The transverse current–voltage characteristics in the cases A, B, and C

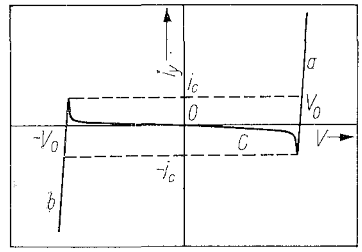


Fig. 9. The transverse current–voltage characteristics in the case D. (a) M^+-structure, (b) M^--structure, (c) M^+M^--structure

the rise of δV_2 gradually slows down with the increase of λ and the contribution of δV_1 predominates as shown on Fig. 8, curve A.

In the cases B and C, if the one-domain structure is maintained, the "extraordinary" resistance is absent; $V_0^{(B,C)} = \pm 2da^+\zeta^{(+)}E_x$ and the current–voltage characteristics take the form shown on Fig. 8 (B and C). But if the S^-M^+- and M^-S^+-structures arise step-like, the characteristics are close to the ones in case A.

In case D three variants of the current–voltage characteristics corresponding to three (stable at $|\lambda| < \lambda_c$) states are possible. They are shown on Fig. 9. The current i_c corresponding to $M^+ \rightleftarrows M^-$-transitions in the order of magnitude is

$$i_c \sim en_0a^+\mu E_x\lambda_C = en_0\mu\alpha \frac{kT}{c} \frac{1}{(k^{(0)} + \frac{1}{2})\sqrt{k^{(0)}}}. \tag{35}$$

The characteristic c on Fig. 9 corresponding to the M^+M^--structure at $\lambda = 0$ is the only one from all the considered ones, for which the Erlbach effect (i.e. negative transverse conductivity) appears. We did not succeed in exactly calculating the negative resistance at $\lambda = 0$

$$|R| > \frac{V_0}{i_c} \sim \frac{2dE_x}{en_0D\alpha}. \tag{36}$$

The homogeneity of E_x and i_y supposed above is difficult to realize experimentally (see Fig. 6b, c). On the other hand the easily realized experimental scheme (Fig. 6a) cannot be described theoretically in any simple manner. There are some considerations according to which the transverse current characteristics in some voltage range are qualitatively similar to the ones on Fig. 9.

References

[1] H. Reik and H. Risken, Phys. Rev. **126**, 1737 (1962).
[2] Z. S. Gribnikov, V. A. Kochelap, and V. V. Mitin, Zh. eksper. teor. Fiz. **59**, 1828 (1970).
[3] Z. S. Gribnikov and V. V. Mitin, Zh. eksper. teor. Fiz., Prisma **14**, 272 (1971).
[4] E. Elrbach, Phys. Rev. **132**, 1976 (1963).
[5] C. Hummer, Phys. Rev. B 4, 2560 (1971).
[6] T. K. Gaylord and T. A. Robson, Phys. Letters A **38**, 493 (1972).

[7] N. N. Grigorev, I. M. Dikman, and P. M. Tomchyk, Fiz. Tekh. Poluprov. 8, 1083 (1974).
[8] H. Shyam and H. Kroemer, Appl. Phys. Letters **12**, 283 (1968).
[9] J. C. McGroddy, M. I. Nathan, and J. E. Smith, IBM J. Res. Developm. **13**, 543 (1969).
[10] Yu. A. Astrov and A. A. Kastalskii, Fiz. Tekh. Poluprov. **6**, 978 (1972); Proc. Internat. Conf. Phys. Semicond., Warsaw 1972.
[11] S. Tosima and T. Hattori, J. Phys. Soc. Japan **19**, 2022 (1964).
[12] E. I. Rashba, Zh. eksper. teor. Fiz. **48**, 1427 (1965).
[13] V. A. Kochelap and V. N. Sokolov, Ukr. fiz. Zh. **16**, 1082 (1971).
[14] Z. S. Gribnikov, V. A. Kochelap, and E. I. Rashba, Zh. eksper. teor. Fiz. **51**, 266 (1966).
[15] Z. S. Gribnikov and V. V. Mitin, Trudy simposiuma po fizike plasmy i elektr. neust. v tverdykh telakh, Vilnius 1972 (p. 130).

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