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Galvanomagnetic effects in deformed p-type Ge

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A theoretical analysis is made of galvanomagnetic effects in weakly and strongly deformed p-type Ge. It is shown that under uniaxial deformation conditions when the stretching or compression axis is perpendicular to the current the transverse magnetoresistance $\Delta\sigma/\sigma_0$ is different in the cases when the magnetic field is parallel to the deformation axis $\Delta\sigma_\parallel/\sigma_0$ and perpendicular to this axis $\Delta\sigma_\perp/\sigma_0$. The magnetoresistance anisotropy $[(\Delta\sigma_\parallel/\sigma_0)/(\Delta\sigma_\perp/\sigma_0) - 1]$ changes its sign on transition from weak to strong deformation. This anisotropy should be exhibited by p-type Ge films which are deformed in the as-prepared state. The matrix element of the scattering of holes by acoustic lattice vibrations is calculated for an arbitrary compression (elongation) of p-type Ge.

1. The recent experimental observations of the magnetoresistance anisotropy of p-type Ge films[1,2] have been attributed either to the inhomogeneity across the film thickness (accumulation layers near the surface in contact with the substrate)[1] or to the inhomogeneity and appearance (in a thin accumulation layer) of the size effect over distances of the order of the mean free path.[2] We shall show that the magnetoresistance anisotropy can also be due to the initial deformation of the p-type Ge films.[3,4] We shall do this by considering the galvanomagnetic effects in semiconductors such as p-type Ge subjected to a uniaxial homogeneous deformation when the dispersion law of holes becomes[5,6]

$$E_l(\mathbf{k}) = Ak^2 - (-1)^l \sqrt{B^2k^4 + |B|\eta(k^2 - 3k_z^2) + \eta^2}, \tag{1}$$

where $l = 1, 2$ is the number of the band; k is the wave vector; E_l is the hole energy in the l-th band;

$$\eta = b_1(\varepsilon_{xx} - \varepsilon_{zz}), \quad b_1 = \frac{bB}{|B|}; \tag{2}$$

ε_{ik} are the components of the strain tensor expressed in terms of the x, y, and z axes selected[1)] so that $\varepsilon_{xx} = \varepsilon_{yy} \neq \varepsilon_{zz}$, $\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{xy} = 0$. The constants A, B, and b are given in refs. 6 and 7. [Following ref. 7, we shall assume that if $\eta = 0$ the semiconductor is isotropic, we shall introduce average constants $\bar{b} = \bar{d}/\sqrt{3}$, $\bar{B} + \bar{D}/\sqrt{3}$, and $\bar{C}_{44} = \frac{1}{2}(\bar{C}_{11} - \bar{C}_{12})$, and we shall drop the averaging bar over these constants.]

Uniaxial deformation lifts the band degeneracy at $k = 0$ in accordance with Eq. (1) and it gives rise to a dependence of the effective masses in each of the bands $(m_\beta^{(l)})$ on the direction:

$$\frac{1}{m_\beta^{(l)}} = \frac{3}{4\pi} \int_\Omega \frac{k_\beta}{\hbar^2 k^2} \frac{\partial E_l}{\partial k_\beta} d\Omega. \tag{3}$$

Here, β = x, y z; $d\Omega = \sin\theta d\theta d\varphi$; θ and φ are the polar and azimuthal angles in a spherical system of coordinates with a polar axis z. Figure 1 shows the dependence of the effective masses on deformation obtained from Eqs. (1) and (3). Since the effective mass anisotropy $\zeta_l [\zeta_l = m_z^{(l)} \cdot (m_x^{(l)})^{-1} - 1 \equiv (-1)^l\ (\eta/|\eta|)\,|(m_z^{(l)}/m_x^{(l)}) - 1]$ has different signs in different bands [see Eqs. (1) and (3) and Fig. 1)], the band of the holes dominating a given galvanomagnetic effect has a strong influence on the anisotropy of this effect.

In the absence of deformation the main contribution to the transverse magnetoresistance $\Delta\sigma/\sigma_0$ in weak magnetic fields is made by the light holes.[7] If the deformation is weak ($|\bar{\eta}| \ll 1$, where $\bar{\eta} = \eta/\varkappa T$, $\varkappa$ is the Boltzmann constant, and T is temperature), so that the redistribution of the holes between the bands can be ignored to within terms linear with respect to $\bar{\eta}$ (refs. 5 and 6), the magnetoresistance is still governed primarily by the holes in the first band and it becomes anisotropic because of the anisotropy of these holes.[2)]

If the deformation is strong ($|\bar{\eta}| \gg 1$), all the holes are located in the second band,[5,6] so that the magnetoresistance anisotropy is governed by the anisotropy of the effective mass in this band ζ_2 and its sign is opposite to the sign of the magnetoresistance anisotropy in the weak deformation case, i.e., when the deformation is increased from $|\bar{\eta}| \ll 1$ to $|\bar{\eta}| \gg 1$, the sign of the magnetoresistance anisotropy changes.

In classically strong magnetic fields there is no magnetoresistance anisotropy in weakly and strongly deformed samples.

Since the conductivity σ_0 in H = 0 is dominated by the second-band holes even in the absence of deformation,[7] it follows that throughout the full range of values of η the conductivity anisotropy $\vartheta = [\sigma_0(\mathbf{j}\parallel x)/\sigma_0(\mathbf{j}\parallel z)] - 1$ is governed by the anisotropy of the second-band holes and when the deformation is increased, only the magnitude of ϑ changes.

Figure 2 shows qualitatively the dependence of the magnetoresistance $\Delta\sigma/\sigma_0$ and of the conductivity σ_0 on the deformation in the case when the momentum is dissipated by the scattering on acoustic lattice vibrations. The curves in Fig. 2 are plotted using the results obtained below for weak and strong deformations [Eqs. (7)-(14)] and on the basis of the dependences $m_\beta^{(l)}(\eta)$ (Fig. 1).

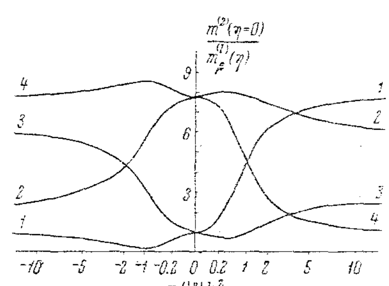

Fig. 1. Dependences of the effective masses on the uniaxial deformation of p-type Ge. The elongation or compression axis coincides with the z axis: 1) $m^{(2)}(\eta=0)/m_z^{(2)}(\eta)$; 2) $m^{(2)}(\eta=0)/m_{x,y}^{(1)}(\eta)$; 3) $m^{(2)}(\eta=0)/m_{x,y}^{(2)}(\eta)$; 4) $m^{(2)}(\eta=0)/m_z^{(1)}(\eta)$.

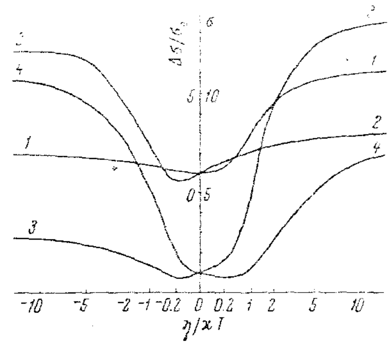

Fig. 2. Dependences of the magnetoresistance (1, 2) and electrical conductivity (3, 4) on the uniaxial deformation of p-type Ge. The stretching or compression axis coincides with the z axis: T = const: 1) $\Delta\sigma/\sigma_0$ (j ∥ x; H ∥ y; η)/($\Delta\sigma/\sigma_0$)(η = 0); 2) ($\Delta\sigma/\sigma_0$)(j ∥ H ∥ z; η)/($\Delta\sigma/\sigma_0$)(η = 0; 3) $\sigma_z(\eta)/\sigma(\eta = 0)$; 4) $\sigma_{x,y}(\eta)/\sigma(\eta = 0)$.

2. In weak electric fields the distribution function $\mathscr{F}_i(k)$ of holes in the i-th band can be represented in the form $\mathscr{F}_i(k) = f_0(E_i)[1 + kR^{(i)}(E_i)]$, where $f_0(E_i)$ is the equilibrium Boltzmann distribution function which depends only on the hole energy: $f_0(E_i) = Ce^{-E_i/\varkappa T}$, where $C = p\cdot[2/(2\pi)^3\sum_{i=1}^{2}\int e^{-E_i/\varkappa T}d^3k]^{-1}$, $kR^{(i)}(E_i) \ll 1$, and p is the hole density. Equations for the functions $R^{(i)}$ are obtained in the elastic approximation from the transport equation (see, for example, ref. 7) and they are of the form

$$-\frac{e\hbar}{\varkappa T}\frac{F_\beta}{m_\beta^{(i)}}+\frac{e}{c}\frac{[\mathbf{H}\mathbf{R}^{(i)}(E_i)]_\beta}{m_\beta^{(i)}}=-\frac{R_\beta^{(i)}(E_i)}{\tau_\beta^{(i)}(E_i)}+\frac{R_\beta^{(j)}(E_i)}{\tau_\beta^{(ij)}(E_i)}, \tag{4}$$

where F is the electric field intensity; $1/\tau_\beta^{(i)}$ and $1/\tau_\beta^{(ij)}$ are the components of the diagonal – in terms of the x, y, and z axes – tensors of the reciprocals of the intraband and interband relaxation times, given by

$$\frac{1}{\tau_\beta^{(i)}}=\frac{3}{4\pi}\frac{2}{(2\pi)^3}\left[\sum_{j=1}^{2}\int d\Omega\frac{k_\beta^2}{k^2}\int d^3k' P'_{ij}\delta(E_i(\mathbf{k})-E_j(\mathbf{k}')) - \int d\Omega\frac{k_\beta}{k^2}\int d^3k' k'_\beta P'_{ii}\delta(E_i(\mathbf{k})-E_i(\mathbf{k}'))\right], \tag{5}$$

$$\frac{1}{\tau_\beta^{(ij)}}=\frac{3}{4\pi}\frac{2}{(2\pi)^3}\int d\Omega\frac{k_\beta}{k^2}\int d^3k' k'_\beta P'_{ji}\delta(E_i(\mathbf{k})-E_j(\mathbf{k}'))(1-\delta_{ij}), \tag{6}$$

$P_{ij}(k, k') = P'_{ij}\delta[E_i(k)-E_j(k')]$ is the probability of a transition from a state with a wave vector k in the band i to a state with a wave vector k' in the band j; $P_{ij}(k, k')$ depends on the deformation and this dependence is calculated in the Appendix for the scattering by acoustic lattice vibrations.

We shall solve the system (4) for weak $|\bar{\eta}| \ll 1$ ($|\eta| \ll Bk^2$) and strong $|\bar{\eta}| \gg 1$ ($|\eta| \gg Bk^2$) deformations.

In the case of strong deformations it follows from Eq. (1) and the Boltzmann distribution that all the holes are in the second band so that we can ignore the last term in the system (4) and solve it only for i = 2.

If $\gamma^2 = (m_1)/m_2 \equiv (A-|B|)/A + |B| \ll 1$ [$m_i = m_\beta^{(i)}(\eta = 0)$] and $\eta = 0$, the equations (4) for different bands are independent to within terms $\sim\gamma^2$ if H = 0 and to within terms $\sim\gamma$ if H ≠ 0 (ref. 7). Therefore, in the case of weak deformations, when we shall be interested in small deviations of the quantities $m_\beta^{(i)}$, $\tau_\beta^{(i)}$, and $\tau_\beta^{(ij)}$ from their values in the $\eta = 0$ case, we can ignore the terms $\sim\gamma^2$ and also the last term in Eq. (4).

After dropping the interband scattering terms, Eq. (4) can be solved easily for each of the bands (see, for example, the Appendix in ref. 8). The current is expressed in the usual way in terms of the distribution functions [$V_\beta^{(i)} = dE_i/dk_\beta$, $j = 2e/(2\pi)^3\sum_{i=1}^{2}\int\mathscr{F}_i(k)V^{(i)}d^3k$] and the galvanomagnetic effects are calculated easily.

If the dissipation of the momentum is due to the scattering by acoustic phonons, the conductivity in the absence of a magnetic field ($j_\alpha = \sigma_\alpha F_\alpha$) is

$$\sigma_\alpha(|\bar{\eta}|\ll 1)=\sigma'_0(1+\bar{\eta}N_\alpha),\quad \sigma_\alpha(|\bar{\eta}|\gg 1)=\frac{2}{3}\frac{e^2p\bar{\tau}_\alpha^0}{\bar{m}_\alpha\Gamma\left(\frac{3}{2}\right)}=\sigma'_0N'_\alpha. \tag{7}$$

$$\sigma'_0=\frac{2}{3}\frac{e^2p\tau_2^0}{m_2\Gamma\left(\frac{3}{2}\right)}\left(1+\gamma\frac{\tau_1^0}{\tau_2^0}\right),\quad N_\alpha=\frac{s_\alpha^{(2)}-t_\alpha^{(2)}+\gamma\frac{\tau_1^0}{\tau_2^0}(s_\alpha^{(1)}-t_\alpha^{(1)})}{1+\gamma\frac{\tau_1^0}{\tau_2^0}}, \tag{8}$$
$$N'_\alpha=\frac{m_2}{\bar{m}_\alpha}\frac{\bar{\tau}_\alpha^0}{\tau_2^0}\left(1+\gamma\frac{\tau_1^0}{\tau_2^0}\right)^{-1}.$$

Here, $\tau_i^0 = \tau_i(E_i)\sqrt{E_i/\varkappa T} = \tau_\beta^{(i)}(E_i, \eta = 0)\sqrt{E_i/\varkappa T}$, $t_\alpha^{(i)} = \{(1/m_\alpha^{(i)})[\partial m_\alpha^{(i)}/\partial(\eta/E_i)]\}_{\eta=0}$, $s_\alpha^{(i)} = \{(1/\tau_\alpha^{(i)})[\partial\tau_\alpha^{(i)}/\partial(\eta\cdot E_i)^{-1}]\}_{\eta=0}$, τ_i^0 and m_i are defined in ref. 7. It follows from Eqs. (1) and (3) that $(1/2)t_z^{(1)} = -t_x^{(1)} = -(1/2)t_z^{(2)} = t_x^{(2)} = 0.3$. The expressions for $s_\alpha^{(i)}$ are obtained in the Appendix [Eqs. (A.7) and (A.8)] and for a semiconductor with the parameters of p-type Ge[$\gamma = 0.36$, $b_i/a = 0.33$, $C_L^2/C_T^2 = 2.76$ (see ref. 7), where C_L and C_T are the velocities of longitudinal and transverse sound, a is the deformation potential constant] these expressions are $(1/2)s_z^{(1)} = -s_x^{(1)} = 0.001$, $-(1/2)s_z^{(2)} = s_x^{(2)} = 0.05$. Since $|S_\alpha^{(i)}| \ll |t_\alpha^{(i)}|$, in the case of weak deformations it is satisfactory to use the approximation employed in refs. 6 and 9: $\tau_\alpha^{(i)} = \tau_i[E_i(k, \eta)]$, i.e., $s_\alpha^{(i)} = 0$.

In the case of strong deformations there is one anisotropic band characterized by the relationships $\bar{m}_\alpha = m_\alpha^{(2)}\cdot(|\bar{\eta}| \gg 1)$, $1/\bar{m}_x = 2/\hbar^2(A-1/2|B|\eta/|\eta|)$, $1/\bar{m}_z = 2/\hbar^2(A+|B|\eta/|\eta|)$, $\tau_\alpha^{(2)}(|\bar{\eta}| \gg 1) = \bar{\tau}_\alpha^0\sqrt{\varkappa T/E}$, and it follows from the results in the Appendix that $\bar{\tau}_\alpha^0$ can be taken from ref. 10, where we have to substitute $\bar{\tau}_x^0 = \tau_\perp^0$, $\bar{\tau}_z^0 = \tau_\parallel^0$, $C^* = 0$, $C_L/C_{44} = C_L^2/C_T^2$, $\Xi_U = 3/2|b|(\varepsilon_{xx}-\varepsilon_{zz})/|\varepsilon_{xx}-\varepsilon_{zz}|$, $\Xi_d = a-(1/3)\Xi_U$.

For a semiconductor with the parameters of p-type Ge, we have

$$\frac{1}{2}N_z=-N_x=0.122,\quad N'_x(\eta>0)=7.06,\quad N'_x(\eta<0)=10.67,$$
$$N'_z(\eta>0)=13.74,\quad N'_z(\eta<0)=2.8. \tag{9}$$

Figure 2 shows qualitatively the dependence of the conductivity of p-type Ge (T = const) on the deformation, plotted on the basis of Eqs. (7)-(9) and of the dependence $m_\beta^{(l)}(\eta)$ (Fig. 1).

If the current flows along the α axis, the magnetic field perpendicular to the current is directed along the β axis and the magnetoresistance MC $\Delta\sigma_{\alpha\beta}/\sigma_0 = [j_\alpha(H_\beta = 0) - j_\alpha(H_\beta)]/[j_\alpha(H_\beta = 0)]$ in weak magnetic fields is given by[8]

$$\frac{\Delta\sigma_{\alpha\beta}}{\sigma_0}(|\eta|\ll 1)=\frac{\Delta\sigma}{\sigma_0}(0)\left[1+\eta L_{\alpha\beta}\ln\frac{1}{|\eta|}\right],\quad \frac{\Delta\sigma_{\alpha\beta}}{\sigma_0}(|\eta|\gg 1)=\frac{\Delta\sigma}{\sigma_0}(0)\,L'_{\alpha\beta},\tag{10}$$

where

$$\frac{\Delta\sigma}{\sigma_0}(0)=\frac{9\pi}{16}\left(\frac{c'_0 H}{epc}\right)^2\left(1-\frac{\pi}{4}\right)R,\quad R=\frac{1-\frac{\pi}{4}\gamma\frac{\tau_1^0}{\tau_2^0}\left[1+\gamma\left(\frac{\tau_2^0}{\tau_1^0}\right)^2\right]^2\Big/\left(1+\gamma\frac{\tau_1^0}{\tau_2^0}\right)}{\gamma^3\left(\frac{\tau_2^0}{\tau_1^0}+\gamma\right)^3\left(1-\frac{\pi}{4}\right)},\tag{11}$$

$$\left.\begin{aligned} L_{\alpha\beta}&=\frac{2s_\alpha^{(1)}-2t_\alpha^{(1)}+s_\gamma^{(1)}-t_\gamma^{(1)}}{1-\frac{\pi}{4}\gamma\frac{\tau_1^0}{\tau_2^0}\left[1+\gamma\left(\frac{\tau_2^0}{\tau_1^0}\right)^2\right]^2\Big/\left(1+\gamma\frac{\tau_1^0}{\tau_2^0}\right)},\\ L'_{\alpha\beta}&=\left(\frac{\sigma_\alpha(|\eta|\gg 1)}{\sigma'_0}\right)^2\frac{\tau_\gamma^0\bar m_\alpha}{\tau_\alpha^0\bar m_\gamma R},\quad \alpha\neq\beta\neq\gamma. \end{aligned}\right\}\tag{12}$$

It is clear from Eqs. (10)-(12) that when the current flows along the x axis the magnetoresistance of a deformed semiconductor is different in fields H ∥ y and H ∥ z. Since in the weak deformation case the main contribution to the magnetoresistance is made by holes in the first band and in the strong deformation case the magnetoresistance is dominated by holes in the second band [Eqs. (10)-(12)], it follows that when the deformation is increased the magnetoresistance anisotropy MC [$\zeta_{xyz} = (\Delta\sigma_{xy}/\sigma_0)/\Delta\sigma_{xy}\cdot(\sigma_0)^{-1}-1$] changes its sign on transition from weak to strong deformations. For a semiconductor with the parameters of p-type Ge, we have

$$L_{xy}=0,\ L_{xz}=-L_{zx}=1.54,\ L'_{xy}(\eta>0)=L'_{zx}(\eta>0)=6.2,$$
$$L'_{xy}(\eta<0)=L'_{zx}(\eta<0)=1.9,\ L'_{xz}(\eta>0)=3.18,\ L'_{xz}(\eta<0)=7.27.\tag{13}$$

Figure 2 shows qualitatively the dependence of the magnetoresistance of p-type Ge on η.

In strong magnetic fields the magnetoresistance becomes isotropic relative to the rotation of the magnetic field and it is given by the expressions

$$\frac{\Delta\sigma_{\alpha\beta}}{\sigma_0}(|\eta|\ll 1)=\left(1-\frac{9\pi}{32}\frac{1}{1+\gamma\frac{\tau_1^0}{\tau_2^0}}\right)\left(1+\frac{\eta}{z}\frac{9\pi\left(t_\alpha^{(2)}-s_\alpha^{(2)}+2N_\alpha\right)}{32\left(1+\gamma\frac{\tau_1^0}{\tau_2^0}\right)-9\pi}\right),$$

$$\frac{\Delta\sigma_{\alpha\beta}}{\sigma_0}(|\eta|\gg 1)=1-\frac{9\pi}{32}.\tag{14}$$

We shall conclude by pointing out that although the main attention has been concentrated on the magnetoresistance anisotropy, the expressions (7)-(14) have a wider range of validity. For example, when the current flows along the compression axis ($\eta > 0$, j ∥ z), the magnetoresistance is isotropic but it is interesting to note the nonmonotonic nature of the dependence of the magnetoresistance on the deformation in weak magnetic fields H [see Eqs. (10) and (13)]: $\Delta\sigma/\sigma_0(|\eta|\ll 1) < \Delta\sigma/\sigma_0(0) < \Delta\sigma/\sigma_0\cdot(|\eta|\gg 1)$, and so on.

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APPENDIX

We shall now calculate the transition probabilities in the scattering of holes by lattice vibrations in deformed p-type Ge using the theory of the deformation potential of semiconductors with a complex band structure and applying the Luttinger–Kohn method.[11]

Ignoring the deformation-induced changes in the deformation potential constants, which give rise to corrections of the order of $E/V_0 \ll 1$ (V_0 is the atomic potential) to the transition probability, compared with the deformation-induced changes in the carrier spectrum, we find that the electron–phonon interaction operator of a deformed semiconductor is the same as that of an undeformed semiconductor [see Eqs. (1.29) and (1.37) in ref. 12]. However, if the eigenfunctions of the noninteracting electron–phonon system are represented, as usual, as products of the electron functions F and the phonon functions χ, we find that the functions F should be obtained, in contrast to ref. 12, from the equation for electrons in a deformed crystal:

$$(\hat S(\mathbf{k})+\hat D(\varepsilon))F=i\hbar\frac{\partial F}{\partial t}.\tag{A.1}$$

Solving Eq. (A.1) in the same way as in refs. 5 and 12 (the matrices S and D are given in ref. 12), we obtain the dispersion law, eigenfunctions, and expressions for $P^{lm}_{k,k'(\pm)q_{1\nu}}$, which are the probabilities of the scattering of an electron from a state of energy $E_l(k)$ to a state with energy $E_m(k')$ accompanied by the emission (+) or absorption (−) of an acoustic phonon whose polarization is ν and whose wave vector is $q_{1\nu}$. In the elastic approximation, we find in the same way as in Eq. (2.12), in ref. 12, that $P^{lm}_{k,k'(\pm)q_{1\nu}}$ is given by

$$P^{lm}_{\mathbf{k},\mathbf{k}'(\pm)\mathbf{q}_{1\nu}}=P'^{lm}_{\mathbf{k},\mathbf{k}'(\pm)\mathbf{q}_{1\nu}}\delta\,[E_l(\mathbf{k})-E_m(\mathbf{k}')],\quad P'^{lm}_{\mathbf{k},\mathbf{k}'(\pm)\mathbf{q}_{1\nu}}=\frac{\pi\varkappa T}{\hbar\rho C_\nu^2}\frac{W^{lm}_{\mathbf{k},\mathbf{k}',\nu}}{q^2},\tag{A.2}$$

where C_ν is the velocity of so[illegible] of polarization ν,

$$W^{lm}_{\mathbf{k},\mathbf{k}',\nu}=\delta E_{l,\nu}(\mathbf{k},\eta)\,\delta E_{m,\nu}([illegible]\quad -\delta E^{(0)}_{1,\nu}\delta E^{(0)}_{2,\nu}\Psi^{lm}_{\mathbf{k},\mathbf{k}'}(\eta),\tag{A.3}$$

$$\delta E_{l,\nu}([illegible]$$

$$=a(\mathbf{e}_\nu\mathbf{q})-(-1)^l b_1\frac{|B|\,[3(\mathbf{e}_\nu\mathbf{k})(\mathbf{qk})-k^2(\mathbf{e}_\nu\mathbf{q})]+\eta\,[(\mathbf{e}_\nu\mathbf{q})-3e_{\nu z}q_z]}{2\sqrt{B^2k^4+|B|\,\eta\,(k^2-3k_z^2)+\eta^2}},\tag{A.4}$$

$$\delta E^{(0)}_{1\nu}\delta E^{(0)}_{2,\nu}=(\mathbf{e}_\nu\mathbf{q})^2\left(a^2-\frac{1}{4}b^2\right)-\frac{3}{4}b^2q^2,\tag{A.5}$$

$$\Psi^{lm}_{\mathbf{k},\mathbf{k}'}(\eta)=\frac{1}{2}\left[1-(-1)^{l+m}\right.$$
$$\left.\times\frac{B^2[3(\mathbf{kk}')^2-k^2k'^2]+|B|\,\eta\,(k^2-3k_z^2+k'^2-3k_z'^2)+2\eta^2}{2\sqrt{B^2k^4+|B|\,\eta\,(k^2-3k_z^2)+\eta^2}\sqrt{B^2k'^4+|B|\,\eta\,(k'^2-3k_z'^2)+\eta^2}}\right],\tag{A.6}$$

$\mathbf{e}_\nu$ is a unit polarization vector, and a is the deformation potential constant.[12]

If the deformation is weak, we find from Eq. (5), where $P'_{ij} = \sum_\nu\left(P'^{ij}_{k,k'(+)q_{1\nu}} + P'^{ij}_{k,k'(-)q_{1\nu}}\right)$, and from Eqs. (A.2)-(A.6) that $\tau_a^{(i)} = \tau_i[E_i(k,\eta)](1+s_a^{(i)}\cdot\eta/E_i)$,

$$-\frac{1}{2}s_z^{(1)}=s_x^{(1)}=0.02\left[(1-y)^2-\frac{3}{2}\frac{C_L^2}{C_T^2}y^2\right]\left[(1-y)^2+\frac{3}{2}\frac{C_L^2}{C_T^2}y^2\right]^{-1},\ y=\frac{b_1}{a},\tag{A.7}$$

$$M=0.02+0.1y-0.02y^2+\frac{A-|B|}{2|B|}(-0.32+y-0.02y^2),$$

$$-\frac{1}{2}s_z^{(2)}=s_x^{(2)}=\frac{M+\frac{3}{4}y^2\frac{C_L^2}{C_T^2}\left(0.02+0.64\frac{A-|B|}{2|B|}\right)}{(1-y)^2+\frac{3}{4}y^2\left(1+\frac{C_L^2}{C_T^2}\right)}.\tag{A.8}$$

In the strong deformation case the relaxation times calculated using Eqs. (5), (A.2)-(A.6) are identical with the relaxation times obtained from the theory of anisotropic scattering in ref. 10. However, in our case the deformation potential constants introduced in ref. 10 depend on whether the semiconductor is stretched or compressed: $\Xi_U = (3/2)|b|(\varepsilon_{xx} - \varepsilon_{zz})/|\varepsilon_{xx} - \varepsilon_{zz}|$, $\Xi_d = a - 1/3\,\Xi_U$.

1) In the case of films these axes are selected in such a way that the z axis is perpendicular to the plane of the film. Since films of p-type Ge on GaAs and on sapphire are compressed[3] ($\varepsilon_{xx} < 0$, $|\varepsilon_{xx}| > \varepsilon_{zz} > 0$) and $bB > 0$ (ref. 7), it follows that for these films we have $\eta < 0$.

2) Here the magnetoreistance anisotropy is defined as

$$\xi_{\alpha\beta\gamma} = \frac{\frac{\Delta\sigma}{\sigma_0}(\mathbf{j}\parallel\alpha,\ \mathbf{H}\parallel\beta)}{\frac{\Delta\sigma}{\sigma_0}(\mathbf{j}\parallel\alpha,\ \mathbf{H}\parallel\gamma)} - 1,$$

where $\alpha \neq \beta \neq \gamma$, H is the magnetic field intensity, and j is the current density.

3) In Eq. (10) the expression $(\Delta\sigma_{\alpha\beta}/\sigma)(|\bar{\eta}| \ll 1)$ denotes that the terms linear in $\bar{\eta}$ are omitted because $\ln(1/|\bar{\eta}|) \gg 1$.

1. O. P. Gorodnichii, Yu. A. Tkhorik, and Yu. M. Shvarts, Abstracts of papers presented at Conf. on Structure of and Galvanomagnetic Effects in Films, L'vov, 1973 [in Russian].
2. B. P. Zot'ev, A. F. Kravchenko, É. M. Skok, and K. T. Ermaganbetov, Fiz. Tekh. Poluprovodn., 6, 1077 (1972) [Sov. Phys. – Semicond., 6, 937 (1972)].
3. M. Berkenblit, A. Reisman, and T. B. Light, J. Electrochem. Soc., 115, 966 (1968).
4. L. A. Matveeva, Fiz. Tekh. Poluprovodn., 5, 2207 (1971) [Sov. Phys. – Semicond., 5, 1923 (1972)].
5. G. E. Pikus and G. L. Bir, Fiz. Tverd. Tela, 1, 1642 (1959) [Sov. Phys. – Solid State, 1, 1502 (1960)].
6. G. L. Bir and G. E. Pikus, Symmetry and Deformation Effects in Semiconductors [in Russian], Nauka, Moscow (1972).
7. G. L. Bir, É. Normantas, and G. E. Pikus, Fiz. Tverd. Tela, 4, 1180 (1962) [Sov. Phys. – Solid State, 4, 867 (1962)].
8. V. V. Mitin, Fiz. Tekh. Poluprovodn., 5, 1739 (1971) [Sov. Phys. – Semicond., 5, 1519 (1972)].
9. G. E. Pikus and G. L. Bir, Fiz. Tverd. Tela, 1, 1828 (1959) [Sov. Phys. – Solid State, 1, 1675 (1960)].
10. C. Herring and E. Vogt, Phys. Rev., 101, 944 (1956).
11. W. Kohn and J. M. Luttinger, Phys. Rev., 97, 869 (1955).
12. G. L. Bir and G. E. Pikus, Fiz. Tverd. Tela, 2, 2287 (1960) [Sov. Phys. – Solid State, 2, 2039 (1961)].