Galvanomagnetic effects in deformed p-type Ge

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A theoretical analysis is made of galvanomagnetic effects in weakly and strongly deformed p-type Ge. It is shown that under axial deformation conditions when the stretching or compression axis is perpendicular to the current the transverse magnetoresistance \( \Delta \rho / \rho \) is different in the cases when the magnetic field is parallel to the deformation axis, \( \rho = \rho \parallel \) and perpendicular to this axis, \( \rho = \rho \perp \). The magnetoresistance anisotropy \( (\rho \parallel / \rho \perp - 1) \) changes its sign on transition from weak to strong deformation. This anisotropy should be exhibited by p-type Ge films which are deformed in the asprepared state. The matrix element of the scattering of holes by acoustic lattice vibrations is calculated for an arbitrary compression (elongation) of p-type Ge.

1. The recent experimental observations of the magnetoresistance anisotropy of p-type Ge films\(^1,2\) have been attributed either to the inhomogeneity across the film thickness or accumulation layers near the surface in contact with the substrate\(^1\) or to the inhomogeneity and appearance of a thin accumulation layer of the size effect over distances of the order of the mean free path.\(^2\) We shall show that the magnetoresistance anisotropy can also be due to the initial deformation of the p-type Ge films.\(^3,4\) We shall do this by considering the galvanomagnetic effects in semiconductors such as p-type Ge subjected to a uniaxial homogeneous deformation when the dispersion law of holes becomes \( E_\parallel = \omega_0^2 - (l + 1)^2 \omega_0^2 \), \( l = 1, 2 \) is the number of the band, \( k \) is the wave vector; \( E_\parallel \) is the hole energy in the \( l \)-th band;

\[
\eta = \epsilon_{1x} (x_1 - y_1), \quad \eta_l = \frac{\epsilon_{kx}}{B^1},
\]

where \( \epsilon_{1x} \) and \( \epsilon_{2x} \) are the components of the strain tensor expressed in terms of the \( x \), \( y \), and \( z \) axes selected\(^3\) so that \( \epsilon_{XX} = \epsilon_{YY} = \epsilon_{ZZ} \). The constants \( A, B, \) and \( C \) are given in refs. 6 and 7. (Following ref. 7, we shall assume that if \( \eta = 0 \) the semiconductor is isotropic, we shall introduce average constants \( \bar{B} = \bar{A}/\bar{C}, \bar{B} = \bar{D}/\bar{C}, \) and \( \bar{C}_{11} = \bar{C}_{12}/\bar{C}_{11} \).) and we shall drop the averaging bars over these constants.

Uniaxial deformation lifts the band degeneracy at \( k = 0 \) in accordance with Eq. (1) and it gives rise to a dependence of the effective masses in each of the bands (\( m_{1/2}^\parallel / m_{1/2}^\parallel \)) on the direction:

\[
\frac{m_{1/2}^\parallel}{m_{1/2}^\parallel} = \frac{8}{\beta^2} \frac{\epsilon_0}{B^1} \frac{\partial E_\parallel}{\partial \epsilon}.
\]

Here, \( \beta = x, y, z \); \( \bar{\Omega} = \sin \theta \cos \phi \); \( \theta \) and \( \phi \) are the polar and azimuthal angles in a spherical coordinate system of coordinates with a polar axis \( z \). Figure 1 shows the dependence of the effective masses on deformation obtained from Eqs. (1) and (3). Since the effective mass anisotropy \( m_{1/2}^\parallel / m_{1/2}^\parallel - 1 \) has different signs in different bands [see Eqs. (1) and (3)], the dependence of the holes dominating a given galvanomagnetic effect has a strong influence on the anisotropy of this effect.

In the absence of deformation the main contribution to the transverse magnetoresistance \( \Delta \rho / \rho \) is made by the light holes.\(^1\) If the deformation is weak (\( |\eta| \ll 1 \)), \( \eta = \eta / \gamma T \), \( \gamma \) is the Boltzmann constant, and \( T \) is temperature, so that the redistribution of the holes between the bands can be ignored to within terms linear in \( \eta \) (refs. 5 and 6), the magnetoresistance is still governed primarily by the holes in the first band and it becomes anisotropic because of the anisotropy of these holes.\(^2\)

If the deformation is strong (\( |\eta| \gg 1 \)), all the holes are located in the second-band holes even in the absence of deformation.\(^1\) It follows that throughout the full range of values of \( \eta \) the conductivity anisotropy \( \delta = (\rho_1 (\parallel) / \rho_1 (\perp)) - 1 \) is governed by the anisotropy of the second-band holes and when the deformation is increased, only the magnitude of \( \delta \) changes.

Figure 2 shows qualitatively the dependence of the magnetoresistance anisotropy \( \Delta \rho / \rho \) and of the conductivity \( \rho_1 \) on the deformation in the case when the momentum is dissipated by the scattering on acoustic lattice vibrations. The curves in Fig. 2 are plotted using the results obtained below for weak and strong deformations [Eqs. (7)-(14)] and on the basis of the dependences \( m_{1/2}^\parallel (\eta) \) (Fig. 1).

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Fig. 1. Dependences of the effective masses on the uniaxial deformation of p-type Ge. The deformation axis coincides with the \( z \) axis.

1. \( m_{1/2}^\parallel (\eta) = \epsilon_0 / \bar{C} \bar{C}_{11} \). 2. \( m_{1/2}^\parallel (\eta) = \epsilon_0 / \bar{C} \bar{C}_{12} \). 3. \( m_{1/2}^\parallel (\eta) = \epsilon_0 / \bar{C} \bar{C}_{11} \). 4. \( m_{1/2}^\parallel (\eta) = \epsilon_0 / \bar{C} \bar{C}_{12} \).

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2. In weak electric fields the distribution function \( f^\text{eq}(k) \) of the \( i \)-th band can be represented in the form
\[
\frac{2}{(2\pi)^3} \sum_{\mathbf{k}} \left[ e^{-\varepsilon_i E_\mathbf{k}/kT} \chi^i \right]^{-1}, \quad \text{where} \quad \chi^i = \text{constant}, \quad \eta \sim 0,
\]
and \( \chi^i \) is the hole density. Equations for the functions \( R^i(\tau) \) are obtained in the elastic approximation from the transport equation (see, for example, ref. 7) and they are of the form
\[
\frac{e \varepsilon_i E_0}{kT} \frac{\partial N^i}{\partial \tau} = \frac{\partial}{\partial x}\left[ R^i(\tau) - \frac{\partial}{\partial \xi} \right] E_0(\xi) E_0(\xi - \xi_0 - \eta), \quad i = 1, 2.
\]
where \( F_0 \) is the electric field intensity, \( 1/\tau^1(\xi) \) and \( 1/\tau^2(\xi) \) are the components of the reciprocal of the intraband and interband relaxation times, given by
\[
\frac{1}{\tau^1(\xi)} = 3 \frac{2}{4\pi^2} \sum_{\mathbf{k}} \left\{ d^2 \frac{k^2}{2\hbar^2} \left[ \frac{\partial^2}{\partial \xi^2} \right] E_0(\xi) E_0(\xi - \xi_0 - \eta) \right\}, \quad \tau^2(\xi) = \frac{2}{4\pi^2} \sum_{\mathbf{k}} \left\{ d^2 \frac{k^2}{2\hbar^2} \left[ \frac{\partial^2}{\partial \xi^2} \right] E_0(\xi) E_0(\xi - \xi_0 - \eta) \right\}
\]
(5)

Here \( \nu^1(\xi) \) and \( \nu^2(\xi) \) are the relaxation times for the intraband and interband processes, respectively.

3. In the case of strong deformations \( \eta \gg 0 \), the equations (4) are reduced to
\[
\frac{e \varepsilon_i E_0}{kT} \frac{\partial N^i}{\partial \tau} = \frac{\partial}{\partial x}\left[ R^i(\tau) - \frac{\partial}{\partial \xi} \right] E_0(\xi) E_0(\xi - \xi_0 - \eta), \quad i = 1, 2.
\]
where \( \tau^1(\xi) \) and \( \tau^2(\xi) \) are the relaxation times for the intraband and interband processes, respectively.

We shall solve the system (4) for weak \( \eta \ll 1 \)
\[
\eta = \text{constant}, \quad \eta \ll \text{Bi}^2
\]
and for strong \( \eta \gg 1 \)
\[
\eta = \text{constant}, \quad \eta \gg \text{Bi}^2
\]

In the case of strong deformations, it follows from Eq. (1) and the Boltzmann distribution that all the holes are in the second band and that we can ignore the last term in the system (4) and solve it only for \( i = 2 \).

If \( \eta = 0 \), the equations (4) for different bands are independent to within terms \( \sim \eta^2 \) if \( H = 0 \) and to within terms \( \sim \eta \) if \( H \neq 0 \) (ref. 7). Therefore, in the case of weak deformations, when we shall be interested in small deviations of the quantities \( \nu^1(\xi) \), \( \nu^2(\xi) \), and \( \tau^1(\xi) \), \( \tau^2(\xi) \) from their values in the case \( \eta = 0 \), we can ignore the terms \( \sim \eta^2 \) and also the last term in Eq. (4).

After dropping the interband scattering terms, Eq. (4) can be solved exactly for each of the bands (see, for example, the Appendix in ref. 8). The current is expressed in the usual way in terms of the distribution functions
\[
\frac{dE_0}{dE_0} F^i(\xi) \int \left[ \tau^i(\xi) \right] V^i(\xi) d\xi
\]
and the galvanomagnetic effects are calculated easily.

If the dissipation of the momentum is due to the scattering by acoustic phonons, the conductivity in the absence of a magnetic field \( \sigma_{ij} = \sigma_{ij}^0 \) is
\[
\sigma_{ij} = \frac{\tau_{ij}^0}{\rho_0} \left[ \frac{1}{\tau_{ij}^0} + \frac{\tau_{ij}^0}{\tau_{ij}^0} \right], \quad \tau_{ij}^0 = \frac{2}{3} \frac{\eta^2}{\tau_{ij}^0} \left( 1 + \frac{\tau_{ij}^0}{\tau_{ij}^0} \right)
\]
(6)

Here \( \tau_{ij}^0 = \frac{1}{\tau_{ij}^0} \left[ \frac{1}{\tau_{ij}^0} + \frac{\tau_{ij}^0}{\tau_{ij}^0} \right] \eta^2 \eta_0 \), \( \sigma_{ij}^0 = \frac{1}{\tau_{ij}^0} \left[ \frac{1}{\tau_{ij}^0} + \frac{\tau_{ij}^0}{\tau_{ij}^0} \right] \eta^2 \eta_0 \), and \( \tau_{ij}^0 \) are the deformation potential constants, these expressions are \( (1/2) \eta_0 b_1(\xi) = -b_2(\xi) \), \( 1/2 \eta_0 b_2(\xi) = -b_1(\xi) \), \( \eta_0 \). Since \( \eta_0 \) is small, we can neglect the terms \( \sim \eta^2 \) in the case of weak deformations, it is satisfactory to use the approximation employed in refs. 1, 7, and 9:

\[
\tau_{ij}^0 = \tau_{ij}^0 \left[ \frac{1}{\tau_{ij}^0} + \frac{\tau_{ij}^0}{\tau_{ij}^0} \right] \eta_0
\]

(7)

In the case of strong deformations the coefficient of the conductivity is \( \sigma_{ij} = \sigma_{ij}^0 \), \( (\eta \gg 0) \), and \( \sigma_{ij} = \sigma_{ij}^0 \), \( (\eta \ll 0) \), (ref. 7, 9).

Figure 2 shows qualitatively the dependence of the conductivity of p-type Ge (\( T = \text{const.} \)) on the deformation, plotted on the basis of Eqs. (7)–(9) and of the dependence \( m_{ij}^1(\eta) \) (Fig. 1).

If the current flows along the surface of the crystal, the magnetic field is perpendicular to the current, the magnetic field is given by
\[
B = \text{constant}, \quad \eta \ll \text{Bi}^2
\]
It is clear from Eqs. (10)-(12) that when the current flows along the x axis the magnetoresistance of a deformed semiconductor is different in fields $H \parallel y$ and $H \parallel z$. Since in the weak deformation case the main contribution to the magnetoresistance is made by holes in the first band and in the strong deformation case the magnetoresistance is dominated by holes in the second band (Eqs. (10)-(12)), it follows that when the deformation is increased the magnetoresistance anisotropy $\Delta \rho_{xy} = (\Delta \rho_{yy} / \rho_{xx}) / \Delta \rho_{xy} \cdot \rho_{xx}$1-1 changes its sign on transition from weak to strong deformations. For a semiconductor with the parameters of p-type Ge, we have

$$L_{sx} = 0, \quad L_{sy} = -L_{sz} = 1.54, \quad L_{sy}(\eta > 0) = L_{sz}(\eta > 0) = 6.2,$$

$$L_{sy}(\eta < 0) = L_{sz}(\eta < 0) = -1.45, \quad L_{sz}(\eta > 0) = 3.85, \quad L_{sz}(\eta < 0) = -7.27.$$  

Figure 2 shows qualitatively the dependence of the magnetoresistance of p-type Ge on $\eta$.

In strong magnetic fields the magnetoresistance becomes isotropic relative to the rotation of the magnetic field and it is given by the expressions

$$\Delta \rho_{0}/\rho_{0} (|\eta| \ll 1) = \left(1 - \frac{9\eta}{32} + \frac{7}{8}\right) \left(1 + \frac{9\eta}{2} \right) \frac{1}{\left(1 + \frac{7}{8}\right)^2} - \frac{9\eta}{32}, \quad \Delta \rho_{0}/\rho_{0} (|\eta| \gg 1),$$  

$$k_{0}/\rho_{0} (|\eta| \gg 1) = 1 - \frac{9\eta}{32}.$$  

We shall conclude by pointing out that although the main attention has been concentrated on the magnetoresistance anisotropy, the expressions (7)-(14) have a wider range of validity. For example, when the current flows along the compression axis ($\eta > 0$, $j \parallel \hat{z}$), the magnetoresistance is isotropic but it is interesting to note the non-monotonic nature of the dependence of the magnetoresistance on the deformation in weak magnetic fields $H$ (see Eqs. (10) and (13)).

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APPENDIX

We shall now calculate the transition probabilities in the scattering of holes by lattice vibrations in deformed p-type Ge using the theory of the deformation potential of semiconductors with a complex band structure and applying the Luttinger–Kohn method.\textsuperscript{11}

Ignoring the deformation-induced changes in the deformation potential constants, which give rise to corrections of the order of $E/V_0 \ll 1$ ($V_0$ is the atomic potential) to the transition probability, compared with the deformation-induced changes in the carrier spectrum, we find that the electron–phonon interaction operator of a deformed semiconductor is the same as that of an undeformed semiconductor [see Eqs. (1.29) and (1.37) in ref. 12]. However, if the eigenfunctions of the noninteracting electron–phonon system are represented, as usual, as products of the electron functions $F$ and the phonon functions $\chi$, we find that the functions $F$ should be obtained, in contrast to ref. 12, from the equation for electrons in a deformed crystal:

$$\left[\hbar \beta \frac{\delta}{\delta \chi} \right] F = i \hbar \frac{\delta F}{\delta \chi}.$$  

Solving Eq. (A.1) in the same way as in refs. 5 and 12 (the matrices $S$ and $D$ are given in ref. 12), we obtain the dispersion law, eigenfunctions, and expressions for $P^{\text{In}}_{k,k'(\pm \xi)q_{1+\nu}}$, which are the probabilities of the scattering of an electron from a state of energy $E_\nu(k)$ to a state with energy $E_{\nu'}(k')$ accompanied by the emission ($+$) or absorption ($-$) of an acoustic phonon whose polarization is $\nu$ and whose wave vector is $q_{1+\nu}$. In the elastic approximation, we find in the same way as in Eq. (2.12), in ref. 12, that $P^{\text{In}}_{k,k'(\pm \xi)q_{1+\nu}}$ is given by

$$P^{\text{In}}_{k,k'(\pm \xi)q_{1+\nu}} = P^{\text{In}}_{k,k'(\pm \xi)q_{1+\nu}}, \quad E_{\nu}(k) - E_{\nu'}(k'),$$

where $\nu$ is the polarization of $\nu$-vector

$$W_{k,k'}^{\text{In}}(\eta, \gamma, \eta', \gamma') = \delta_{\nu, \nu'} \rho^{\text{In}}(k, \nu) \rho^{\text{In}}(k', \nu'),$$

$$\rho^{\text{In}}(k, \nu) = \frac{e}{\theta} \left(\frac{v}{\beta}\right)^{\frac{1}{2}}$$

$$\times \frac{E_{\nu}(k) - E_{\nu}(k')}{\left(\frac{v}{\beta}\right)^{\frac{1}{2}} + E_{\nu}(k) - E_{\nu}(k')}.$$  

$\nu$ is a unit polarization vector, and $\alpha$ is the deformation potential constant.\textsuperscript{12}

If the deformation is weak, we find from Eq. (5), where

$$P^{\text{Pl}}_{1|j} = \sum_{k,k')} \left( P^{\text{Pl}}_{k,k'}(+)q_{1+\nu} + P^{\text{Pl}}_{k,k'}(-)q_{1+\nu} \right),$$

(A.2)-(A.6) that

$$\tau_{2|\alpha} = (\tau_{1|k}(\eta, \gamma)) (1 + s_{\nu}(\eta) / \eta / E_{\nu}),$$

$$-\frac{1}{2} \beta_{12}^{(1)} = \frac{1}{2} \beta_{12} = 0.02 \left(1 - y^{\frac{1}{2}} - \frac{3}{2} \frac{C_s}{C_p} y^{\frac{1}{2}} \right) \left(1 - y^{\frac{1}{2}} + \frac{3}{2} \frac{C_s}{C_p} y^{\frac{1}{2}} \right)^{-1} y^{\frac{1}{2}},$$

$$M = 0.15, 0.15 - 0.02 y - \frac{A - B}{2 B} (0.32 - y - 0.02 y),$$

$$-\frac{1}{2} \beta_{12}^{(2)} = \frac{1}{2} \beta_{12}^{(2)} = \frac{M}{y^{\frac{1}{2}} + \frac{3}{2} \frac{C_s}{C_p} y^{\frac{1}{2}} + \frac{3}{2} \frac{C_s}{C_p} y^{\frac{1}{2}}},$$

$$M = 0.02 + 0.02 y - \frac{A - 2 B}{2 B} (0.32 - y - 0.02 y),$$

$$-\frac{1}{2} \beta_{12}^{(1)} = \frac{1}{2} \beta_{12}^{(1)} = \frac{M}{y^{\frac{1}{2}} + \frac{3}{2} \frac{C_s}{C_p} y^{\frac{1}{2}} + \frac{3}{2} \frac{C_s}{C_p} y^{\frac{1}{2}}}.$$
In the strong deformation case the relaxation times calculated using Eqs. (5), (A.2)-(A.6) are identical with the relaxation times obtained from the theory of anisotropic scattering in ref. 10. However, in our case the deformation potential constants introduced in ref. 10 depend on whether the semiconductor is stretched or compressed: \( \Sigma_U = (3/2) b |(\varepsilon_{XX} - \varepsilon_{ZZ})/|\varepsilon_{XX} - \varepsilon_{ZZ}|, \quad \Sigma_d = a - 1/3 \Sigma_U \).

\( \Sigma_d \) in the case of films these axes are selected in such a way that the 2 axis is perpendicular to the plane of the film. Since films of p-type Ge on GaAs and on sapphire are compressed\( \left( \varepsilon_{XX} < 0, \varepsilon_{ZZ} > 0 \right) \) and \( b > 0 \) (ref. 7), it follows that for these films we have \( \eta < \frac{a}{2} \).

\( \delta \) Here the magnetoresistance anisotropy is defined as
\[
\delta = \frac{\Delta \rho}{\rho} \left( j \cdot H, j \cdot H \right) - 1,
\]
where \( \sigma = x, y, z \) is the magnetic field intensity, and \( j \) is the current density.

\( \delta \) In Eq. (10) the expression \( \left( \Delta \rho_{x,y} / \rho_{x,y} \right) \ll 1 \) denotes that the terms linear in \( \tau \) are omitted because \( \ln(1/\tau) \ll 1 \).

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