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Size Dependence of Magnetoresistance of Many-Valley Semiconductors

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The dependence of the magnetoresistance (MR) is studied of a many-valley semiconductor with thickness $2\,d$ $(-\,d \leqq y \leqq d)$ comparable with the length of the intervalley scattering and/or the cooling length when the correlation between them is arbitrary. If a weak electric current flows along the x-axis the MR monotonously reduces in a magnetic field $H = H_z$ when the thickness decreases and for $d \to 0$ it becomes equal to zero. In the magnetic field $H = H_y$ the MR can both increase and reduce and it always remains non-negative when the thickness decreases.

Изучена зависимость магнетосопротивления многодолинного полупроводника от толщины $2\,d$ $(-\,d \leqq y \leqq d)$, когда она сравнима с длиной междудолинного рассеяния и/или длиной остывания, а слабый электрический ток протекает вдоль оси x. Показано, что в магнитном поле $H = H_z$ магнетосопротивление с уменьшением толщины монотонно падает и при $d \to 0$ обращается в нуль. В магнитном поле $H = H_y$ при уменьшении толщины магнетосопротивление может как падать, так и расти, оставаясь всегда неотрицательным.

1. Introduction

A transverse magnetic field $\boldsymbol{H}$ does not cause a deviation from the equilibrium carrier distribution in the volume, when a weak electric current flows through the sample which is thick compared with all characteristic lengths. Quite another situation takes place in a sample, the thickness of which is comparable with any of the characteristic lengths. In this case the carrier distribution in the volume of the sample can deviate from the equilibrium distribution which results in a size-dependence (SD) of the magnetoresistance (MR). Two types of deviation from the equilibrium distribution are known in monopolar semiconductors. Namely, the deviation from the equilibrium distribution of electrons with respect to the energy (valleys) in samples with thickness comparable with the cooling length of electrons $L_\varepsilon = \sqrt{D\,\tau_\varepsilon}$ (the length of the intervalley scattering is $L_\mathrm{i} = \sqrt{D\,\tau_\mathrm{i}}$) where D is the diffusion coefficient, τ_ε is the energy relaxation time of electrons, τ_i is the intervalley scattering time.

The first deviation from the equilibrium and the SDMR connected with it was theoretically considered only for the one-valley semiconductor with isotropic spectrum of carriers [1 to 3]; while the second one was studied in papers [4 to 6] for many-valley semiconductors where due to the condition $L_\mathrm{i} \gtrapprox d \gg L_\varepsilon$ the authors neglected the redistribution of electrons with respect to the energy (here $2\,d$ is the thickness of the sample).

A theoretical study of the MR of a many-valley semiconductor is carried out below for an arbitrary correlation between d, L_ε, and L_i, but it is supposed (like in [1 to 6]) that the free path is small compared to d, L_ε, and L_i, and hence the size-effect on the free path is absent.

2. The Basic Equation

This paper deals with the MR of a rectangular many-valley semiconductor in which the current flows along the x-axis, a transverse magnetic field $\boldsymbol{H}$ ($H_y = H \sin\gamma$, $H_z = H \cos\gamma$) is present and only one size of the sample is limited $-d \leqq y \leqq d$. In this case the distribution function $F_{\boldsymbol{p}}^{\alpha}$ in the α-th valley, the field E_y, and the current j_x depend on the coordinate y only. The fields E_x and E_z do not depend on y due to the condition rot $\boldsymbol{E} = 0$. When quasi-elastic mechanisms of intra-valley scattering are predominant, $F_{\boldsymbol{p}}^{(\alpha)}(y) = F_0^{(\alpha)}(\varepsilon, y) + F_1^{(\alpha)}(\varepsilon, y)$, $F_0^{(\alpha)} = \sum\limits_{\boldsymbol{p}} F_{\boldsymbol{p}}^{(\alpha)}(y)\, \delta(\varepsilon - \varepsilon_{\boldsymbol{p}}) / \big(\sum\limits_{\boldsymbol{p}} \delta(\varepsilon - \varepsilon_{\boldsymbol{p}})\big)$ and $F_1^{(\alpha)} \ll F_0^{(\alpha)}$, so the equation for $F_0^{(\alpha)}$ is as follows:

$$\left(\frac{\partial}{\partial x_i} + e\, E_i \frac{\partial}{\partial \varepsilon}\right) j_i^{(\alpha)}(\varepsilon) = \hat{\mathcal{J}}^{(\alpha)} F_0^{(\alpha)}; \tag{1}$$

$$j_i^{(\alpha)}(\varepsilon) = -\sigma_{ik}^{*(\alpha)}(\varepsilon, h) \left(\frac{\partial}{\partial x_k} + e\, E_k \frac{\partial}{\partial \varepsilon}\right) F_0^{(\alpha)}, \qquad \sigma_{ik}^{*(\alpha)} = \sigma_{ik}^{(\alpha)} + \sigma_{ikj}^{(\alpha)} h_j + \tilde{\sigma}^{(\alpha)} h_i h_k, \tag{2}$$

where

$$\hat{\mathcal{J}}^{(\alpha)} F_0^{(\alpha)} \equiv \frac{1}{2} \sum_{\boldsymbol{p}} \delta(\varepsilon - \varepsilon_{\boldsymbol{p}})\, \hat{I}^{(\alpha)} (F_{\boldsymbol{p}}^{(\alpha)} + F_{-\boldsymbol{p}}^{(\alpha)}) \approx g(\varepsilon)\, \hat{I}^{(\alpha)} F_0^{(\alpha)},$$

$$\hat{I}^{(\alpha)} F_{\boldsymbol{p}}^{(\alpha)} = \sum_{\beta=1}^{\lambda} \sum_{\boldsymbol{p}'} \left(W_{\boldsymbol{p}'\boldsymbol{p}}^{(\beta,\alpha)} F_{\boldsymbol{p}'}^{(\beta)} - W_{\boldsymbol{p}\boldsymbol{p}'}^{(\alpha,\beta)} F_{\boldsymbol{p}}^{(\alpha)}\right).$$

λ is the number of valleys, $W_{\boldsymbol{p}'\boldsymbol{p}}^{(\beta,\alpha)}$, $W_{\boldsymbol{p}'\boldsymbol{p}}^{(\alpha,\alpha)}$ are the probabilities of the intra- and intervalley scattering, respectively. The $\sigma_{ik}^{(\alpha)}$, $\sigma_{ikj}^{(\alpha)}$, $\tilde{\sigma}^{(\alpha)}$ can be expressed in the system of main axis of the ellipsoidal energy surface of the α-th valley as follows:

$$\left.\begin{aligned}
&\sigma_{ik}^{(\alpha)} = \frac{2}{3} g(\varepsilon)\, \varepsilon \frac{(\tau/m)_{ik}\, \delta_{ik}}{1 + (\omega_\alpha \tau_\alpha)^2}, \qquad \sigma_{ikj}^{(\alpha)} = \frac{2}{3} g(\varepsilon)\, \varepsilon \frac{(\tau/m)_{ii}\, (\tau/m)_{kk}}{1 + (\omega_\alpha \tau_\alpha)^2} \delta_{ikj}, \\
&\tilde{\sigma}^{(\alpha)} = \frac{2}{3} g(\varepsilon)\, \varepsilon \frac{(\tau/m)_{11}\, (\tau/m)_{22}^2}{1 + (\omega_\alpha \tau_\alpha)^2}, \qquad g(\varepsilon) = \sum_{\boldsymbol{p}} \delta(\varepsilon - \varepsilon_{\boldsymbol{p}}) \approx g_0\, \varepsilon^{1/2}, \\
&(\omega_\alpha \tau_\alpha)^2 = \frac{\tau_{22}}{m_2} \left[h_1^2 \frac{\tau_{22}}{m_2} + (h_2^2 + h_3^2) \frac{\tau_{11}}{m_1}\right]^{(\alpha)}; \qquad h = \frac{e}{c} H.
\end{aligned}\right\} \tag{3}$$

Here the axis 1 is directed along the symmetry axis of the ellipsoidal energy surface of the α-th valley, e is the charge of the electron, τ_{ik} are the components of the momentum relaxation time tensor.

The fields E_y and E_z are determined by the conditions

$$j_y \equiv \sum_{\alpha=1}^{\lambda} j_y^{(\alpha)} = 0, \qquad \int_{-d}^{d} j_z\, \mathrm{d}y \equiv \int_{-d}^{d} \mathrm{d}y \sum_{\alpha=1}^{\lambda} j_z^{(\alpha)} = 0, \tag{4}$$

where

$$j_i^{(\alpha)} = \int\limits_0^\infty j_i^{(\alpha)}(\varepsilon)\,\mathrm{d}\varepsilon\,.$$

It is clear that equations (4) are true if the quasi-neutrality condition takes place, i.e., if the screening length is small compared with the characteristic lengths L_ε and L_i and the thickness of the sample.

In the following we shall restrict ourselves to the linear approximation with respect to the external field E_x (weak electric current j_x).

3. Temperature Approximation

If the time of the electron–electron collision τ_{ee} is considerably less than τ_ε, τ_i it becomes possible to introduce the electronic temperature T, which will be the same in all the valleys,[1]) i.e.

$$F_0^{(\alpha)} = n_\alpha f_0\,, \qquad f_0 = \frac{\mathrm{e}^{-\varepsilon/kT}}{g_0(kT)^{3/2}\,\Gamma(3/2)}\,.$$

In order to find the temperature T and the concentration n_α in the α-th valley we obtain from (1) the total heat continuity equation (the equations (1) are multiplied by ε, integrated with respect to the energy, and summed with respect to the valley) and the current continuity equations in each valley (the equations (1) are integrated with respect to the energy). In the linear approximation with respect to the E_x these equations are as follows:

$$\zeta_{yy}^* \frac{1}{T_0}\frac{\mathrm{d}^2 T}{\mathrm{d}y^2}\left(\frac{\eta_{yy}^*}{\zeta_{yy}^*} - \frac{\zeta_{yy}^*}{D_{yy}^*}\right) + \frac{1}{\lambda\, n_0}\,\zeta_{yy}^* \sum_{\alpha=1}^{\lambda} \frac{\mathrm{d}^2 n_\alpha}{\mathrm{d}y^2}\left(\frac{\zeta_{yy}^{*(\alpha)}}{\zeta_{yy}^*} - \frac{D_{yy}^{*(\alpha)}}{D_{yy}^*}\right) =$$

$$= -\,S(T) \equiv -\int\limits_0^\infty \frac{\varepsilon}{kT_0}\,\hat{\mathcal{J}}\, f_0\,\mathrm{d}\varepsilon\,, \tag{5}$$

$$\frac{1}{n_0}\sum_{\beta=1}^{\lambda}\frac{\mathrm{d}^2 n_\beta}{\mathrm{d}y^2} D_{yy}^{*(\beta)}\left(\delta_{\alpha\beta} - \frac{1}{\lambda}\,\frac{D_{yy}^{*(\alpha)}}{D_{yy}^*}\right) + \zeta_{yy}^*\,\frac{1}{T_0}\,\frac{\mathrm{d}^2 T}{\mathrm{d}y^2}\left(\frac{\zeta_{yy}^{*(\alpha)}}{\zeta_{yy}^*} - \frac{D_{yy}^{*(\alpha)}}{D_{yy}^*}\right) = \sum_{\beta=1}^{\lambda}\frac{n_\alpha - n_\beta}{n_0\,\tau_{\alpha\beta}}\,. \tag{5}$$

Moreover, $-S(T) = -S(T_0) - (\mathrm{d}S/\mathrm{d}T)_{T_0}(T - T_0) \equiv (T - T_0)/\tau_\varepsilon T_0$ where T_0 is the lattice temperature and $S(T_0) \equiv 0$. In the semiconductors considered by us all $\tau_{\alpha\beta}$ are equal and we shall denote $\tau_{\alpha\beta} = \lambda\,\tau_\mathrm{i}$.

[1]) It is supposed that the electrons of different valleys collide as frequently as the electrons of the same valley. That is why the increase of the concentration of electrons leads to an increase of the intervalley energy change and hence to equal temperatures in the valleys. Minor differences in the distributions with respect to the energy in the valley are not taken into account because they do not lead to new qualitative results, and the corrections to the quantative characteristics are of the order $\tau_{\mathrm{ee}}/\tau_\varepsilon$ and $\tau_{\mathrm{ee}}/\tau_\mathrm{i}$. It is essential to note that the approach of the total electronic temperature is far better than the difference electronic temperature in the valleys as it is follows from [7] for slight heating.

The equations (5) and (6) ought to be supplemented by the boundary conditions on the surfaces $y = \pm d$:

$$U_y(\pm d) = \pm S_\varepsilon^\pm k [T(\pm d) - T_0] \lambda n_0 , \qquad U_i = \sum_{\alpha=1}^{\lambda} U_i^{(\alpha)} = \sum_{\alpha=1}^{\lambda} \int_0^\infty \varepsilon j_i^{(\alpha)}(\varepsilon) \, d\varepsilon , \tag{7}$$

$$j_y^{(\alpha)}(\pm d) = \pm \sum_{\beta=1}^{\lambda} S_{\alpha\beta}^\pm [n_\alpha(\pm d) - n_\beta(\pm d)] . \tag{8}$$

Here

$$D_{ik}^{*(\alpha)} = \int_0^\infty \sigma_{ik}^{*(\alpha)} f_0 \, d\varepsilon , \quad \zeta_{ik}^{*(\alpha)} = \int_0^\infty \frac{\varepsilon}{kT_0} \sigma_{ik}^{*(\alpha)} f_0 \, d\varepsilon , \quad \eta_{ik}^{*(\alpha)} = \int_0^\infty \left(\frac{\varepsilon}{kT_0}\right)^2 \sigma_{ik}^{*(\alpha)} f_0 \, d\varepsilon ,$$

$$b_{ik}^* = \frac{1}{\lambda} \sum_{\alpha=1}^{\lambda} b_{ik}^{*(\alpha)} . \tag{9}$$

$S_\varepsilon^\pm$ and $S_{\alpha\beta}^\pm$ are the surface speeds of the energy scattering and the intervalley scattering, respectively.

Analysing the value $A \equiv \zeta_{yy}^{*(\alpha)}/\zeta_{yy}^* - D_{yy}^{*(\alpha)}/D_{yy}^*$ it is easy to obtain that:

$$\left.\begin{array}{l} \text{1. } A \equiv 0 \text{ if } H = H_z \text{ and the } z\text{-axis is directed along the axis with} \\ \quad \text{respect to which all the valleys are situated symmetrically (e.g.} \\ \quad \text{(100) type axis in Ge).} \\ \text{2. } A \sim H^2 \text{ in weak magnetic field } (\omega_\alpha \tau_\alpha)^2 \ll 1, \text{ if } \tau_{ik} = \tau_{ik}^0 (\varepsilon/kT_0)^s . \\ \text{3. } A \sim H^{-2} \text{ in strong magnetic field } (\omega_\alpha \tau_\alpha)^2 \gg 1, \text{ if } \tau_{ik} = \tau_{ik}^0 (\varepsilon/kT_0)^s . \end{array}\right\} \tag{10}$$

In these three cases the second terms on the l.h.s. of equations (5) and (6) are not essential because in the first case they are equal to zero and in the two others they give a correction of higher order (with respect to H in weak field and to H^{-1} in strong field) to n_α and T than the first terms on the l.h.s. of equations (5) and (6). So the equations for defining the temperature (5) and the concentrations (6) are independent. The same situation takes place for the boundary conditions (7) and (8) too.

Consequently in these three cases the size effect is the algebraic sum of two effects studied before.

a) The size effect on the cooling length L_ε [1, 2] $\left(L_\varepsilon^2 = \tau_\varepsilon \zeta_{yy}^* (\eta_{yy}^*/\zeta_{yy}^* - \zeta_{yy}^*/D_{yy}^*) \text{ from (5)}\right)$.

b) The size effect on the intervalley length L_i [5, 6] (due to the quasi-neutrality condition in (6) there exist the $(\lambda - 1)$ intervalley length $L_\alpha^2 = \tau_\mathrm{i} D_{yy}^{*(\alpha)} [1 - (D_{yy}^{*(\alpha)} - D_{yy}^{*(\lambda)})/\lambda D_{yy}^*]$, $\alpha = 1$ to $(\lambda - 1)$. In the following we shall mention simply the intervalley length L_i having in mind one of L_α and take that inequality which will be fulfilled in the best way).

As the method of equations decision (5), (6) with the boundary condition (7) to (8) is stated in [1, 2, 4], below (Sections 3.1 and 3.2) we shall give only the results for some particular cases supposing that the conditions (10) are fulfilled.

Quite another situation takes place when the conditions (10) are not fulfilled and (5) and (6) do not split. Then the decisions for T and n_α can be easily obtained only in the infinitely thin semiconductor,[2]) where the r.h.s on (5) to (8)

[2]) In Sections 3.1 and 3.2 such a semiconductor for which the conditions d/L_ε, d/L_i $(S_\varepsilon^\pm \tau_\varepsilon d)/L_\varepsilon^2$, $(S_\mathrm{i}^\pm \tau_\mathrm{i} d)/L_\mathrm{i}^2 \ll 1$ are fulfilled we call infinitely thin. But here $d/L_\mathrm{ee} \gg 1$ unlike to Section 4 where $d/L_\mathrm{ee} \ll 1$ too.

can be neglected. It appears that the MR of the infinitely thin many-valley semiconductor is non-negative. Moreover for $H = H_z$ the MR is always less than the MR of the thick sample ($d \gg L_\varepsilon$, L_i) and for $H = H_y$ it can be both less and more than in the thick sample.

3.1 One-valley semiconductor with anisotropic spectrum of electrons

If the directional cosines of the axis 1 relative to the axes x, y, z are $\cos\varphi$; $\sin\varphi\cos\alpha$; $\sin\varphi\sin\alpha$, respectively, we shall obtain the following expression from the equations (5) and (7) for the MR in weak magnetic fields ($b_0^2 \ll 1$):

$$\frac{\Delta\sigma}{\sigma_0} \equiv \frac{\Delta\varrho}{\varrho_\mathrm{H}} \equiv$$

$$\equiv \frac{\sigma(0) - \sigma(h)}{\sigma(0)} = b^2 (1 - a^2) \left[\Delta_{0\varepsilon} \Psi + (1 - f^2) \frac{(1 + a) \sin^2\gamma}{1 + a(1 - 2\sin^2\varphi\cos^2\alpha)} \right]; \quad (11)$$

$$\left.\begin{aligned} \Psi &= \frac{\{\cos\gamma + a[\cos\gamma - 2\sin^2\varphi\cos\alpha\cos(\alpha + \gamma)]\}^2}{(1 + a\cos 2\varphi)[1 + a(1 - 2\sin^2\varphi\cos^2\alpha)]}, \\ \Delta_{0\varepsilon} &= 1 - f^2\left(1 + \frac{\tanh\delta_\varepsilon^0}{\delta_\varepsilon^0}\zeta_\varepsilon^0 \frac{s^2}{5/2 + s}\right). \end{aligned}\right\} \quad (12)$$

Here $\sigma(h)$ is the conductivity of the sample,

$$\left.\begin{aligned} &b_0 = (\tau^0/m)\,h, \qquad b^2 = b_0^2\,\big(\Gamma(5/2 + 3s)\big)/\big(\Gamma(5/2 + s)\big), \\ &\zeta_\varepsilon = \frac{1 + 1/2\,(q_\varepsilon^+ + q_\varepsilon^-)\coth\delta_\varepsilon}{1 + q_\varepsilon^+ q_\varepsilon^- + (q_\varepsilon^+ + q_\varepsilon^-)\coth 2\delta_\varepsilon}, \qquad q_\varepsilon^\pm = \frac{S_\varepsilon^\pm \tau_\varepsilon}{L_\varepsilon}, \\ &L_\varepsilon^2 = \tau_\varepsilon D_\varepsilon, \qquad \delta_\varepsilon = \frac{d}{L_\varepsilon}, \qquad \left(\frac{\tau}{m}\right) = \frac{1}{2}\left(\frac{\tau_{11}}{m_1} + \frac{\tau_{22}}{m_2}\right), \\ &a = \left(\frac{\tau_{22}}{m_2} - \frac{\tau_{11}}{m_1}\right)\left(\frac{\tau_{22}}{m_2} + \frac{\tau_{11}}{m_1}\right)^{-1}, \qquad f^2 = \frac{\Gamma^2(5/2 + 2s)}{\Gamma(5/2 + s)\,\Gamma(5/2 + 3s)}. \end{aligned}\right\} \quad (13)$$

In the considered case of weak magnetic fields

$$\left.\begin{aligned} &D_\varepsilon = D_\varepsilon^{(0)} \equiv D_\varepsilon(h = 0) = D^{(0)}(5/2 + s)[1 + a(1 - 2\sin^2\varphi\cos^2\alpha)]; \\ &D^{(0)} = \int_0^\infty \frac{2}{3} g(\varepsilon)\,\varepsilon\left(\frac{\tau}{m}\right) f_0\,\mathrm{d}\varepsilon\,. \end{aligned}\right\} \quad (13')$$

Index zero at δ_ε and ζ_ε in (12) signifies that the meaning of these values is taking for $h = 0$.

As (11) shows, the essential difference of the regarded case of the anisotropic semiconductor from the isotropic is the SDMR not only for $H = H_z$ but also for $H = H_y$. The dependence of the MR on the thickness of the sample is defined by the value $\Delta_{0\varepsilon}$ which coincides with the value Δ multiplied by f^2. The value Δ was introduced and analysed in detail in [1].

The MR reduces when the thickness decreases and in the infinitely thin sample it tends to a value depending on s. The MR of the infinitely thin sample as a function of s has a minimum when $s = 1$ as it follows from (11) and the table in [1]. Moreover the minimum of $\Delta\sigma/\sigma_0$ for $H = H_y$ is positive but for $H = H_z$ it is equal to zero (the latter takes place in the isotropic semiconductor too [1]).

The even Hall effect in the anisotropic semiconductor depends on the thickness too (in the isotropic case the even Hall effect is absent) and for $b_0^2 \ll 1$ we obtain

$$\frac{E_z}{E_x} = \frac{a \sin 2\varphi \sin\alpha}{1 + a\cos 2\varphi} - \frac{b\, f\,(1 - a^2)}{1 + a \cos 2\varphi} \sin\gamma - \\ - \frac{b^2 (1 - a^2)\, a \sin 2\varphi \sin(\gamma + \alpha)}{(1 + a \cos 2\varphi)^2} \Delta_{0\varepsilon} [(1 + a)\cos\gamma - 2a \sin^2\varphi \cos\alpha \cos(\alpha + \gamma)]. \tag{14}$$

In the strong magnetic fields we shall distinguish two cases: a) $H = H_z$ and b) $H = H_y$.

a)

$$\frac{\Delta\sigma}{\sigma_0} = \\ = \frac{1 - f_2\left(1 + \dfrac{\tanh\delta_\varepsilon}{\delta_\varepsilon}\zeta_\varepsilon \dfrac{s^2}{5/2 - s}\right)}{1 + f_2\left(1 + \dfrac{\tanh\delta_\varepsilon}{\delta_\varepsilon}\zeta_\varepsilon\dfrac{s^2}{5/2 - s}\right) a^2 \sin^2 2\varphi \sin^2\alpha\, (1 - a)^{-1} [1 + a(1 - 2\sin^2\varphi\cos^2\alpha)]^{-1}}, \tag{15}$$

$$\frac{E_z}{E_x} = \left(1 - \frac{\Delta\sigma}{\sigma_0}\right) \frac{a \sin 2\varphi \sin\alpha}{1 + a\cos 2\varphi}. \tag{16}$$

Here

$$\left.\begin{aligned} &(1 - a^2)\, b_0^2 \gg 1\,, \qquad f_2 = \frac{\Gamma^2(5/2)}{\Gamma(5/2 + s)\,\Gamma(5/2 - s)}, \\ &D_\varepsilon = D_\varepsilon^{(0)} \left\{ b_0^2 \frac{\Gamma(7/2 + s)}{\Gamma(7/2 - s)} (1 + a)\,[1 - a\,(1 - 2\sin^2\varphi\sin^2\alpha)]\right\}^{-1}. \end{aligned}\right\} \tag{17}$$

b)

$$\left.\begin{aligned} &\frac{\Delta\sigma}{\sigma_0} = 1 - \frac{f^2\,(1 + N)}{1 + f_2\, N\left(1 + \dfrac{\tanh\delta_\varepsilon}{\delta_\varepsilon}\zeta_\varepsilon\dfrac{s^2}{5/2 + s}\right)}, \\ &N = \frac{a^2 \sin^2 2\varphi \cos^2\alpha}{(1 - a)\,[1 + a\,(1 - 2\sin^2\varphi\sin^2\alpha)]}, \end{aligned}\right\} \tag{18}$$

$$\frac{E_z}{E_x} = -\frac{1 - a\,(1 - 2\sin^2\varphi\cos^2\alpha)}{1 + a\,(1 - 2\sin^2\varphi\sin^2\alpha)} \frac{b_0\,(1 + a)\,\dfrac{\Gamma(5/2)}{\Gamma(5/2 - s)}}{1 + f_2\,N\left(1 + \dfrac{\tanh\delta_\varepsilon}{\delta_\varepsilon}\zeta_\varepsilon\dfrac{s^2}{5/2 + s}\right)}. \tag{19}$$

Here

$$(1 - a^2)\, b_0^2 \gg 1\,, \qquad D_\varepsilon = (1 - a^2)\, D_\varepsilon^{(0)} [1 - a^2 (1 - 2\sin^2\varphi\cos^2\alpha)^2]^{-1}. \tag{20}$$

Attention should be paid to two essential differences of the cases a) (15) to (17) and b) (18) to (20). In the first place for $H = H_z$ the MR reduces when the thickness decreases and in the infinitely thin samples it becomes equal to zero when $s = 1$, but the MR increases for $H = H_y$. In the second place $L_\varepsilon \to 0$ (17), (13) when $H_z \to \infty$, i.e., the observation of the SDMR is impossible when $H_z \to \infty$, but L_ε tends to the finite value (20), (13) which slightly differs from $L_\varepsilon^{(0)}$ when $H_y \to \infty$.

3.2 Many-valley semiconductors

It is difficult to calculate the SDMR for a semiconductor with any number of valleys without specification of the sample orientation, therefore we have considered some particular cases. We shall give below the expression for the MR in weak and strong magnetic fields for the case when the condition (10) is fulfilled.

1. There are two valleys which have the directional cosines of the rotational axis (axis 1) relative to the x, y, z axes as follows: $\cos\varphi$; $\pm\sin\varphi$; 0.

In weak magnetic field ($b_0^2 \ll 1$)

$$\frac{\Delta\sigma}{\sigma_0} = b^2 (1 + a) [1 - \Phi^0(\delta_i)]^{-1} \Big\{(1 - a) \cos^2\gamma \Big[\frac{1 - a^2}{1 - a^2 \cos^2 2\varphi} \Delta_{0\varepsilon} + \Phi(\delta_i = 0, \zeta_i = 1) - \Phi^0(\delta_i)(1 + \eta^0)\Big] + (1 - a\cos 2\varphi)\sin^2\gamma \times$$
$$\times \Big[1 - \Phi^0(\delta_i)\Big(1 + \eta^0 + \frac{1 - a^2}{1 - a^2\cos^2 2\varphi}(1 - \eta^0)\Big) + f^2 \big(2\,\Phi^0(\delta_i) - 1 - \Phi^{(0)2}(\delta_i)\big)\Big]\Big\}, \tag{21}$$

where

$$\eta^0 = \frac{\left(1 - \frac{2\delta_i}{\sinh 2\delta_i}\right)\tanh\delta_i}{q_i^+ + q_i^- + 2\tanh\delta_i} + \frac{q_i^+ q_i^- \tanh 2\delta_i + \frac{1}{2}(q_i^+ + q_i^-)\left(1 - \frac{4\delta_i}{\sinh 4\delta_i}\right)}{(1 + q_i^+ q_i^-)\tanh 2\delta_i + q_i^+ + q_i^-}, \tag{22}$$

$$L_i^2 = \tau_i D_i\,, \qquad D_i^{(0)} = D^{(0)}(1 + a\cos 2\varphi)\,, \qquad D_\varepsilon^{(0)} = D_i^{(0)}(5/2 + s)\,,$$

$$\Phi(\delta_i) = \frac{a^2 \sin^2 2\varphi}{1 - a^2\cos^2 2\varphi}\,\frac{\tanh\delta_i}{\delta_i}\,\zeta_i\,.$$

When $H = H_z$ and $b_0^2 (1 - a^2) \gg 1$

$$\frac{\Delta\sigma}{\sigma_0} = 1 - f_2 \frac{1 - a^2}{1 - a^2\cos^2 2\varphi}\,\frac{1 + \frac{\tanh\delta_\varepsilon}{\delta_\varepsilon}\zeta_\varepsilon\frac{s^2}{5/2 - s}}{1 - \Phi^0(\delta_i)}\,, \quad D_\varepsilon = \frac{D_\varepsilon^{(0)}\,\Gamma(7/2 - s)}{b_0^2 (1 - a^2)\,\Gamma(7/2 + s)}\,. \tag{23}$$

When $H = H_y$ and $b_0^2 (1 - a^2) \gg 1$

$$\left.\begin{aligned} \frac{\Delta\sigma}{\sigma_0} &= 1 - f_2 [1 - \Phi^0(\delta_i)]^{-1}\left[1 + f_2\,\Phi(\delta_i)\frac{1 - a^2\cos^2 2\varphi}{1 - a^2}\right]^{-1}, \\ D_i &= D_i^{(0)}\frac{1 - a^2}{1 - a^2\cos^2 2\varphi}\,. \end{aligned}\right\} \tag{24}$$

2. Two valleys have the directional cosines of the axes 1: $\cos\varphi$; 0; $\pm\sin\varphi$. In this case the intervalley redistribution is absent in any magnetic field, that is why such semiconductor in the sense of the size effect is equivalent to the one-valley semiconductor. The SDMR will occur only when $H = H_z$ and it can be obtained from (11) to (17) supposing that in (11), (12), (14) to (16) $\varphi = 90°$, $\alpha = 90°$, $\gamma = 0$ but in (13) and (17) $\varphi = 0$.

3. If in two-valley semiconductors the axis 1 of one valley coincides with the y-axis and, of another, with the z-axis, analogously to the previous subsection the SDMR is obtained from (11) to (17) when $\varphi = 90°$, $\alpha = 45°$, $\gamma = 0$.

4. In three-valley n-Si when the x, y, z-axes coincide with the fourth-order axes the size effect will take place only when $H = H_z$.

In weak magnetic field ($b_0^2 \ll 1$), when $S_i^+ = S_i^-$, we have

$$\frac{\Delta\sigma}{\sigma_0} = \frac{b^2}{3+a^2}\left\{(3-a)^2(1+a)\,\Delta_{0\varepsilon} + 2a^2\left[(1-a)\left(1 - f^2\frac{\tanh\delta_{1i}}{\delta_{1i}}\zeta_{1i}\right) + (3+a)\left(1 - f^2\frac{\tanh\delta_{2i}}{\delta_{2i}}\zeta_{2i}\right)\right]\right\}, \tag{25}$$

$$D_\varepsilon = D^{(0)}\frac{3+a}{3}(5/2+s)\,, \qquad D_{1,2i} = D^{(0)}\frac{1+a}{3+a}(3-a\pm 2a) \tag{26}$$

but when $b_0^2(1-a^2) \gg 1$

$$\frac{\Delta\sigma}{\sigma_0} = \frac{9}{9-a^2}\left\{(1-a^2)\left[1 - f_2\left(1 + \frac{\tanh\delta_\varepsilon}{\delta_\varepsilon}\zeta_\varepsilon\frac{s^2}{5/2-s}\right)\right] + \frac{8}{9}a^2\left[1 - f_2\frac{\tanh\delta_{2i}}{\delta_{2i}}\zeta_{2i}\right]\right\}, \tag{27}$$

$$\frac{D_\varepsilon\,\Gamma(7/2-s)}{D_\varepsilon^{(0)}\,\Gamma(7/2+s)} = \frac{D_{2i}\,\Gamma(5/2-s)}{D_{2i}^{(0)}\,\Gamma(5/2+s)} = \frac{3-a}{3(1-a)\,b^2}\,\frac{\Gamma(5/2+3s)}{\Gamma(5/2+s)}\,, \qquad S_i^+ = S_i^-\,. \tag{28}$$

5. For four-valley n-Ge where the current flows along the [100] axis and $y \parallel [011]$, $z \parallel [0\bar{1}1]$ we have

$$\begin{aligned}\frac{\Delta\sigma}{\sigma_0} = {} & \frac{b(1+a)}{2\left[3 + a - \dfrac{\tanh\delta_{1i}^0}{\delta_{1i}^0}\zeta_{1i}^0\dfrac{4a^2}{3-a}\right]} \times \\ & \times\left[2\frac{(1+a)(3-a)^2}{3+a}\left\{1 - f^2\left(1 + \frac{\tanh\delta_\varepsilon}{\delta_\varepsilon}\zeta_\varepsilon\frac{s^2}{5/2+s}\cos^2\gamma\right)\right\} + \right. \\ & + \frac{8a^2}{3-a}\left\{\left(1 - a + \frac{4}{3}a\sin^2\gamma\right)\left[1 - \frac{\tanh\delta_{1i}^0}{\delta_{1i}^0}\zeta_{1i}^0(1+\eta^0)\right] + \right. \\ & + \frac{6(1-a^2)}{3-a}\sin^2\gamma\,(1-\eta^0) + \frac{4}{3}\frac{a^2\cos^2\gamma}{3+a}\left[1 - f^2\frac{\tanh\delta_{2i}^0}{\delta_{2i}^0}\zeta_{2i}^0\right] + \\ & + \frac{4a^2(1+a)}{9-a^2}\sin^2\gamma\left[1 - f^2\left(\frac{\tanh\delta_{1i}^0}{\delta_{1i}^0}\zeta_{1i}^0\right)^2\right] + \\ & \left.\left. + \frac{2(1+a)(3-a)}{3+a}\sin^2\gamma\left(f^2\frac{\tanh\delta_{1i}^0}{\delta_{1i}^0}\zeta_{1i}^0 - 1\right) - \frac{4a}{3+a}\sin^2\gamma\right\}\right], \end{aligned} \tag{29}$$

where

$$\left.\begin{aligned} b_0^2 \ll 1\,, \quad D_\varepsilon^{(0)} = D^{(0)} \frac{3+a}{3}(5/2+s)\,, \quad D_{1i}^{(0)} = D^{(0)} \frac{3-a}{3}\,, \\ D_{2i}^{(0)} = D^{(0)} \frac{(1+a)(3-a)}{3+a}\,. \end{aligned}\right\} \tag{30}$$

When $H = H_z$ and $b_0^2(1-a^2) \gg 1$

$$\frac{\Delta\sigma}{\sigma_0} = \\ = 1 - f_2 \frac{9(1-a^2)(3-a)}{9-5a^2} \frac{1 + \dfrac{\tanh\delta_\varepsilon}{\delta_\varepsilon}\zeta_\varepsilon \dfrac{s^2}{5/2-s} + \dfrac{\tanh\delta_{2i}}{\delta_{2i}}\zeta_{2i}\dfrac{16a^4}{9(1-a^2)(9-a^2)}}{3 + a - \dfrac{\tanh\delta_{1i}^0}{\delta_{1i}^0}\zeta_{1i}^0 \dfrac{4a^2}{3-a}}\,, \tag{31}$$

where

$$\frac{D_{2i}}{D_{2i}^{(0)}} = \frac{3(3+a)\,\Gamma(5/2-s)}{9-5a^2\,\Gamma(5/2+s)} \frac{1}{b_0^2(1+a)}\,, \quad \frac{D_\varepsilon}{D_\varepsilon^{(0)}} = \frac{\Gamma(7/2+s)}{\Gamma(7/2-s)} \frac{9-5a^2}{(1-a^2)(3+a)^2 b_0^2}\,. \tag{32}$$

When $H = H_y$ and $b_0^2(1-a^2) \gg 1$

$$\left.\begin{aligned} \frac{\Delta\sigma}{\sigma_0} &= \\ &= 1 - f_2 \frac{9(1-a^2)(3+a)}{(9-5a^2)\left[1 + f_2 \dfrac{4a^2}{9-5a^2} \dfrac{\tanh\delta_{1i}}{\delta_{1i}}\zeta_{1i}\right]\left[3 + a - \dfrac{\tanh\delta_{1i}^0}{\delta_{1i}^0}\zeta_{1i}^0\dfrac{4a^2}{3-a}\right]}\,, \\ &\frac{D_{1i}}{D_{1i}^0} = \frac{9(1-a^2)}{9-a^2}\,. \end{aligned}\right\} \tag{33}$$

So the SDMR takes place in the field $H = H_y$ too for the many-valley semiconductor and for the anisotropic one-valley semiconductor. Moreover the MR can both increase and reduce, but it always remains non-negative when the thickness decreases.

4. Non-Temperature Approach

The effective electronic temperature approach is not justified, if the concentration of electrons are small, or the sample is so thin that the condition $d \gg L_{ee}$ is not fulfilled. In the second case the electron–electron collision term in the kinetic equation is not the main one even for $\tau_{ee} \ll \tau_\varepsilon$, τ_i. In a weak electric field it is necessary to suppose that $F_0^{(\alpha)} = n_0 f_0 [1 + f_1^{(\alpha)}(\varepsilon, y)]$. Excluding the fields E_y and E_z we shall obtain from (1) the equations for $f_1^{(\alpha)}$ which should be completed with the boundary conditions

$$j_y^{(\alpha)}(\varepsilon, \pm d) = \pm S_\alpha^\pm(f_1^{(\alpha)}, f_1^{(\beta)})\,. \tag{34}$$

Here the functionals $S_\alpha^\pm(f_1^{(\alpha)}, f_1^{(\beta)})$ characterize the relaxation of $f_1^{(\alpha)}(\varepsilon, y)$ on the surfaces $y = \pm d$ and generally speaking, they have the same structure as the functionals $S_\alpha^{(0)}(f_1^{(\alpha)}, f_1^{(\beta)})$. The functionals $S_\alpha^{(0)}$ appear on the r.h.s. of equations (1) after the substitution of $F_0^{(\alpha)} = n_0 f_0 (1 + f_1^{(\alpha)})$ and they characterize the relaxation of the functions $f_1^{(\alpha)}$ in the volume [3].

Let us consider the simplest but quite important case of the infinitely thin semiconductor (see the footnote on page 812). Then we can neglect the functionals S_α in (1) and in the boundary conditions (34). So the decisions of the equations (1) are as follows $f_1^{(\alpha)}(\varepsilon, y) = \Phi^{(\alpha)}(\varepsilon)\, y/d$, where the functions $\Phi^{(\alpha)}(\varepsilon)$ can be easily found from (34). For an arbitrary magnetic field having the components H_y and H_z and for an arbitrary dependence of $\tau_{ik}(\varepsilon)$ in the λ-valley semiconductor we obtain

$$\frac{\Delta\sigma}{\sigma_0} = 1 - \frac{\sigma(h)}{\sigma(0)}, \tag{35}$$

where

$$\sigma(h) = \frac{\sigma_0^{(0)}}{\lambda \int\limits_0^\infty x^{3/2}\,\mathrm{e}^{-x}\,\tau_{22}(x)\,\mathrm{d}x} \sum_{\alpha=1}^{\lambda} \left[\int\limits_0^\infty \frac{x^{3/2}\,\mathrm{e}^{-x}\,\tau_{22}(x)\,\mathrm{d}x}{P_\alpha(x, h)} \left\{ 1 - a\,(1 - 2\sin^2\varphi_\alpha \sin^2\alpha_\alpha) - \right.\right.$$

$$- \left[a \sin 2\,\varphi_\alpha \sin\alpha_\alpha - (1 - a)\, h\, \frac{\tau_{22}(x)}{m_2} \sin\gamma \right] \times$$

$$\left.\left. \times \frac{\sum\limits_{\beta=1}^{\lambda} \int\limits_0^\infty \frac{x^{3/2}\,\mathrm{e}^{-x}\,\tau_{22}(x)\,\mathrm{d}x}{P_\beta(x, h)} \left[a \sin 2\,\varphi_\alpha \sin\alpha_\alpha + (1 - a)\, h\, \frac{\tau_{22}(x)}{m_2} \sin\gamma \right]}{\sum\limits_{\beta=1}^{\lambda} \int\limits_0^\infty \frac{x^{3/2}\,\mathrm{e}^{-x}\,\tau_{22}(x)\,\mathrm{d}x}{P_\beta(x, h)} (1 + a \cos 2\,\varphi_\beta)} \right\} \right]. \tag{36}$$

$\sigma_0^0 = (\mathrm{e}^2/kT)\, \lambda\, n_0\, D^{(0)}$ is the isotropic conductivity; $x = \varepsilon/kT$; $\cos\varphi_\alpha$, $\sin\varphi_\alpha \cos\alpha_\alpha$, and $\sin\varphi_\alpha \sin\alpha_\alpha$ are the directional cosines of the axis 1 of the α-th valley with respect to the axes x, y, z; $P_\alpha(x, h) = 1 + a\,(1 - 2\sin^2\varphi_\alpha \cos^2\alpha_\alpha) + (1 - a)\, h^2 \left(\tau_{22}(x)/m_2\right)^2 \sin^2\gamma$.

From (35) to (36) we note first of all that the MR does not depend on the z-component of the magnetic field, i.e. the MR become equal to zero when $H = H_z$ and $d \to 0$. When $d \to 0$ and the component $H = H_y$ is present the MR can both become equal to zero and remain the finite depending on φ_α and α_α.

5. Conclusion

It was shown in the Sections 3 and 4 that the SDMR takes place not only for $H = H_z$ but also for $H = H_y$ in the many-valley and one-valley anisotropic semiconductors. It is essential that the MR in the first case become equal to zero when $d \to 0$ but in the second case it is non-negative (zero, or positive) (35), (36). The characteristic lengths L_ε and L_i tend to a finite value (20), (24), (33) when $H = H_y \to \infty$ while $L_\varepsilon \to 0$ and $L_\mathrm{i} \to 0$ when $H = H_z \to \infty$, i.e. for $H = H_z \to \infty$ the SDMR disappears.

If the lengths L_ε and L_i essentially differ there will be two regions ($d \approx L_\varepsilon$ and $d \approx L_\mathrm{i}$) of noticable change of the MR when the thickness decreases but for $L_\varepsilon \approx L_\mathrm{i}$ there will be only one region ($d \approx L_\varepsilon \approx L_\mathrm{i}$). In case when $L_{\mathrm{ee}} \ll L_\varepsilon$,

L_i there will be one more region of change of the MR from (11) to (33) and (35) to (36) when $d \approx L_{ee}$.

It is interesting to note that the usage of an anisotropic deformation in many-valley semiconductors can considerably facilitate the determination of the characteristic lengths L_ε and L_i. The task gets simplified only if the electrons are in the minima which become the lowest after the deformation. In such a case n-Si which has been considered in Section 3.2, point 4 becomes a two-valley semiconductor (Section 3.2, point 3) when it is strained along the x-axis, and becomes a one-valley semiconductor with $\varphi = 0$ (Section 3.1) when it is compressed along the x-axis. In the case of compression of n-Ge (Section 3.2, point 5) along the y- or z-axis we obtain the two-valley semiconductor of the Subsection 3.2, point 2 or 3.2, point 1.

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