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Galvanomagnetic Effects in Many-Valley Semiconductors for Strong Magnetic and Heating Electric Fields

By

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The effect of the heating of electrons on the conductivity of many-valley semiconductors in strong (but not quantitizing) transverse magnetic fields is studied. It is shown that at low temperatures the current–voltage characteristic is of S-type, when the main mechanism of the intervalley scattering of the hot electrons is the emission of inter-valley phonons. The anisotropy of conductivity is not only reduced but also changes its sign when the electric field grows. The conductivity in strong magnetic fields can be greater than σ_0 (σ_0 is σ for $H \equiv 0$, $E_x \to 0$) within a certain region of the electric field strength. All these effects take place as a result of the influence of the Hall field on the heating of electrons and their redistribution with respect to the valleys.

Изучено влияние разогрева электронов на проводимость многодолинного полупроводника при наличии перпендикулярного к току сильного (но не квантующего) магнитного поля. Показано, что при низких температурах, когда с разогревом электронов основным механизмом междудолинного рассеяния становится испускание междудолинных фононов, вольтамперная характеристика — S-образная. Анизотропия проводимости при разогреве уменьшается и даже изменяет знак. В некоторой области электрических полей проводимость в сильном магнитном поле может быть больше чем слабополевая проводимость σ_0 при $E_x \to 0$ и $H \equiv 0$. Все рассмотренные эффекты обусловлены влиянием холловского поля на разогрев электронов и их распределение по долинам.

1. Introduction

Some theoretical works dealing with the influence of the Hall field on the heating of electrons and their redistribution over the valleys in many-valley semiconductors have been published lately. This problem is very complex and similar calculations have been performed mainly numerically (see e.g. [1, 2]). But such calculations do not give conclusive information on the galvanomagnetic effects. Therefore in a preceding paper [3] we have analysed qualitatively the galvanomagnetic effects in arbitrary electric and transverse (not quantitizing) magnetic fields for the simplest case of the bi-valley semiconductor, the magnetic field being applied parallel to the symmetry axis of the valleys. The investigation of the galvanomagnetic effects in many-valley semiconductors is continued in this paper.

It is known (see e.g. [4]) that the conductivity of many-valley semiconductors is rather anisotropic in strong magnetic fields, when the electrons are not heated, i.e. it depends on the orientation of the current and the transverse magnetic field relative to the crystal axes. As will be shown hereafter, at low temperatures the anisotropy of conductivity does not only reduce but also changes its sign when electrons are heated. The conductivity increases with increasing electric field strength, and as in [5] current–voltage (I–U) characteristics of S-type are possible.

2. Theoretical Foundation

The kinetic equation may be given for the distribution function $F_{\boldsymbol{p}}^{(\alpha)}$ in each α-th valley and the collision term does not include the inter-valley scattering when inter-valley scattering is negligible for the relaxation of energy and momentum with respect to the intra-valley scattering. The inter-valley scattering is only considered for the redistribution of carriers among the valleys. Usually, the distribution function $F_{\boldsymbol{p}}^{(\alpha)}$ in the α-th valley will be represented as a sum of symmetrical ($F_0^{(\alpha)}$) and asymmetrical ($F_1^{(\alpha)}$) parts: $F_{\boldsymbol{p}}^{(\alpha)} = F_0^{(\alpha)}(\varepsilon) + F_1^{(\alpha)}(\boldsymbol{p})$. When quasielastic mechanisms of intra-valley scattering are predominant we have $F_1^{(\alpha)} \ll F_0^{(\alpha)}$. An ordinary relation for $F_1^{(\alpha)}$ and $F_0^{(\alpha)}$ is obtained if the scattering of the momentum is characterized by a relaxation time tensor [6]. The equation for $F_0^{(\alpha)}$ in the system of main axes of the ellipsoidal energy surface of the α-th valley is as follows:

$$-\frac{2}{3} e^2 \frac{\partial}{\partial \varepsilon} \left\{ \frac{\sum\limits_{i=1}^{3} E_i^2 \frac{\tau_{ii}}{m_i} \varepsilon\, g(\varepsilon) \frac{\partial F_0}{\partial \varepsilon}}{1 + \frac{e^2}{c^2} \frac{\tau_{22}}{m_2} \left[H_1^2 \frac{\tau_{22}}{m_2} + (H_2^2 + H_3^2) \frac{\tau_{11}}{m_1} \right]} \right\}^{(\alpha)} = \hat{\hat{I}}\, F_0^{(\alpha)} , \tag{1}$$

where

$$\sum_{\boldsymbol{p}} \hat{I}\, F_{\boldsymbol{p}}^{(\alpha)}\, \delta(\varepsilon - \varepsilon_{\boldsymbol{p}}) \approx \sum_{\boldsymbol{p}} \hat{I}\, F_0^{(\alpha)}\, \delta(\varepsilon - \varepsilon_{\boldsymbol{p}}) \equiv \hat{\hat{I}}\, F_0^{(\alpha)} ,$$

$\hat{I}$ is the operator of intra-valley scattering, the axis 1 is directed along the symmetry axis of the ellipsoidal energy surface of the α-th valley.

For the electron current in the α-th valley the following expression is obtained[1] if we use the relation of $F_1^{(\alpha)}$ and $F_0^{(\alpha)}$:

$$j_i^{(\alpha)} = -n_\alpha (\mu_{ik}^{(\alpha)} + \mu_{ikj}^{(\alpha)} H_j)\, E_k , \tag{2}$$

where n_α is the number of electrons in the α-th valley, and $\mu_{ik}^{(\alpha)}$ and $\mu_{ikj}^{(\alpha)}$ can be expressed in the system of the main axes of the ellipsoidal energy surface of the α-th valley as follows:

$$\mu_{ii}^{(\alpha)} = \left\{ \frac{2}{3} e \int\limits_0^\infty \frac{\varepsilon\, g(\varepsilon) \frac{\tau_{ii}}{m_i} \left(-\frac{\partial F_0}{\partial \varepsilon} \right) \mathrm{d}\varepsilon}{1 + \frac{e^2}{c^2} \frac{\tau_{22}}{m_2} \left[H_1^2 \frac{\tau_{22}}{m_2} + (H_3^2 + H_3^2) \frac{\tau_{11}}{m_1} \right]} \right\}^{(\alpha)} , \tag{3}$$

$$\mu_{ikj}^{(\alpha)} = \left\{ \frac{2}{3} \frac{e^2}{c} \int\limits_0^\infty \frac{\varepsilon\, g(\varepsilon) \frac{\tau_{ii}}{m_i} \frac{\tau_{kk}}{m_k} \frac{\partial F_0}{\partial \varepsilon} \delta_{ikj}}{1 + \frac{e^2}{c^2} \frac{\tau_{22}}{m_2} \left[H_1^2 \frac{\tau_{22}}{m_2} + (H_2^2 + H_3^2) \frac{\tau_{11}}{m_1} \right]} \right\}^{(\alpha)} . \tag{4}$$

We consider the case of strong magnetic fields, namely

$$1 \ll \frac{e^2}{c^2} \frac{\tau_{22}}{m_2} \left[H_1^2 \frac{\tau_{22}}{m_2} + (H_2^2 + H_3^2) \frac{\tau_{11}}{m_1} \right],$$

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[1]) In the following we shall assume that the condition $\boldsymbol{E} \perp \boldsymbol{H}$ is fulfilled. It is sufficient for the fulfilment of the last condition in Ge and Si that the magnetic field should be directed along [100] and [110] type axes.

i.e. if unity may be neglected in the denominator of the expressions (1), (3), and (4) for all energies of interest. For example, at a temperature of 77 °K for pure germanium this condition is fulfilled for a magnetic field $H > 40$ kG and the quantum effect can be ignored up to $H = 200$ kG.

If $\tau_{ii}(\varepsilon)/\tau_{kk}(\varepsilon) = \text{const}$ (hereafter we assume that this condition is valid), it follows from (4) that $\mu^{(\alpha)}_{xyz}$ does not depend on heating and on the number of the valley and for any orientation of axes $x\ y\ z$

$$\mu^{(\alpha)}_{xyz} = -\mu^{(\alpha)}_{yxz} = -\frac{c}{H^2} \equiv \tilde{\mu} . \tag{5}$$

The following expressions for the tangent of the anisotropy angle $\operatorname{tg} \vartheta = \theta \equiv \equiv E_y/E_x$ (from $j_y = 0$) and the conductivity of the sample σ^* are obtained from (2) if the main H-dependent term is preserved:

$$\theta = \frac{\mu\, n\, H}{\sum\limits_{\alpha=1}^{\lambda} n_\alpha \mu^{(\alpha)}_{yy}} ; \qquad \sigma^* = e\, n\, \tilde{\mu}\, H\, \theta \equiv e\, n \frac{n\, c^2}{H^2 \sum\limits_{\alpha=1}^{\lambda} n_\alpha \mu^{(\alpha)}_{yy}} . \tag{6}$$

Here λ is the number of valleys. The current is parallel to the x-axis and $\boldsymbol{H}$ is oriented along the z-axis.

Expression (6) leads to the independence of the Hall constant ($R = -1/e\, n\, c$) on the electric field because in strong magnetic fields the Hall current is not connected with the scattering of electrons [7]. Below we shall analyse only one of the values θ or σ^* because of their relationship (6).

The dependence of the conductivity of the sample on the external electric field E_x and on the orientation of $\boldsymbol{E}_x$ and $\boldsymbol{H}$ relative to the crystal axes is fully determined by the transverse conductivity of the sample (6):

$$\sigma_\perp = e \sum_{\alpha=1}^{\lambda} n_\alpha \mu^{(\alpha)}_{yy} . \tag{7}$$

Expression (3) for strong magnetic fields yields

$$\mu^{(\alpha)}_{yy} = \frac{2}{3} \frac{c^2}{e H^2} \int\limits_0^\infty \frac{m_2}{\tau_{22}(\varepsilon)}\, \varepsilon\, g(\varepsilon) \left(-\frac{\partial F_0}{\partial \varepsilon}\right) \mathrm{d}\varepsilon\, \Pi_\alpha , \tag{8}$$

$$\Pi_\alpha = \left[\frac{\beta^2_{y1} + K\, (\beta^2_{y2} + \beta^2_{y3})}{K\, \beta^2_{z1} + \beta^2_{z2} + \beta^2_{z3}}\right]^{(\alpha)} , \tag{9}$$

where β_{ik} are the directional cosines of the axes x, y, z relative to the main axes 1, 2, 3 of the ellipsoidal energy surface of the α-th valley and $K = (\tau_{22}/m_2)\,(m_1/\tau_{11})$.

The symmetrical part of the distribution functions in any valley α is of Maxwellian type:

$$F^{(\alpha)}_0 = C\, \mathrm{e}^{-\varepsilon/kT} , \qquad n_\alpha = n_0 = \frac{1}{\lambda} n , \qquad \mu^{(\alpha)}_{yy} = \mu_0\, \Pi_\alpha , \qquad \sigma_\perp = e\, n_0\, \mu_0 \sum_{\alpha=1}^{\lambda} \Pi_\alpha$$

at low electric field when no heating of electrons occurs. Here the anisotropy of conductivity can be culculated easily (see e.g. [4]).

We shall analyse the change in conductivity due to the heating of electrons when an inter-valley redistribution arises. For this purpose we rewrite equation (1) as follows:

$$-\frac{2}{3}\frac{c^2}{H^2}\frac{\partial}{\partial\varepsilon}\left\{\frac{m_2}{\tau_{22}}\varepsilon\, g(\varepsilon)\frac{\partial}{\partial\varepsilon}F_0^{(\alpha)}\right\}\Pi_\alpha\,\theta^2\,E_x^2\left[1+\frac{\mu_{yx}^{(\alpha)}}{\mu_{yy}^{(\alpha)}\,\theta}+\frac{\mu_{xx}^{(\alpha)}}{\mu_{yy}^{(\alpha)}\,\theta^2}\right]=\hat{I}\,F_0^{(\alpha)}\,. \tag{10}$$

As we consider strong magnetic fields, then $\theta \gg 1$ and in (10) the expression in the square brackets differs from unity by terms of the order $1/\theta$ and $1/\theta^2$. By all qualitative arguments we shall consider that the expression in the square brackets of (10) is equal to unity.

3. Semi-Quantitative Considerations

First we shall consider the case that the momentum scattering time does not depend on energy:

$$\tau_{ik}(\varepsilon)=\tau_{ik}^0=\text{const}(\varepsilon)\,. \tag{11}$$

Thus (8) becomes

$$\mu_{yy}^{(\alpha)}=\mu_0\,\Pi_\alpha\,,\qquad \mu_0=\frac{c^2\,m_2}{e\,H^2\,\tau_{22}^0}\,, \tag{12}$$

and the dependence of σ^* on E_x is fully defined by inter-valley redistribution.

If all Π_α are equal (9), the distribution function and mobilities $\mu_{yy}^{(\alpha)}$ are equal. There are no inter-valley redistributions and σ^* does not depend on the electric field if the approximation (11) is valid.

We shall investigate the case that the inter-valley scattering time decreases due to the increase of the mean energy [8] (the reason for this limitation will be explained later); i.e. the ordinary Sasaki effect takes place [9, 10]: viz. electron transfer from hot to cold valleys. The case of low temperatures $kT \ll \hbar\,\omega$ is of great interest; here $\hbar\,\omega$ is the energy of the inter-valley phonon. For electric fields in which the average energy of electrons $\bar{\varepsilon}$ is less than $\hbar\,\omega$, the inter-valley scattering time τ_α changes rapidly due to heating, and such a strong inter-valley redistribution is possible that practically all electrons transfer to the coldest valley.

As it follows from (10) the electrons of the valley for which Π_α is greater, i.e. for which the mobility in transverse direction (12) is greater, are hotter.

As electrons leave the hot valley, in $\sigma_\perp = e\,\mu_0 \sum_{\alpha=1}^{\lambda} n_\alpha\,\Pi_\alpha$ the factor n_α before Π_α, which contributes mainly to $\sigma_\perp$, is reduced and $\sigma_\perp$ decreases[2]), whereas the conductivity $\sigma^*(E_x)$ increases with increasing E_x, as long as the inter-valley redistribution grows. The conductivity reaches a maximum and then drop if the inter-valley redistribution decreases. The inter-valley redistribution decreases because at high heatings for $\bar{\varepsilon} \gg \hbar\,\omega$ the inter-valley scattering time as function of mean energy becomes more gradual (see e.g. [8] where the inter-valley scattering time versus temperature is represented).

The maximum possible value of conductivity for each orientation can be estimated providing that all electrons are transferred to the coldest valley:

$$\sigma^*_{\text{max.}}=e\,n\,\frac{c^2}{\mu_0\,H^2}\,\frac{1}{\Pi_{\text{min.}}}=e^2\,n\,\frac{\tau_{22}^0}{m_2}\,\frac{1}{\Pi_{\text{min.}}}\,. \tag{13}$$

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[2]) When the inter-valley redistribution increases, $\sigma_\perp$ decreases because the weights of all Π_α are equal ($n_\alpha = n_0$) when $E_x \to 0$.

Table 1

$\boldsymbol{H}\|$ axis	$\boldsymbol{E}_x\|$ axis	germanium $E_x \to 0$	germanium E_1	silicon $E_x \to 0$	silicon E_1
[001]	[100]	$\frac{(K+2)\,3\,K}{(1+2\,K)^2}$	const (E_x)	$\frac{9\,K}{(K+2)\,(2\,K+1)}$	$\frac{3\,K}{1+2\,K}$
	[110]	$\frac{(K+2)\,3\,K}{(1+2\,K)^2}$	$\frac{3\,K}{1+2\,K}$	$\frac{18\,K^2}{(K+1)\,(1+2\,K)^2}$	$\frac{3\,K}{1+2\,K}$
[011]	[100]	$\frac{9\,K}{K^2+7\,K+1}$	1	$\frac{9\,K}{(K+2)\,(2\,K+1)}$	$\frac{3\,K}{1+2\,K}$
	[01$\bar{1}$]	$\frac{9\,K}{(2+K)\,(2\,K+1)}$	$\frac{3\,K}{1+2\,K}$	$\frac{9\,K\,(K+1)}{(5\,K+1)\,(2\,K+1)}$	$\frac{3\,K}{1+2\,K}$
	[11$\bar{1}$]	$\frac{27\,K}{5\,K^2+17\,K+5}$	$\frac{9\,K}{7\,K+2}$	$\frac{27\,K\,(K+1)}{(K^2+13\,K+4)\,(2\,K+1)}$	$\frac{9\,K\,(K+1)}{(5K+1)\,(2K+1)}$

$K = \frac{\tau_{22}^0}{m_2}\frac{m_1}{\tau_{11}^0}$; for Ge $K \leqq 19.2$, for Si $K \leqq 4.7$.

Table 1 gives the values of σ^*/σ_0 for Ge and Si for several directions of electric and magnetic fields. Here

$$\sigma_0 = e^2\, n\, \frac{1}{3}\left(2\,\frac{\tau_{22}^0}{m_2} + \frac{\tau_{11}^0}{m_1}\right) \tag{14}$$

is the isotropic conductivity if $E_x \to 0$, $H \equiv 0$. E_1 is the value of the electric field E_x for which $\sigma^*(E_x)$ has its maximum value.

Table 1 shows that for a few directions of $\boldsymbol{E}_x$ and $\boldsymbol{H}$ $\sigma^*_{\text{max.}} > \sigma_0$. This is conditioned by the fact that all electrons transfer to the valley where their mobilities in x-direction are greater than the isotropic low-field mobility $\sigma_0/e\,n$. (We remember that electrons transfer to the valley with the smallest Π_α. It follows from (9) that the smallest value of Π_α is equal to unity and (13) gives $\sigma^*_{\text{max., max.}} = e^2\, n\, \tau_{22}^0/m_2$. This value of conductivity is $(2/3 + 1/k)^{-1}$ times greater than σ_0.)

The anisotropy of conductivity reduces and even changes its sign (see Table 1) because, if one of the Π_α (and consequently $\sigma_\perp$ for $E_x \to 0$) increases, the other Π_α, which determine $\sigma^*_{\text{max.}}$, decrease in cubic crystals (9), when the directions of the x-, y-, z-axes relative to the crystal axes are changed.

4. Intra-Valley Scattering by Acoustic Phonons

First we have analysed the case $\tau_{ii}(\varepsilon) = \text{const}$.

Though it is an idealized case all the results obtained are true at low temperatures if the main mechanism of inter-valley scattering of hot electrons is the emission of inter-valley phonons. In this case the inter-valley scattering time τ_α decreases quickly when the electric field grows. Due to this, $\sigma^*(E_x)$ is mainly determined by $\tau_\alpha(E_x)$ and not, as usually, by $\mu_\alpha(E_x)$, because the latter changes gradually with respect to τ_α with increasing electric field. In this case the approximation (11) is justified and all the results obtained in Section 3 are true for any quasi-elastic mechanism of intra-valley scattering.

Section 3 did not deal with the case in which the inter-valley scattering time increases with increasing mean energy $\bar{\varepsilon}$ (inter-valley scattering by impurities [8]) as τ_α changes gradually with growing $\bar{\varepsilon}$ [10, 11].. In the given case, as well as at high temperatures and high heating of the electrons at $\bar{\varepsilon} \gtrsim \hbar\,\omega$, the contributions of inter-valley redistribution and the changes in mobility to the dependence of σ^* on E_x are comparable and the approximation (11) is not justified.

To confirm these qualitative results, we consider a pure semiconductor, in which the energy $(\tau_s = \tau_{s0}\sqrt{kT/\varepsilon})$ and momentum $(\tau_{ii}(\varepsilon) = \tau_{ii}^0\sqrt{kT/\varepsilon})$ are determined by scattering by acoustic lattice vibrations. For such scattering the collision term of the kinetic equation (10) is as follows:

$$\hat{I}\,F_0^{(\alpha)} = \frac{g(\varepsilon)}{\varepsilon\,\tau_s}\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\left[\varepsilon^2\left(1 + kT\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\right)F_0^{(\alpha)}\right]. \tag{15}$$

Then equation (10) can be solved easily (see e.g. [12]) and the distribution function in each α-th valley proves to be a Maxwellian function with effective electronic temperature T_α:

$$T_\alpha = T\left\{1 + \Pi_\alpha\left(\frac{\sigma^*}{\sigma_0^*}\gamma\right)^2\left[1 + \frac{\mu_{yx}^{(\alpha)}}{\mu_{yy}^{(\alpha)}\,\theta} + \frac{\mu_{xx}^{(\alpha)}}{\mu_{yy}^{(\alpha)}\,\theta^2}\right]\right\}, \tag{16}$$

where

$$\gamma = \frac{E_x}{E_c}, \qquad E_c^{-2} = \frac{2}{3}\,\theta_0^2\,\frac{c^2\,\tau_{s0}\,m_2}{kT\,\tau_{22}^0\,H^2}; \tag{17}$$

σ_0^* and θ_0 are σ^* and θ when $E_x \to 0$.

The mobility (7) calculated by means of this distribution function is a power function of T_α:

$$\mu_{yy}^{(\alpha)} = \mu_1\sqrt{\frac{T_\alpha}{T}}\,\Pi_\alpha, \qquad \mu_1 = \frac{8}{3\sqrt{\pi}}\,\frac{c^2\,m_2}{e\,H^2\,\tau_{22}^0}. \tag{18}$$

As in pure semiconductors the inter-valley scattering takes place via lattice vibrations [8, 9], we obtain the following expression for the inter-valley scattering time τ_α (see e.g. [13]):

$$\tau_\alpha = \tau_0\,C_\alpha; \qquad C_\alpha = 2\sqrt{\frac{T_\alpha}{T}}\;\frac{\left(1 + \mathrm{e}^{\frac{\hbar\omega}{kT}}\right)\mathrm{e}^{\frac{\hbar\omega}{2kT}\left(1-\frac{T}{T_\alpha}\right)}}{1 + \left(2 + \mathrm{e}^{\frac{\hbar\omega}{kT}}\right)\mathrm{e}^{-\frac{\hbar\omega}{kT_\alpha}}}\;\frac{\mathrm{K}_1\left(\frac{\hbar\,\omega}{2\,kT}\right)}{\mathrm{K}_1\left(\frac{\hbar\,\omega}{2\,kT_\alpha}\right)}, \tag{19}$$

where $\mathrm{K}_1(z)$ is a Bessel function.

As T_α depends on σ^* (16), we obtain the transcendental equation for σ^* from (6)[3]:

$$\frac{\sigma^*}{\sigma_0^*} = \frac{1}{\lambda}\sum_{\alpha=1}^{\lambda} C_\alpha \sum_{\alpha=1}^{\lambda}\Pi_\alpha\left[\sum_{\alpha=1}^{\lambda} C_\alpha\sqrt{\frac{T_\alpha}{T}}\,\Pi_\alpha\right]^{-1}. \tag{20}$$

We shall analyse the main features of the dependence of σ^*/σ_0^* on E_x.

[3]) Here the equilibrium condition for the inter-valley scattering $\left(\frac{n_\alpha}{\tau_\alpha} = \frac{1}{\lambda}\sum_{\beta=1}^{\lambda}\frac{n_\beta}{\tau_\beta}\right)$ was used. We obtain $n_\alpha = \dfrac{n\,\tau_\alpha}{\sum_{\beta=1}^{\lambda}\tau_\beta}$ from the equilibrium condition taking into consideration that $\sum_{\alpha=1}^{\lambda} n_\alpha = n$.

If $\Pi_\alpha = \Pi$ then $T_\alpha \approx T_e$, $C_\alpha = C_e$ and (20) gives

$$\frac{\sigma^*}{\sigma_0^*} = \left(\frac{\sqrt{1 + 4\,\Pi\,\gamma^2} - 1}{2\,\Pi\,\gamma^2}\right)^{1/2}, \tag{21}$$

i.e. σ^*/σ_0^* decreases when E_x grows.

For high temperatures or high electron heating ($kT_\alpha \gg \hbar\,\omega$)

$$\frac{\sigma^*}{\sigma_0^*} = \frac{1}{\lambda} \sum_{\alpha=1}^{\lambda} \sqrt{\frac{T}{T_\alpha}}, \tag{22}$$

i.e. σ^*/σ_0^* decreases with increasing electric field as in the case (21) and for $T_\alpha \gg T$ (16), $\dfrac{\sigma^*}{\sigma_0^*} = \left(\dfrac{1}{\lambda\sqrt{\gamma}} \sum\limits_{\alpha=1}^{\lambda} \dfrac{1}{\Pi_\alpha}\right)^{1/2}$.

But at low temperatures ($kT_\alpha \ll \hbar\,\omega$), from (19) follows

$$C_\alpha = 2\left(1 + e^{\frac{\hbar\omega}{kT}\left(1 - \frac{T}{T_\alpha}\right)}\right)^{-1}. \tag{23}$$

C_α changes rapidly with respect to T_α (and also $\mu_{yy}^{(\alpha)}$) when the electric field increases. Therefore an increase of the electric field leads to the reduction of the factor C_α of the term with the greatest $\sqrt{T_\alpha/T}\,\Pi_\alpha$, the greatest term $C_\alpha\sqrt{T_\alpha/T}\,\Pi_\alpha$ thus decreases and σ^*/σ_0^* increases.

From (20) it follows that if C_α is determined by expression (23) and not all Π_α are equal (in alternative case see (21)), σ^*/σ_0^* reaches its maximum value approximately for the electric field E_1 when

$$\frac{\sum\limits_{\alpha=1}^{\lambda} C_\alpha}{\sum\limits_{\alpha=1}^{\lambda} C_\alpha \sqrt{\frac{T_\alpha}{T}}\,\Pi_\alpha} \approx \sqrt{\frac{T}{T_{\beta,\mathrm{m}}}}\,\frac{1}{\Pi_\beta}. \tag{24}$$

Here and hereafter the β valley should be the coldest valley. Then σ^*/σ_0^* decreases with increasing E_x.

If Π_α differs significantly in different valleys, as it is the case for all orientations given in Table 1 except for $\boldsymbol{H} \parallel [001]$, $\boldsymbol{E}_x \parallel [100]$ in Ge, we obtain $C_\beta \gg C_{\alpha\neq\beta}$ in low electric fields when $(T_\beta - T)/T \ll 1$ (but $(T_{\alpha\neq\beta} - T)/T \approx 1$). So the maximum value of σ^*/σ_0^* is

$$\left(\frac{\sigma^*}{\sigma_0^*}\right)_{\mathrm{max.}} = \frac{\sum\limits_{\alpha=1}^{\lambda} \Pi_\alpha}{\lambda \sum\limits_{\alpha=1}^{\lambda} \sqrt{\frac{T_{\beta,\mathrm{m}}}{T}}\,\Pi_\beta} \approx \frac{\sum\limits_{\alpha=1}^{\lambda} \Pi_\alpha}{\lambda\,\Pi_\beta}. \tag{25}$$

The ratio $(\sigma^*/\sigma_0^*)_{\mathrm{max.}}$ is determined by the same expression (25) as (13), when condition (11) is fulfilled. Thus the expressions for σ^*/σ_0, when $E_x \to 0$ and $E_x = E_1$, are the same as in Table 1 if $\hbar\,\omega \gg kT$, the only difference is that all values of σ^*/σ_0 are $32/9\,\pi \approx 1.13$ times less than the corresponding values in Table 1. This factor is determined by the dependence of the momentum scattering time on energy. Now if we consider $K = 1$ (isotropic semiconductor), the magnetoresistance will not be equal to zero as it is the case in relation (11).

$\sigma^*_{\mathrm{max.,\,max}} > \sigma_0$ if energy and momentum are scattered by acoustic phonons because $32/9\,\pi < 3\,K/(1 + 2\,K)$. But if $H \equiv 0$, the conductivity $\sigma(E)$ decreases

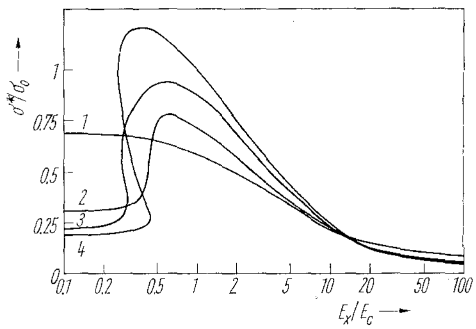

Fig. 1. Conductivity σ^* vs. E_x in strong magnetic fields for n-type Ge at 20 °K, $E_c = 8.6$ V/cm; (1): $\boldsymbol{H} \parallel [001]$, $\boldsymbol{E}_x \parallel [100]$; (2), (3), (4): $\boldsymbol{H} \parallel [011]$; $\boldsymbol{E}_x \parallel [100], [01\bar{1}], [11\bar{1}]$

because the mobility reduces, and additionally the electrons are transferred to the coldest valley (valley with lowest mobility in the direction of the current), with increasing electric field [9, 13, 14]. At low temperatures the decrease of σ can be so sharp due to the growth of E_x that the I–U characteristics can be of N-type [14]. As the electric field grows, σ decreases more for that direction of current for which σ^* increases, and negative magnetoresistance occurs not only via relation (11) but also in the general case of $kT \ll \hbar\,\omega$ if inter-valley scattering by lattice vibrations is predominant.

As σ^* grows due to the increase of the electric field, the differential conductivity $\sigma_d = \mathrm{d}j/\mathrm{d}E$ may extend to infinity [5]. Applying (20), we can write down the following condition for $\sigma_d = \infty$:

$$\frac{\sum\limits_{\alpha=1}^{\lambda} \Pi_\alpha}{\lambda} \, \frac{\partial}{\partial\left(\dfrac{\sigma^*}{\sigma_0^*}\right)} \left[\frac{\sum\limits_{\alpha=1}^{\lambda} C_\alpha}{\sum\limits_{\alpha=1}^{\lambda} C_\alpha \sqrt{\dfrac{T_\alpha}{T}}\, \Pi_\alpha}\right]_{E_x=\mathrm{const}} = 1\,. \tag{26}$$

Here the dependences of C_α and T_α on σ^*/σ_0^* are defined by the expressions (19) and (16). If not all Π_α are equal, the equation (26) has two solutions at low temperatures (C_α from (23)), which means that the I–U characteristics are of S-type.

Fig. 1 and 2 represent the dependences of σ^*/σ_0^* on E_x for Ge at 20 and 78 °K for some directions of $\boldsymbol{E}_x$ and $\boldsymbol{H}$, which have been obtained from (20). As Fig. 1 shows, I–U characteristics of S-type are realized indeed. At 78 °K (Fig. 2) $\sigma^*(E_x)$ goes through a maximum, the anisotropy of conductivity changes its sign, but $\sigma^* < \sigma_0$ and I–U characteristics of S-type are absent. This is due to the fact that the inter-valley scattering time is a relatively weak function of heating and equation (26) has no solutions.

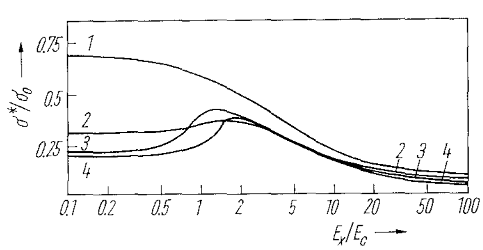

Fig. 2. Conductivity σ^* vs. E_x in strong magnetic fields for n-type Ge at 78 °K. $E_c = 33.6$ V/cm; (1): $\boldsymbol{H} \parallel [001]$, $\boldsymbol{E}_x \parallel [100]$; (2), (3), (4): $\boldsymbol{H} \parallel [011]$; $\boldsymbol{E}_x \parallel [100], [01\bar{1}], [11\bar{1}]$

In strong magnetic fields I–U characteristics of S-type occur for the same directions of the current for which they were of N-type when $H \equiv 0$. The cause of the existence of I–U characteristics of S- and N-types are the same, viz. the inter-valley redistribution of electrons. Thus we can proceed from I–U characteristics of N-type to I–U characteristics of S-type by increasing the magnetic field.

In conclusion we shall state that the Sasaki effect is "turned over" in strong magnetic fields in the sense that the valley which was the coldest for $H \equiv 0$ will necessarily become the hottest for $H \to \infty$. The cause of this phenomenon lies in the fact that the effective heating field is directed along the y-axis ($E_y \gg E_x$) when $H \to \infty$, while it is directed along the x-axis when $H \equiv 0$.

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