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SOME FEATURES OF THE HALL EFFECT IN A MANY-VALLEY SEMICONDUCTOR

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An analysis is made of the dependence of the Hall angle and of the conductivity of a many-valley semiconductor on an external electric field in the case when the intervalley scattering time depends strongly (compared with all other quantities) on the heating power. In this case the electric-field dependences of the Hall angle and the conductivity are not determined by the energy dependence of the momentum dissipation time (which is the usual situation) but by the dependence of the intervalley scattering time on the heating power. At certain critical values of the magnetic field, the dependences considered here change basically from curves with minima to curves with maxima. An increase in the magnetic field applied to a two-valley semiconductor gives rise to a situation in which the average energies of the valleys change in such a way that the energy which is the lower of the two in $H = 0$ becomes the higher energy. Moreover, there are always two values of the magnetic field (one positive and one negative) in which the average energies of the valleys become equal. Reversal of the magnetic field gives rise to a new state, i.e., the conductivity and the Hall angle exhibit effects which are odd with respect to the magnetic field.

1. The calculation of the galvanomagnetic coefficients of many-valley semiconductors in heating electric fields is rather complicated in the case when the effect of the Hall field on the heating of carriers is taken into account. For a semiconductor with λ valleys, the distribution functions corresponding to different valleys and the Hall field should be obtained self-consistently, i.e., by solving a system of $\lambda + 1$ equations (λ transport equations and the condition that the transverse current vanishes). Such a problem was solved numerically for each valley using the effective temperature approximation and assuming that the magnetic field is weak [1-3]. In previous papers, it was only pointed out that the effect of the Hall field on the valley distribution functions should be taken into account (see, for example, [4]) or the whole effect was neglected altogether [5, 6]. Since the numerical solutions clearly cannot provide complete information about the dependences of the Hall field and the magnetoresistance on the electric and magnetic fields, we decided to tackle this problem using a different method.

Our aim is to investigate qualitatively the Hall effect in arbitrary electric and arbitrary magnetic (but nonquantizing) fields using a method which was developed in [7]. We shall consider a two-valley semiconductor and assume that the orientation of the electric field with respect to the symmetry axis of the valleys is arbitrary [Eq. (4)]. It is found that an increase in the magnetic field gives rise to a situation in which the population of the valleys changes in such a way that the valley which is cooler for $H = 0$ and, therefore, contains more electrons if the intervalley scattering is due to phonons [8, 9] (or fewer electrons if the scattering is due to impurities [8, 10]) becomes hotter for $H \to \infty$, which means that it becomes less (or, alternatively, more) populated than the other valley. Therefore, by influencing the Hall field, the magnetic field leads to a transfer of energy, and, therefore, also to a transfer of carriers between the valleys. Moreover, for an arbitrary external electric field, there are two values of the magnetic field (one positive and one negative), depending on the angle φ between the current and the symmetry axis of the valley, in which the heating and the population of the two valleys become equal [Eq. (13)]. In general, the reversal of the magnetic field ($H \to -H$) gives rise to a new state, i.e., the effects in question become odd with respect to the magnetic field. If the momentum dissipation time is independent of the energy [$\tau_{ii}(\varepsilon)$ = const] then, in contrast to isotropic semiconductors which do not exhibit any magnetoresistance, the magnetoresistance of many-valley semiconductors (which is due to a transfer of carriers caused by heating) can be either positive or negative depending on the magnetic field and the angle φ. If the intervalley scattering time depends strongly on the heating power [Eq. (25)], the dependences of the tangent of the anisotropy angle ($\tan\vartheta = \Theta = E_y/E_x$) and of the conductivity (σ^*) on the electric field are governed by the dependence of the intervalley scattering time on the heating power rather than by the energy dependence [$\tau_{ii}(\varepsilon)$] of the momentum dissipation time (which is the usual situation). Depending on the magnetic field, these quantities can exhibit either maxima or minima. For every angle φ, there are three critical values of the magnetic field at which the form of the dependences $\Theta(E_x)$ and $\sigma^*(E_x)$ changes radically.

2. If the intervalley scattering time is considerably longer than all the intravalley scattering times, the electrons belonging to a given valley can be regarded as an essentially isolated group of carriers. Therefore, in the case $\mathbf{E} \perp \mathbf{H}$, the current of electrons in a valley α in an arbitrary magnetic field is given by

$$j_i^{(\alpha)} = -n_\alpha \left(\mu_{ik}^{(\alpha)} + \mu_{ikj}^{(\alpha)} H_j\right) E_k, \qquad (1)$$

where n_α is the number of electrons in the valley α and the quantities $\mu_{ik}^{(\alpha)}$ and $\mu_{ikj}^{(\alpha)}$ have the following form in the coordinate system corresponding to the principal axes of the mass ellipsoid:

$$\mu_{ii}^{(\alpha)} = \left\{\frac{2}{3} e \int_0^\infty \frac{\varepsilon g(\varepsilon)\frac{\tau_{ii}}{m_i}\left(-\frac{\partial F_0}{\partial \varepsilon}\right) d\varepsilon}{1 + \frac{e^2}{c^2}\frac{\tau_{33}}{m_3}\left[H_1^2 \frac{\tau_{22}}{m_2} + (H_2^2 + H_3^2)\frac{\tau_{11}}{m_1}\right]}\right\}^{(\alpha)}, \qquad (2)$$

$$\mu_{ikj}^{(\alpha)} = \left\{\frac{2}{3}\frac{e^2}{c} \int_0^\infty \frac{\varepsilon g(\varepsilon)\frac{\tau_{ii}}{m_i}\frac{\tau_{kk}}{m_k}\frac{\partial F_0}{\partial \varepsilon} d\varepsilon\, \delta_{ikj}}{1 + \frac{e^2}{c^2}\frac{\tau_{33}}{m_3}\left[H_1^2 \frac{\tau_{22}}{m_2} + (H_2^2 + H_3^2)\frac{\tau_{11}}{m_1}\right]}\right\}^{(\alpha)}. \qquad (3)$$

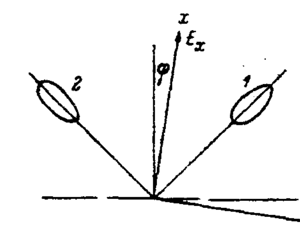

Fig. 1. Current is in the direction of the x axis. The magnetic field $H = H_z$ is perpendicular to the plane of the figure.

Here, the axis 1 is in the direction of the symmetry axis of the mass ellipsoid and e is the magnitude of the electron charge.

For simplicity, we shall consider a two-valley semiconductor (Fig. 1) and assume that the external electric field E_x is applied in the valley plane a an arbitrary angle φ to the symmetry axis. The magnetic field $H = H_z$ is perpendicular to the valley plane, i.e., it is oriented along the second symmetry axis, which implies that the denominators in Eqs. (2) and (3) are identical for both valleys [for example, this can occur in the case of n-type Ge if the electric field is in the (100) plane and H is applied along the four-fold symmetry axis]. Therefore

$$\begin{gathered}\mu_{xx}^{(1,2)} = \mu_{1,2}(1 \pm a \sin 2\varphi), \quad \mu_{yy}^{(1,2)} = \mu_{1,2}(1 \mp a \sin 2\varphi), \\ \mu_{xy}^{(1,2)} = \pm\mu_{1,2} a \cos 2\varphi, \quad \mu_{xyz}^{(1,2)} = -\mu_{yxz}^{(1,2)} = \tilde{\mu}_{1,2}\end{gathered} \qquad (4$$

and, for a two-valley semiconductor, we find $\mu = \frac{1}{2}(\mu_{11} + \mu_{22})$,

$$\tilde{\mu} = \mu_{123}, \quad a = \frac{\mu_{22} - \mu_{11}}{\mu_{22} + \mu_{11}}.$$

For Ge we obtain

$$\mu = \frac{1}{3}(\mu_{11} + 2\mu_{22}), \quad \tilde{\mu} = \frac{1}{3}(\mu_{231} + 2\mu_{123}), \quad a = \frac{\mu_{22} - \mu_{11}}{\mu_{11} + 2\mu_{22}}.$$

Since $\mu_{ikj} = -\mu_{kij}$, the heating power per α electron is given by

$$P_\alpha = -\frac{e\mathbf{j}_\alpha \mathbf{E}}{n_\alpha} = e\mu_{ik}^{(\alpha)} E_i E_k = e\mu_\alpha \Pi_\alpha, \qquad ($$

where

$$\Pi_{1,2} = E_x^2 [1 \pm a \sin 2\varphi \pm 2a \cos 2\varphi\, \Theta + (1 \mp a \sin 2\varphi)\Theta^2]. \qquad ($$

If a is independent of the electric field and th magnetic field is specified, the quantity Π_α define completely the state of electrons in the α valley [7] (see Appendix), i.e.,

$$\mu_{1,2} = \mu(\Pi_{1,2}), \quad \tilde{\mu}_{1,2} = \tilde{\mu}(\Pi_{1,2}), \quad \tau_{1,2} = \tau(\Pi_{1,2}) \qquad ($$

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[1]In particular, we shall assume that $\varphi > 0$.

(τ_α is the intervalley scattering time for electrons belonging to the valley α).

The transverse field E_y is obtained from the condition that the transverse current vanishes:[2]

$$\Theta = \frac{H(\Psi_1 + \Psi_2) - a\cos 2\varphi\,(\Phi_1 - \Phi_2)}{\Phi_1 + \Phi_2 - a\sin 2\varphi\,(\Phi_1 - \Phi_2)}. \tag{8}$$

Here, $\Phi_\alpha = \tau_\alpha\mu_\alpha$, $\Psi_\alpha = \tau_\alpha\tilde{\mu}_\alpha$ and we have made use of the condition of detailed balance of intervalley transitions, which implies

$$n_\alpha = n\frac{\tau_\alpha}{\sum\limits_{\beta=1}^{2}\tau_\beta}. \tag{9}$$

It follows from Eq. (8) that, in general, the transverse field is a sum of two fields: the true Hall field, which is represented by the first term in the numerator in Eq. (8), and the Sasaki field, which is represented by the second term. Equations (6) and (7) indicate that the Sasaki field also depends on the magnetic field. Even in the case when the two valleys are symmetric with respect to E_x ($\varphi = 0$) in zero magnetic field, i.e., equally heated[3] and equally populated, which implies that the Sasaki field vanishes, it follows from Eq. (6) that, in an applied magnetic field, the heating of the valleys is different, which gives rise to a finite Sasaki field.

The system of transcendental equations (6) and (8) is a closed system which governs the dependences of $\Pi_{1,2}$ and Θ on E_x and H. Since all the parameters in the present problem are functions of Π_α [Eq. (7)], it is of great interest to investigate the dependence $\Pi_\alpha(\Theta)$. It follows from Eq. (6) that the expression

$$\Pi_1 - \Pi_2 = aE_x^2\,[2\cos 2\varphi\Theta - \sin 2\varphi\,(1 - \Theta^2)] \tag{10}$$

represents a parabola which opens downwards and intersects the Θ axis at the points

$$\Theta_{1,2} = \frac{\cos 2\varphi \pm 1}{\sin 2\varphi}, \quad \Pi_1 - \Pi_2 = 0. \tag{11}$$

It also follows from Eq. (6) that the two valleys are equally heated at the points $\Theta = \Theta_{1,2}$:

$$\Pi_1(\Theta_{1,2}) = \Pi_2(\Theta_{1,2}) = 2E_x^2\frac{1 \pm \cos 2\varphi}{\sin 2\varphi}$$

[illegible]

and Eq. (8) takes the form

$$\Theta_{1,2} = H\frac{\tilde{\mu}}{\mu}\ \text{[illegible]} \tag{12}$$

Using Eqs. (11) and (12), we obtain the equation

$$H_{c,1,2} = \frac{\mu}{\tilde{\mu}}\frac{\cos 2\varphi \pm 1}{\sin 2\varphi} \tag{13}$$

defining the curve $H_c(E_x)$ along which the [illegible] (11) of equal heating of the valleys is satisfied [illegible] for an arbitrary angle φ and an arbitrary [illegible] field E_x, there are two critical values of the magnetic field $H_{c,1}(E_x\varphi)$, $H_{c,2}(E_x\varphi)$, in which the two valleys are equally populated [in general, the quantity $\mu/\tilde{\mu}$ in Eq. (13) depends on the electric field; if $\tau_{ii}(\varepsilon)$ = const, then $\mu/\tilde{\mu}$ is independent of the electric field and $H_{c,1} = H_{c,1}(\varphi)$, $H_{c,2} = H_{c,2}(\varphi)$]. The vertex of the parabola defined by Eq. (10) is located at the point

$$\Theta_3 = \operatorname{ctg} 2\varphi, \quad (\Pi_1 - \Pi_2)_{\max} = \frac{2a}{\sin 2\varphi}E_x^2. \tag{14}$$

Equations (8) and (14) yield the equation

$$\operatorname{ctg} 2\varphi = H_{c,3}\frac{\Psi_1 + \Psi_2}{\Phi_1 + \Phi_2} \tag{15}$$

which defines the curve $H_{c,3}(E_x, \varphi)$ along which the quantity $\Pi_1 - \Pi_2$ is a maximum [as before, for $\tau_{ii}(\varepsilon)$ = const, we obtain $H_{c,3}(\varphi)$].

Therefore, we have

$$\Pi_1 > \Pi_2 \text{ for } \Theta > \Theta_1,\ \Theta < \Theta_2, \tag{16}$$

$$\Pi_2 > \Pi_1 \text{ for } \Theta_2 < \Theta < \Theta_1, \tag{17}$$

i.e., in the case defined by Eq. (16), valley 1 is hotter than valley 2, and, therefore, less populated ($n_1/n_2 < 1$) if the intervalley scattering is due to phonons [8, 9] (or $n_1/n_2 > 1$ if the intervalley scattering is due to impurities [8, 10]); in the case defined by Eq. (17), valley 2 is hotter and n_1/n_2 [illegible] (or, alternatively, $n_1/n_2 < 1$). Since an increase [illegible] the magnetic field always leads (with the exception of the case [illegible]) to the transition from the situation described by Eq. (16) (small $|\Theta|$) to that described by Eq. (17) (large $|\Theta|$), it follows that by influencing the Hall field, the magnetic field essentially leads to a transfer of energy, i.e., also to a transfer of carriers between the valleys. Therefore, the magnetic field influences considerably the heating of carriers in the valleys. Equations

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[2] [illegible] that [illegible] is [illegible] Eq. [illegible]

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(6) and (8) indicate that the reversal of the field $H \to -H$ does not change the state of a semiconductor only in the cases $\varphi = 0$, $\pi/4$, whereas for $0 < \varphi < \pi/4$, the reversal should lead to effects which are odd with respect to the magnetic field. It follows from Eq. (8) that Θ vanishes if

$$H = a \cos 2\varphi \frac{\Phi_1 - \Phi_2}{\Psi_1 + \Psi_2}, \tag{18}$$

i.e., the true Hall field compensates the Sasaki field.

3. To study the dependence $\Theta(E_x)$, it is convenient to write Eq. (8) in the form

$$\theta = \theta_H A, \tag{19}$$

$$\Theta = H \frac{\Psi_1 + \Psi_2}{\Phi_1 + \Phi_2}, \quad A = \frac{1 - \dfrac{a \cos 2\varphi}{\Theta} \dfrac{\Phi_1 - \Phi_2}{\Phi_1 + \Phi_2}}{1 - a \sin 2\varphi \dfrac{\Phi_1 - \Phi_2}{\Phi_1 + \Phi_2}}, \tag{20}$$

We shall assume that $\tau_{ji}(\varepsilon) = \text{const}$. In this case,[4] the quantities μ_α and $\tilde{\mu}_\alpha$ are independent of the heating, $\Theta_H = \Theta_0 \equiv H\tilde{\mu}_0/\mu_0$, $\Phi_1/\Phi_2 = \tau_1/\tau_2$, and the dependence of the tangent of the anisotropy angle on the electric field $\Theta(E_x)$ is governed by the dependence of the intervalley scattering time on the power $\tau(\Pi)$. If $\Theta_0 = \Theta_1, \Theta_2, \Theta_3$, we find $A \equiv 1$ and Θ is independent of E_x. In the case of intervalley scattering by phonons, we find $d\tau/d\Pi < 0$ and for

$$\Theta_0 > \Theta_1, \; \Theta_0 < \Theta_2, \; 0 < \Theta_0 < \Theta_3 \tag{21}$$

we obtain $A \geq 1$, whereas for

$$\Theta_2 < \Theta_0 < 0, \; \Theta_3 < \Theta_0 < \Theta_1 \tag{22}$$

we obtain $A \leq 1$. Since, for a given lattice temperature T, the dependence $\tau(\Pi)$ follows approximately the dependence $\tau(T)$ for $E_x = 0$, it can be deduced from [8] that, at low temperatures and for pure samples (such as were studied in [11, 12]), the dependence of the scattering time $\tau(\Pi)$ on the heating power is initially strong and then becomes weaker. Therefore, the quantity $b(E_x) = \left|\frac{\tau_1 - \tau_2}{\tau_1 + \tau_2}\right|$ is a nonmonotonic function of E_x. For $E_x = 0$, we find that $b = 0$; with increasing E_x, the quantity b increases, reaches a maximum, and eventually begins to decrease, since the dependence of τ on the power Π becomes weaker. Clearly, in the first case [Eq. (21)], the function $|\Theta|(E_x)$ exhibits a maximum, whereas in the case defined by Eq. (22), it exhibits a minimum. For $\varphi = \pi/4$, the value of Θ corresponding to the maximum (minimum), where b can be of the order of unity, can be twice (or

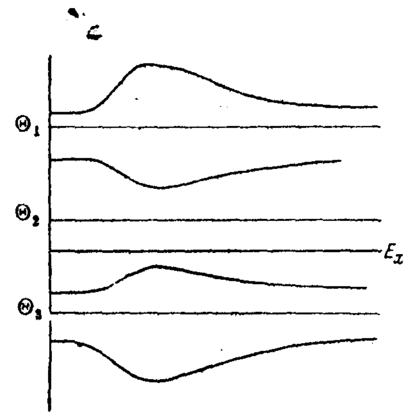

Fig. 2. Qualitative form of the dependence $\Theta(E_x)$ in the case when the intervalley scattering is due to the emission and absorption of intervalley phonons. The straight lines Θ_1, Θ_2, Θ_3 divide the plane ΘE_x into regions with essentially different dependences $\Theta(E_x)$; $\cos 2\varphi = 1/3$.

half) the value corresponding to $E_x = 0$. If the intervalley scattering is due to impurities ($d\tau/d\Pi > 0$), the function $|\Theta|(E_x)$ should exhibit a minimum in the case defined by Eq. (21) and a maximum in the case defined by Eq. (22). These extrema are less pronounced than in the scattering by phonons since the scattering time τ varies more slowly as a function of Π, i.e., the straight lines $\Theta_0 = \Theta_1, \Theta_2, \Theta_3$ (corresponding to the critical magnetic fields $H_{c,1}, H_{c,2}, H_{c,3}$) divide the plane ΘE_x into regions with essentially different dependences $|\Theta|(E_x)$.

Figure 2 shows qualitatively the dependence $\Theta(E_x)$ in the case when the intervalley scattering is due to the emission and absorption of intervalley phonons.

Again, we can study the dependence of the conductivity on the electric field. Therefore, if in the expression for the current we replace H by Θ, we obtain from Eq. (8) the following expressions:

$$I_x = \sigma^* E_x, \quad \sigma^* = \sigma_H B, \tag{23}$$

$$\sigma_H = en \frac{\Phi_1 + \Phi_2}{\tau_1 + \tau_2} (1 + \Theta^2), \quad B = \frac{1 - \dfrac{a^2}{1 + \Theta^2} \left(\dfrac{\Phi_1 - \Phi_2}{\Phi_1 + \Phi_2}\right)^2}{1 - a \sin 2\varphi \dfrac{\Phi_1 - \Phi_2}{\Phi_1 + \Phi_2}}. \tag{24}$$

If $\tau_{ji}(\varepsilon) = \text{const}$, the dependence $\sigma^*(E_x)$ is governed by the dependence of the intervalley scattering time on the heating. In the case of the phonon intervalley scattering and for $\Theta_2 < \Theta_0 < \Theta_1$, we find that $B \leq 1$ and $\sigma^*(E_x)$ exhibits a minimum (the mini-

[4]The subscript zero (for example, μ_0) indicates that the corresponding quantity is taken at $E_x = 0$.

mum value can be half that corresponding to $E_x = 0$); for $\Theta_0 > \Theta_1$, we find that $B \geq 1$ and $\sigma^*(E_x)$ exhibits a maximum. In the case $\Theta_3 < |\Theta_2|$, the conductivity $\sigma^*(E_x)$ reaches a maximum in the region $\Theta_0 < \Theta_2$, whereas in the opposite case, it exhibits a maximum for $\Theta_0 < -\Theta_3$. In the region $-\Theta_3 < \Theta_0 < \Theta_2$, the curve $\sigma^*(E_x)$ exhibits two "humps," provided the quantity b is of the order of unity. Therefore, the dependence $\sigma^*(E_x)$ is essentially different for different values of the parameter Θ_0 (the critical values are $\Theta_0 = \Theta_1, \Theta_2, -\Theta_3$).

If the dependences $\Theta(E_x)$ and $J_x(E_x)$ are known, the Hall factor can be calculated. The form of the dependence of the Hall factor on E_x is also essentially different for different values of the parameter Θ_0.

4. We have assumed that $\tau_{ij}(\varepsilon) = \text{const}$. However, in the case when the dependence $\tau(\Pi)$ is strong compared with all the other functions,

$$\left|\frac{1}{\tau}\frac{d\tau}{d\Pi}\right| \gg \left|\frac{1}{\mu}\frac{d\mu}{d\Pi}\right|, \ \left|\frac{1}{\tilde{\mu}}\frac{d\tilde{\mu}}{d\Pi}\right|, \tag{25}$$

the situation is quite different. In the region of electric fields in which condition (25) is satisfied far from the critical values Θ ($\Theta = \Theta_1, \Theta_2, \Theta_3$), the tangent of the anisotropy angle and the conductivity are not determined by $\tau_{ij}(\varepsilon)$ (which is the usual situation) but by the dependence of the intervalley scattering time on the heating power. This conclusion can be easily verified in the weak heating case. In fact, by expanding in terms of a weak field E_x, we obtain from Eqs. (6), (19), (20), (23), and (24) the following expressions:

$$\Theta = \Theta_0\left\{1 + E_x^2\left[(1+\Theta_0^2)\left(\frac{\tilde{\mu}'}{\tilde{\mu}_0} - \frac{\mu'}{\mu_0}\right) + a^2\frac{\Phi'}{\Phi_0}\left(\sin 2\varphi - \frac{\cos 2\varphi}{\Theta_0}\right)C\right]\right\}, \tag{26}$$

$$\sigma^* = \sigma_0(1+\Theta_0^2)\left\{1 + E_x^2\left[2\Theta_0^2\frac{\tilde{\mu}'}{\tilde{\mu}_0} + (1-\Theta_0^2)\frac{\mu'}{\mu_0} + a^2\sin 2\varphi\frac{\Phi'}{\Phi_0}C\right]\right\}, \tag{27}$$

$$C = \sin 2\varphi + 2\cos 2\varphi\,\Theta_0 - \sin 2\varphi\,\Theta_0^2.$$

The derivatives $\mu' = \frac{d\mu}{d\Pi}$, $\tilde{\mu}' = \frac{d\tilde{\mu}}{d\Pi}$, $\Phi' = \frac{d\Phi}{d\Pi}$ are taken at $E_x = 0$.

Equations (26) and (27) indicate that, far from the critical values of Θ [$\Theta_0 = \Theta_1, \Theta_2, \Theta_3$ are zeros of the coefficients at Φ'/Φ_0 in Eqs. (26) and (27)], when Eq. (25) is satisfied, we find that $\Theta(E_x)$ and $\sigma^*(E_x)$ are governed by the dependence $\tau(\Pi)$. For $\tau' < 0$ and in the case defined by Eq. (21), we obtain $d|\Theta|/dE_x^2 > 0$, whereas in the case defined by Eq. (22), we obtain $d|\Theta|/dE_x^2 < 0$; the quantity $d\sigma^*/dE_x^2$ is positive if condition (17) is satisfied and negative if condition (16) holds.

The magnetoresistance can be calculated only if the actual form of the intravalley scattering mechanism is specified. Negative magnetoresistance in weak magnetic fields was reported in [1-3]. We shall show that the magnetoresistance in strong magnetic fields can become negative if the strong inequality (25) is satisfied. For simplicity, we shall consider the case $\varphi = \pi/4$ and the electric fields in which the quantity b reaches a maximum, i.e., is of the order of unity. For $a \approx 1/2$, we obtain

$$\frac{\Delta\rho}{\rho_0} = \frac{1}{4}\left(\frac{\Delta\rho'}{\rho_0'} - 3\right), \tag{28}$$

where $\Delta\rho'/\rho_0'$ is the positive magnetoresistance (neglecting the intervalley transfer), which is usually smaller than unity.

As already discussed, the special features of the Hall effect in many-valley semiconductors manifest themselves if condition (25) is satisfied, which is usually realized at low temperatures and for sufficiently pure samples. Our results are in agreement with the experimental data [12] obtained at 77°K for $\varphi = 0$. In fact, it follows from Eqs. (19) and (23) that there is only one type of dependence of the Hall coefficient, i.e., $R(E_x)$ exhibits a maximum (in this special case, we find $\Theta_1 = \Theta_3 = \infty$, $\Theta_2 = -\infty$). Unfortunately, the measurements of the Hall effect for an arbitrary angle, when effects odd in H should occur, are not available. They are not even available for $J_x \| [110]$, when the odd effects do not occur, and standard measurement techniques can be used (it should be noted that, for $\varphi \neq 0, \pi/4$, the two measurements carried out during the magnetic field reversal cannot be averaged), and the change from curves $\Theta(E_x)$, $\sigma^*(E_x)$, $R(E_x)$ with minima to curves with maxima occurs in the range of moderate magnetic fields which can be achieved experimentally.

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APPENDIX

If the intervalley scattering time is considerably longer than all the intravalley scattering times, the transport equations for electrons from different valleys*

*Publisher's note: [a b] means a × b.

$$-e\left(E_i+\frac{1}{c}[\mathbf{vH}]_i\right)\frac{\partial}{\partial p_i}F_{\mathbf{p}}^{(\alpha)}=IF_{\mathbf{p}}^{(\alpha)} \quad \text{(A.1)}$$

differ only in the field terms. In the quasielastic approximation, we obtain $F_\mathbf{p} = F^+ + F^- \simeq F_0 + v_i F_i$, where $F^- \ll F^+$ are, respectively, the even and the odd parts (with respect to the momentum) of the distribution function. Introducing the momentum relaxation time tensor and assuming that $\mathbf{E}\perp\mathbf{H}$, we obtain in the system of the principal axes of the mass ellipsoid the following expression:

$$F_i^{(\alpha)}=e\left[\frac{\partial F_0}{\partial \varepsilon_\mathbf{p}}\tau_{ii}\frac{E_i+\delta_{ikj}\frac{e}{c}E_k\frac{\tau_{kk}}{m_k}H_j}{1+\frac{e^2}{c^2}\frac{\tau_{22}}{m_2}\left[H_1^2\frac{\tau_{22}}{m_2}+(H_2^2+H_3^2)\frac{\tau_{11}}{m_1}\right]}\right]^{(\alpha)}. \quad \text{(A.2)}$$

The equation for $F_0^{(\alpha)}$ has the form

$$-\frac{2}{3}e^2\frac{\partial}{\partial\varepsilon}\left[\frac{\sum_i E_i^2\frac{\tau_{ii}}{m_i}\varepsilon g(\varepsilon)\frac{\partial F_0}{\partial\varepsilon}}{1+\frac{e^2}{c^2}\frac{\tau_{22}}{m_2}\left[H_1^2\frac{\tau_{22}}{m_2}+(H_2^2+H_3^2)\frac{\tau_{11}}{m_1}\right]}\right]^{(\alpha)}=\hat{I}F_0^{(\alpha)}, \quad \text{(A.3)}$$

where

$$\hat{I}F_0^{(\alpha)}=\sum_\mathbf{p}\hat{I}F^{+(\alpha)}\delta(\varepsilon-\varepsilon_\mathbf{p})\simeq\sum_\mathbf{p}\hat{I}\delta(\varepsilon-\varepsilon_\mathbf{p})F_0^{(\alpha)}.$$

In the case considered, i.e., when H is parallel to the valley symmetry axis for $\tau_{11}(\varepsilon)/\tau_{22}(\varepsilon)$ = const [in this case, Eq. (4) implies that a = const], we obtain from Eq. (A.3)

$$-\frac{2}{3}e^2\frac{\partial}{\partial\varepsilon}\left[\frac{\left(\frac{\tau}{m}\right)_{\mathrm{is}}\varepsilon g(\varepsilon)\frac{\partial}{\partial\varepsilon}}{1+\frac{1}{3}\frac{e^2}{c^2}H^2\frac{\tau_{22}}{m_2}\left(2\frac{\tau_{11}}{m_1}+\frac{\tau_{22}}{m_2}\right)}\right]F_0^{(\alpha)}\Pi_\alpha=\hat{I}F_0^{(\alpha)}. \quad \text{(A.4)}$$

Here, Π_α are also given by Eq. (6) and $(\tau/m)_{\mathrm{is}} = (1/3)[(\tau_{11}/m_1) + 2(\tau_{22}/m_2)]$, i.e., the transport equations in the first and in the second valleys differ only in the factor Π_α. Consequently, for a given magnetic field H, the quantity $H\Pi_\alpha$ determines completely the distribution function $F_0^{(\alpha)}$ $F_0(\Pi_\alpha)$, and, therefore, all the transport coefficients [Eq. (7)].

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