ELECTRIC PINCH EFFECT IN MANY-VALLEY SEMICONDUCTORS UNDER ELECTRON HEATING CONDITIONS

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A many-valley semiconductor with nearly intrinsic conductivity is considered. It is shown that carrier pinch and strongly rectifying current-voltage characteristics are exhibited by samples which are thin compared with the extended electron-hole recombination length if the conditions on the surfaces are strongly asymmetrical and the conditions of the Sasaki effect obtain. The direction of the pinch is determined by the direction of the current and by the dependence of the intervalley scattering time on the carrier heating. If this time is a nonmonotonic function of the average energy, i.e., if the "anomalous" Sasaki effect [1] occurs in a certain range of fields, the direction of the pinch changes and the forward and reverse branches intersect in the current-voltage characteristic $|I| = f(|E|)$. Under strongly asymmetrical surface conditions, carrier accumulation occurs in a sample if carriers are driven against a surface with a low recombination velocity; in the opposite case, carrier depletion is observed. If the sample is depleted of carriers, negative differential conductance may be observed even in the special case when the recombination time increases with increasing field intensity.

1. In the investigations [1-3], detailed theoretical and experimental studies were made of the pinch effect that accompanies the passage of an electric current through a plate made of an intrinsic anisotropic semiconductor in electric fields that do not heat the carriers. Investigations were then made [4-6] of an electric pinch in many-valley semiconductors with an induced anisotropy. In [4] the anisotropy was induced by the intervalley redistribution near the surfaces [7] and was important only if

$$\frac{eEL}{kT} \gg 1, \qquad (1)$$

where L is the mean free path in intervalley scattering. On the other hand, in [5, 6] the anisotropy was induced by carrier heating by an electric field ("Sasaki anisotropy" [8]), a theoretical analysis being made only for the special case when the surface relaxation rates are low [5] (these being the conditions under which the experiment in [6] was performed).

In the present paper we consider the case (as in [5]) when the anisotropy is induced by the heating effect of an electric field and show that the current-voltage characteristics of samples made of a many-valley semiconductor with an almost intrinsic conduction are rectifying in such an electric field if two conditions are satisfied, namely, the samples are thin compared with the extended electron-hole recombination length and the conditions at the surfaces are strongly asymmetrical. If the second condition is fulfilled, we find that, as in [3], accumulation of carriers occurs if the latter are driven to a surface with a low recombination velocity and depletion takes place in the opposite case. Negative differential conductance is possible if there is carrier depletion even in the special case when the recombination time increases with the field. If the intervalley scattering by impur-

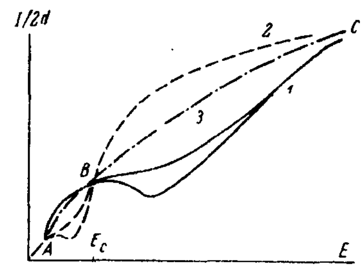

Fig. 1. Approximate form of the current-voltage characteristics of thin samples (1, 2) for $\gamma > 0$ and $\gamma < 0$ and a thick sample (3). $a(E_c) = 0$. The sections with negative differential conductance in curves 1 and 2 are realized under the conditions given in the text.

ities occurs in a definite range of fields (this is the "anomalous" Sasaki effect [9], i.e., the intervalley scattering time has a maximum as a function of the mean energy [10]), the direction of the carrier pinch changes at a certain critical field (corresponding approximately to the maximum of the intervalley scattering time) and the forward and reverse branches intersect in the current-voltage characteristic $|I| = f(|E|)$ (see Fig. 1).

2. We shall consider a homogeneous semiconductor with almost intrinsic conduction ($n \approx p$), a many-valley electron spectrum, and single-valley isotropic hole spectrum[1] (for example, germanium and silicon, in which the hole anisotropy is small compared with the electron anisotropy) in an electric field that heats the carriers. The sample is a planar infinite slab of thickness 2d ($-d \le y \le d$); the electric current flows along the x axis, and the electron conductivity is anisotropic in the xy plane. We shall consider a semiconductor for which

$$L_p \gg L, \tag{2}$$

where L_p is the recombination length [whereas L_p may attain several millimeters [3], L does not exceed a few microns [11] and the condition (2) can be readily satisfied]. We shall consider the case when the anisotropy is induced by intervalley redistribution during heating and one can ignore the intervalley redistribution due to the surfaces [11] and the carrier pinch due to this redistribution [4] (since $d \gtrsim L_p \gg L$).

We shall assume that the induced anisotropy is small and that the contribution to the heating that depends on the y-dependent field component E_y can be neglected. The thermal current can then be ignored and the mobility and diffusion coefficients as well as the recombination and intervalley scattering times, all of which depend on the heating, can be assumed to be independent of y. If the condition (2) is satisfied, the intervalley redistribution is completely determined by the heating and can be found from the condition of balance of the intervalley transitions (see Appendix)

$$n_\alpha = p \frac{\tau_\alpha}{\sum\limits_{\beta=1}^{\lambda} \tau_\beta}, \tag{3}$$

where n_α is the electron density in valley α; τ_α is the intervalley scattering time for electrons in valley α; and λ is the number of valleys.

The electron j_n and hole j_p currents can be expressed in the form[2]

$$j_{n,i} = -p\mu_{n,ik} E_k - D_{n,ik} \frac{dp}{dx_k}, \tag{4}$$

$$j_{p,i} = p\mu_p E_i - D_p \frac{dp}{dx_i}, \tag{5}$$

where

$$\mu_{n,ik} = \frac{\sum\limits_{\alpha=1}^{\lambda} \tau_\alpha \mu_{ik}^{(\alpha)}}{\sum\limits_{\alpha=1}^{\lambda} \tau_\alpha}, \quad D_{n,ik} = \frac{\sum\limits_{\alpha=1}^{\lambda} \tau_\alpha D_{ik}^{(\alpha)}}{\sum\limits_{\alpha=1}^{\lambda} \tau_\alpha}, \tag{6}$$

$\mu_{ik}^{(\alpha)}$ and $D_{ik}^{(\alpha)}$ are the components of the mobility and the diffusion tensors of the electrons in valley α; μ_p and D_p are the hole mobility and diffusion coefficient, respectively.

Under steady-state conditions the hole density distribution along the sample

$$p(y) = p_i \frac{\tau}{\tau_0} \left[1 + \frac{1}{\xi(\gamma)} \left(C_1 e^{\alpha_1 \frac{y}{L_p}} + C_2 e^{\alpha_2 \frac{y}{L_p}} \right) \right] \tag{7}$$

can be found from the continuity equation

$$D \frac{d^2 p}{dy^2} + \frac{a}{2} \frac{eE_x}{kT} D \frac{dp}{dy} - \frac{p}{\tau} + \frac{p_i}{\tau_0} = 0 \tag{8}$$

and the boundary conditions at the surfaces:

$$D \frac{dp}{dy}\bigg|_{y=\pm d} + \frac{a}{2} \frac{eE_x}{kT} Dp(\pm d) \pm s_{2,1} p(\pm d) \mp s_{2,1}^0 p_i = 0. \tag{9}$$

Here, p_i is the equilibrium hole density for $E_x = 0$; τ is the field-dependent recombination time; $\tau_0 = \tau(E_x = 0)$; s_1 and s_2 are the field-dependent recombination velocities at the surfaces $y = \pm d$, $s_{1,2}^0 = s_{1,2}(E_x = 0)$:

[1] Allowance for hole anisotropy does not lead to qualitatively new effects.

[2] As already mentioned, we are neglecting the thermal current.

$$D = \frac{\mu_p D_{n,\,yy} + D_p \mu_{n,\,yy}}{\mu_p + \mu_{n,\,yy}}, \quad a = \frac{2kT\mu_p\mu_{n,\,xy}}{e(\mu_p D_{n,\,yy} + D_p\mu_{n,\,yy})},$$

$$L_p^2 = D\tau, \; \beta = \frac{d}{L_p}, \; \gamma = \frac{aeL_pE_x}{4kT}, \; a_{1,\,2} = -\gamma \pm \sqrt{1+\gamma^2},$$

$$S_{1,\,2} = \frac{L_p s_{1,\,2}}{D}, \quad S_\pm = S_1 \pm S_2, \; f_{1,\,2} = \frac{\tau_0 S^0_{1,\,2}}{\tau S_{1,\,2}}, \; f_\pm = f_1 \pm f_2,$$

$$\xi(\gamma) = [1 + S_1S_2 + S_-\gamma]\,\mathrm{sh}\,2\beta\sqrt{1+\gamma^2} + S_+\sqrt{1+\gamma^2}\,\mathrm{ch}\,2\beta\sqrt{1+\gamma^2}, \tag{10}$$

$$C_1(\gamma) = \Big[-\gamma S_+ + \frac{1}{2}\Big(-S_1S_2f_- - S_+a_1 + \frac{1}{2}S_+f_+a_1 + \frac{1}{2}S_-f_-a_1\Big)\Big]\mathrm{ch}\,a_2\beta$$
$$+ \Big[\gamma(2a_1 + S_-) + \frac{1}{2}\Big(2S_1S_2 - S_1S_2f_+ - S_-a_1 + \frac{1}{2}S_+f_-a_1 + \frac{1}{2}S_-f_+a_1\Big]\mathrm{sh}\,a_2\beta. \tag{11}$$

$$C_2(\gamma) = \Big[\gamma S_+ + \frac{1}{2}\Big(S_1S_2f_- + a_2S_+ - \frac{1}{2}a_2S_+f_+ - \frac{1}{2}a_2S_-f_-\Big)\Big]\mathrm{ch}\,a_1\beta$$
$$+ \Big[-\gamma(2a_2 + S_-) + \frac{1}{2}\Big(-2S_1S_2 + S_1S_2f_+ + S_-a_2 - \frac{1}{2}a_2S_+f_- - \frac{1}{2}a_2S_-f_+\Big)\Big]\mathrm{sh}\,a_1\beta. \tag{12}$$

Knowing the carrier distribution along a sample, we can readily calculate the total current:[3]

$$I_x = e\int_{-d}^{+d}(j_{px} - j_{nx})\,dy$$
$$= 2dE_xe(\mu_p + \mu_{n,\,xx})\,p_i\frac{\tau}{\tau_0}\Big\{1 - \frac{C_1a_2\,\mathrm{sh}\,a_1\beta + C_2a_1\,\mathrm{sh}\,a_2\beta}{\beta\xi(\gamma)}\Big\}. \tag{13}$$

3. We proceed to analyze the expression (13) in the case of large $|\gamma|$ (large $|\gamma|$ can be readily realized even for small a in samples with large L_p [3]).

If the recombination velocity is high at the surface y = +d:

$$S_2 = \infty, \; |\gamma|\max(\beta,\,\beta^{-1}) \gg 1, \tag{14}$$

then carriers are accumulated in the whole of the sample for $\gamma > 0$ and

$$I_x = 2dE_xe(\mu_p + \mu_{n,\,xx})\,p_i\frac{\tau}{\tau_0}\frac{(1 + 2\beta S_1)(\gamma f_2 + \beta)}{\beta(1 + 2\gamma S_1)}. \tag{15}$$

As $S_1 \to 0$ we have

$$I_x = 2dE_xe(\mu_p + \mu_{n,\,xx})\,p_i\frac{\tau}{\tau_0}(\gamma f_2 + \beta)\beta^{-1}. \tag{15'}$$

It follows from Eq. (15') that for $f_2 \ll \beta/\gamma$ the current per unit thickness is the same in a thin sample as in a thick sample.[4] This is because in this case the surface does not furnish carriers for the interior. If $f_2 \gg \beta/\gamma$, the current per unit thickness in a thin sample is larger by a factor of $\gamma f_2/\beta$ than in a thick sample:

$$I_x = ep_i\frac{S_2^0}{S_2}\frac{\mu_p + \mu_{n,\,xx}}{\mu_p + \mu_{n,\,yy}}\mu_p\mu_{n,\,xy}E_x^2\tau\,\mathrm{sign}\,E_x \tag{15''}$$

and, as a rule, the current-voltage characteristic is superlinear. Indeed, even in the special case when an N-type characteristic is observed for thick samples when $\mu_pE_x\tau \sim E_x^{-\alpha}$, $(\alpha > 0)$, the characteristic is superlinear for thin samples when $\frac{S_2^0}{S_2}\mu_{n,\,xy}E_x \sim E_x^\sigma$ and $\sigma > 1 + \alpha$. The N-type characteristic disappears because the surface furnishes many carriers for the interior. It should be noted that if the recombination time and the recombination velocities at the surfaces are independent of the heating, then $f_1 = f_2 = 1 \ll \beta/\gamma$.

If the condition (14) is satisfied, then $\gamma < 0$, carrier depletion occurs in a crystal, and

$$I_x = 2dE_xe(\mu_p + \mu_{n,\,xx})\,p_i\frac{\tau}{\tau_0}\frac{\beta S_1f_1 + \beta^2 + \frac{1}{2}f_2}{\beta(S_1 + 2|\gamma|)}. \tag{16}$$

As $S_1 \to 0$ we have

$$I_x = 2\frac{ep_i}{\tau_0}d^2\frac{(\mu_p + \mu_{n,\,xx})(\mu_p + \mu_{n,\,yy})}{\mu_p\mu_{n,\,xy}}\Big(1 + \frac{f_2}{2\beta^2}\Big)\mathrm{sign}\,E_x. \tag{17}$$

If [see Eq. (17)]

$$\frac{(\mu_p + \mu_{n,\,xx})(\mu_p + \mu_{n,\,yy})}{\mu_p\mu_{n,\,xy}} \propto E^{-g}\ \text{with}\ g > 0, \tag{18}$$

then for $f_2/2\beta^2 < 1$ or for $f_2/2\beta^2 \sim (1/E_x)^n$ with $n > 0$, the current-voltage characteristic is N-type. Note that the condition (18) is always realized at the beginning of heating, when $\mu_{n,\,xy}/\mu_{n,\,yy}$ increases with the field, and the condition for f_2 can be easily satisfied if $f_2 \leq 1$ or if s_2/D increases with the field. Here, the dependence of τ on E is of little significance since the number of carriers in the volume varies as τ because of recombination but their removal from the interior means that their total number varies as $1/L_p^2 = 1/D\tau$. Thus, if the conditions at the surfaces are strongly asymmetrical, the current-voltage characteristic in the case of a pinch under the conditions of the Sasaki effect

[3]Terms $\sim a^2$ are omitted since it is assumed that a is small.

[4]The characteristic length with which the sample thickness is compared is $L_1 = L_p/[(1 + \gamma^2)^{1/2} - |\gamma|]$, the extended recombination length.

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