

5.1.1 SOLUTION:

$$\vec{E} = f(t - z/c_0) \cdot \hat{x} = \exp[-(t - z/c_0)^2 / \tau^2] \cdot \exp[j \cdot 2\pi \nu_0 (t - z/c_0)] \cdot \hat{x} = E_x \cdot \hat{x}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{aligned} \nabla \times \vec{E} &= \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix} = \frac{\partial E_x}{\partial z} \cdot \hat{y} - \frac{\partial E_x}{\partial y} \cdot \hat{z} \\ &= \frac{\partial E_x}{\partial z} \cdot \hat{y} \\ &= E_x \left[\frac{2(t - z/c_0)}{\tau^2 c_0} - \frac{j 2\pi \nu_0}{c_0} \right] \hat{y} \end{aligned}$$

$$\text{So, } \frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = -E_x \left[\frac{2(t - z/c_0)}{\tau^2 c_0} - \frac{j 2\pi \nu_0}{c_0} \right] \hat{y}$$

$$\vec{B} = - \int E_x \left[\frac{2(t - z/c_0)}{\tau^2 c_0} - \frac{j 2\pi \nu_0}{c_0} \right] \cdot dt \cdot \hat{y}$$

$$\text{Where } E_x = \exp[-(t - z/c_0)^2 / \tau^2] \cdot \exp[j \cdot 2\pi \nu_0 (t - z/c_0)] \cdot \hat{x}$$

Therefore, we can find the wave travels along the \hat{z} direction, and the electrical field is decaying.

53. (a) $\vec{E}(\vec{r}) = E_0 \cdot \sin \beta y \cdot \exp(-j\beta z) \cdot \hat{x}$
 $= E_0 \left[\frac{1}{2j} [e^{j\beta y} - e^{-j\beta y}] \right] \cdot \exp(-j\beta z) \cdot \hat{x}$
 $= \frac{E_0}{2j} [e^{-j\beta(z-y)} - e^{-j\beta(z+y)}] \cdot \hat{x}$

So, $\beta \sqrt{2} = \frac{2\pi}{\lambda_0} \Rightarrow \beta = \frac{\sqrt{2} \cdot \pi}{\lambda_0}$

(b) $\nabla \times \vec{E} = -j\omega \mu_0 \vec{H}$
 So, we can get $\vec{H} = \frac{1}{-j\omega \mu_0} \cdot \nabla \times \vec{E}$

$= \frac{1}{-j\omega \mu_0} \left[\frac{\partial E_x}{\partial z} \hat{y} - \frac{\partial E_x}{\partial y} \hat{z} \right]$

$= \frac{1}{-j\omega \mu_0} \left[(-j\beta) \cdot E_0 \cdot \sin \beta y \cdot \exp(-j\beta z) \cdot \hat{y} - \beta E_0 \cdot \cos \beta y \cdot \exp(-j\beta z) \cdot \hat{z} \right]$

$= \frac{\beta E_0 \cdot \sin \beta y \cdot \exp(-j\beta z)}{\omega \mu_0} \cdot \hat{y} - j \frac{\beta E_0 \cdot \cos \beta y \cdot \exp(-j\beta z)}{\omega \mu_0} \cdot \hat{z}$

(c) $\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{\beta E_0^2 \cdot \sin^2 \beta y}{2\omega \mu_0} \cdot \hat{z} - j \frac{\beta E_0^2 \cdot \sin \beta y \cdot \cos \beta y}{\omega \mu_0} \cdot \hat{y}$

the direction of flow of optical power is along \hat{z} direction and along \hat{y} direction, and the phase difference between them is 90° .

And because the optical intensity is the magnitude of the vector $\text{Re}\{\vec{S}\}$. So, the direction is along the \hat{z} direction, and the average optical power flowing along the y direction is zero.

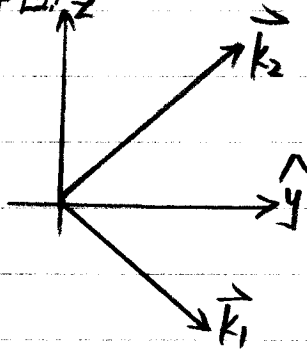
(d) Because $\vec{E}(\vec{r}) = \left[\frac{E_0}{2j} \cdot e^{-j\beta(z-y)} + \frac{E_0}{2} \cdot e^{-j\beta(z+y)} \right] \cdot \hat{x} = E_1 \cdot \hat{x} + E_2 \cdot \hat{x}$

where $E_1 = \frac{E_0}{2j} \cdot e^{-j\beta(z-y)}$ so, $\vec{k}_1 = \beta \cdot \hat{z} - \beta \cdot \hat{y}$

$E_2 = \frac{E_0}{2} \cdot e^{-j\beta(z+y)}$ so, $\vec{k}_2 = \beta \cdot \hat{z} + \beta \cdot \hat{y}$

So, the direction of propagation of E_1, H_1 is.

the direction of propagation of E_2, H_2 is.



$$54.1 \text{ a) } I = P/A = 1/10^8 = 10^8 \text{ W/m}^2, \quad \eta_0 = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} = 120\pi$$

$$I = \frac{|E_0|^2}{2\eta} \Rightarrow E_0 = \sqrt{2\eta I} = 2.746 \times 10^5 \text{ V/m}$$

$$\text{b) } I_0 = \frac{2P}{\pi W_0^2} = \frac{2}{\pi \times 10^{-8}} = \frac{2}{\pi} \times 10^8$$

$$E_0 = \sqrt{2\eta I} = \sqrt{480 \times 10^8} = 2.191 \times 10^5 \text{ V/m}$$