$$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$

$$\nabla \times \vec{h} = \vec{j} + \frac{\partial \vec{d}}{\partial t}$$

$$\nabla \cdot \vec{d} = \rho$$

$$\nabla \cdot \vec{b} = 0$$

Continuity: $\nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$

Constitutive relations:

$$\vec{d} = \epsilon_0 \vec{e} + \vec{p} \qquad \vec{b} = \mu_0(\vec{h} + \vec{m})$$

Let us assume ① no currents or charge.

$$\vec{j} = 0 \qquad \rho = 0.$$

② non-magnetic $\Rightarrow \mu = \mu_0 \quad \vec{m} = 0.$

$$\nabla \times (\nabla \times \vec{e}) = -\frac{\partial (\nabla \times \vec{b})}{\partial t}$$

$$\nabla \times \nabla \times \vec{e} = -\frac{\partial (\nabla \times \mu_0 \vec{h})}{\partial t} = -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial \vec{d}}{\partial t}\right)$$

$$\vec{d} = \epsilon_0 \vec{e} + \vec{p}$$

$$\nabla \times \nabla \times \vec{e} = -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}(\epsilon_0 \vec{e} + \vec{p})\right)$$

$$\nabla(\nabla \cdot \vec{e}) - \nabla^2 \vec{e} = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\vec{e} + \mu_0 \frac{\partial^2}{\partial t^2}\vec{p}$$

$$\nabla^2 \vec{e} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{e}}{\partial t^2} - \nabla(\nabla \cdot \vec{e}) = \mu_0 \frac{\partial^2}{\partial t^2}\vec{p}.$$

Free space : $\nabla \cdot \vec{e} = 0 \qquad (\nabla \cdot \vec{d} = 0)$.

$\vec{p} = 0$

$$\Rightarrow \nabla^2 \vec{e} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{e}}{\partial t^2} = 0.$$

or
$$\nabla^2 \vec{e} - \frac{1}{c_0^2} \frac{\partial^2 \vec{e}}{\partial t^2} = 0 \qquad c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Now suppose $\vec{p} = \epsilon_0 \chi \vec{e}$ (a material with an index)

$$\Rightarrow \nabla^2 \vec{e} - \frac{1}{c_0^2} \frac{\partial^2 \vec{e}}{\partial t^2} - \nabla(\nabla \cdot \vec{e}) = \mu_0 \frac{\partial^2}{\partial t^2} (\epsilon_0 \chi \vec{e})$$

if non-dispersive
$$\frac{\partial^2}{\partial t^2} (\epsilon_0 \chi \vec{e}) = \epsilon_0 \chi \frac{\partial^2 \vec{e}}{\partial t^2}.$$

$$\nabla \cdot \vec{d} = 0 \Rightarrow \nabla \cdot (\epsilon_0 \vec{e} + \epsilon_0 \chi \vec{e}) = 0. \Rightarrow \epsilon_0 (1+\chi) \nabla \cdot \vec{e} = 0$$

if material is isotropic & homogenuus (χ is a scalar) $\Rightarrow \nabla \cdot \vec{e} = 0$.

aside: (Remember inhomogenuus $\Rightarrow \chi = \chi(r)$
anisotropic $\Rightarrow \chi = \overleftrightarrow{\chi} \leftarrow$ tensor)

$\therefore$ for $\vec{p} = \epsilon_0 \chi \vec{e}$.

$$\Rightarrow \nabla^2 \vec{e} - \frac{1}{c_0^2} \frac{\partial^2 \vec{e}}{\partial t^2} - \mu_0 \epsilon_0 \chi \frac{\partial^2 \vec{e}}{\partial t^2} = 0.$$

$$\nabla^2 \vec{e} - \left(\frac{1}{c_0^2} + \epsilon_0 \chi \mu_0 \right) \frac{\partial^2 \vec{e}}{\partial t^2} = 0$$

$$\left.\begin{aligned} &\nabla^2 \vec{e} - \underbrace{\left(\mu_0 \epsilon_0 + \epsilon_0 \chi \mu_0 \right)}_{\frac{1}{c^2}} \frac{\partial^2 \vec{e}}{\partial t^2} = 0. \\ &\frac{1}{c^2} = \mu_0 \underbrace{(\epsilon_0)(1+\chi)}_{\epsilon}. \end{aligned}\right\} \text{Helmholts Eqn.}$$

$$\therefore \frac{1}{c^2} = \mu_0 \epsilon$$

We usually say light travels at $\frac{c_0}{n}$ in a medium $\Rightarrow$ $\frac{1}{c^2} = \frac{n^2}{c_0^2} = \mu_0 \epsilon_0 (1+\chi)$.

or $\boxed{n = \sqrt{1+\chi}.}$

Now lets do non-linear (still non-dispersive, isotropic and homogenuus).

$$\Rightarrow \vec{P} = \vec{P}_{NL} = \vec{P}_0 + a_2 e^2 + a_3 e^3$$

(see page 7 of previous notes).

Inhomogenuus.

$$\Rightarrow \quad \epsilon = \epsilon(\vec{r}) = \epsilon_0 (1 + \chi(\vec{r})).$$

Approximate as locally homogenuus. (say at r_0).
use Helmholtz eqn with $\epsilon = \epsilon(r_0)$

Dispersive: $\epsilon \rightarrow \epsilon(\nu)$. (see page 9 of previous notes).