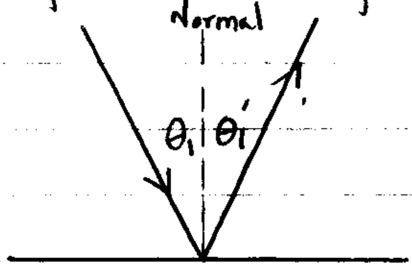
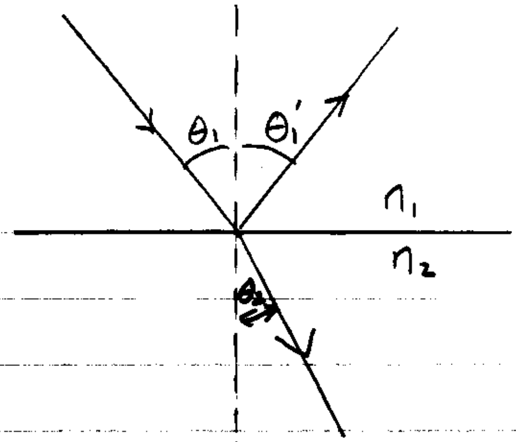
Lenses and Geometric Optics

Reflection and Refraction



Reflection $\theta_1 = \theta_1'$



$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \text{constant}$$

v_2, v_1 is speed of light in medium 2 or 1.

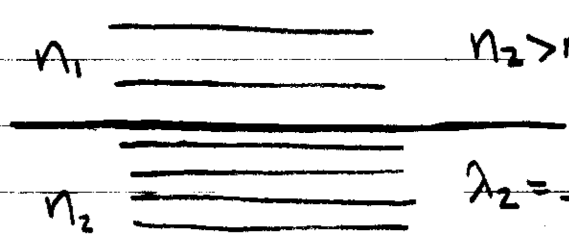
$$v = \frac{c}{n} \Rightarrow \frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}$$

Snell's law. $$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Law of Refraction

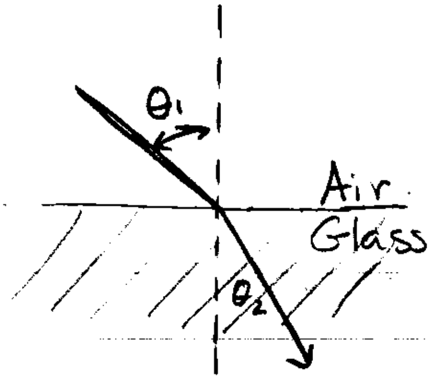
$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}.$$

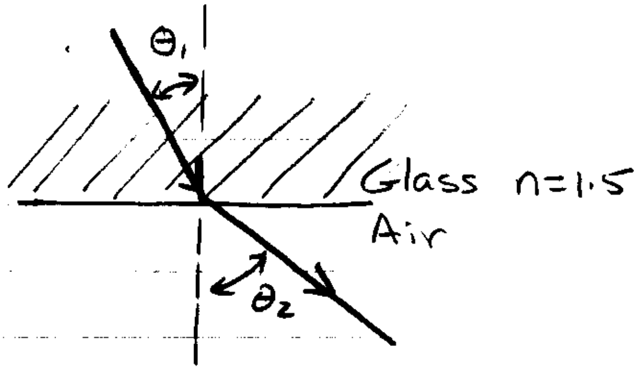
Frequency remains constant $\nu = \frac{c}{\lambda} \Rightarrow \lambda_m = \frac{\lambda}{n}.$



$n_2 > n_1$

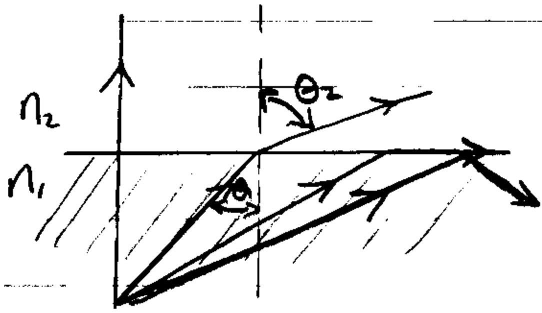
$\lambda_2 = \frac{\lambda}{n_2}.$





$\theta_1 > \theta_2 \quad n_1 < n_2$ $\qquad$ $\theta_2 > \theta_1 \quad n_2 < n_1$

Total Internal Reflection

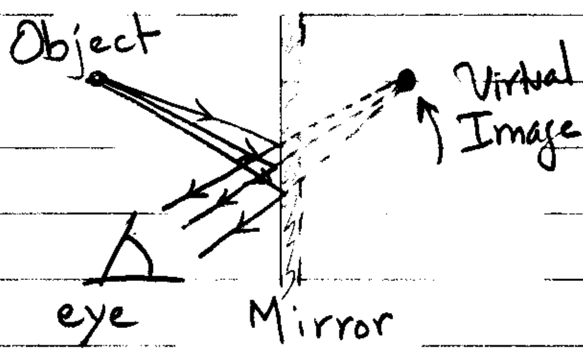


$n_1 > n_2$ $\qquad$ $\sin\theta_c = \frac{n_2}{n_1}$

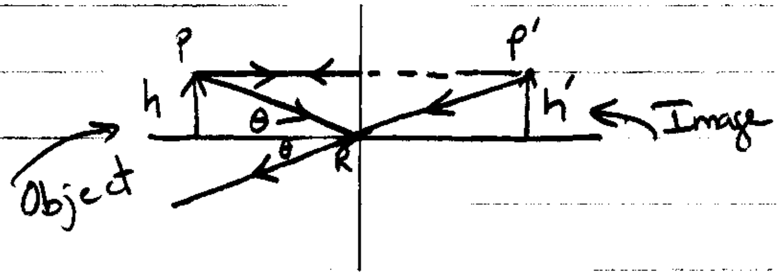
i.e. $\theta_2 = 90^\circ$

For $\theta > \theta_c$ light stays completely within n_1

Geometric Optics



Virtual image – light does not really pass through here



Use at least two rays.

1. perpendicular to mirror
2. Follow PR.

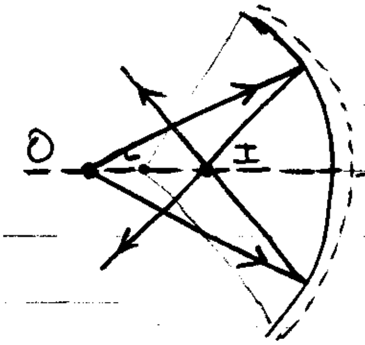
Note: Image Reversed.

Magnification

$$M \equiv \frac{\text{Image Height}}{\text{Object Height}} = \frac{h'}{h}$$

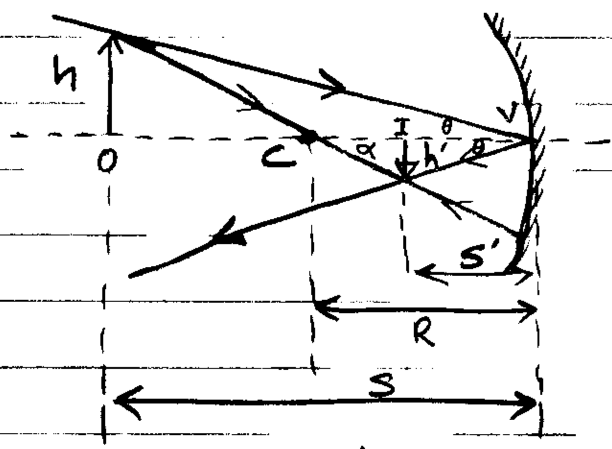
For a mirror $M = 1$

Concave Mirrors.



• Real Image Formed at I

Paraxial rays : close to principal axis.



$$M = \frac{h'}{h} = -\frac{s'}{s}$$

$$\tan\alpha = \frac{h}{s-R}$$

$$\tan\alpha = \frac{-h'}{R-s'}$$

$$\frac{h'}{h} = -\frac{R-s'}{s-R} = -\frac{s'}{s}$$

$$\Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \qquad \text{Mirror Equation}$$

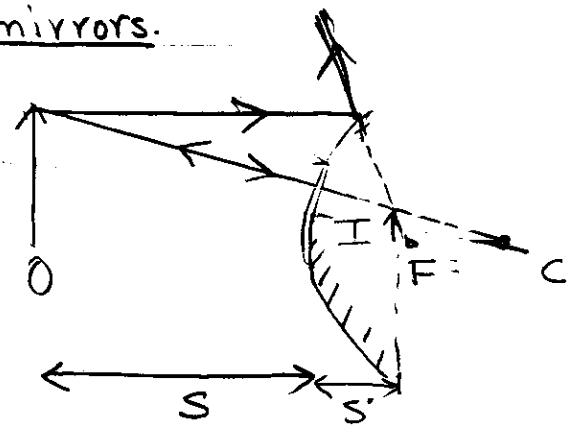
Suppose $s \to \infty \quad = \quad \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$

Image point is half-way between center of curvature

f = focal length.

$$\Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Convex mirrors.



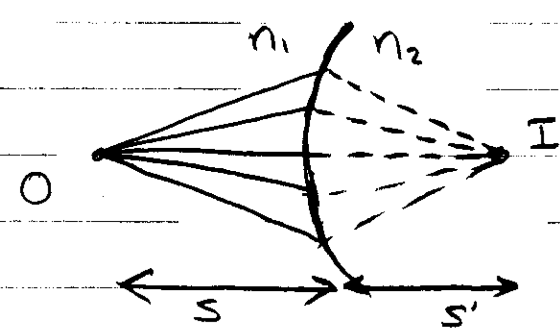
Sign conventions.

s + if object is real.
s − if object is virtual
s' + if image is real
s' − if image is virtual

f and R are + if concave mirror
f and R are − if convex mirror

if $M>0$ image is erect
if $M<0$ image is inverted.

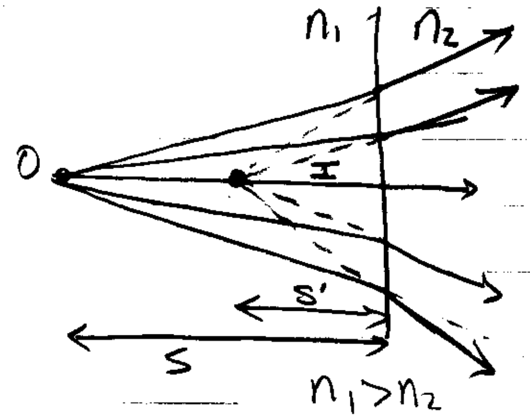
Images formed by refraction.



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

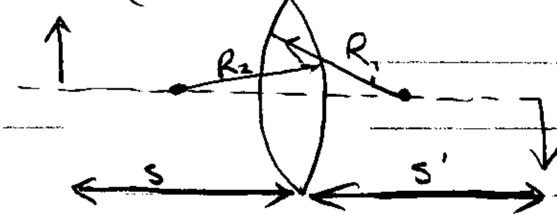
$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}.$$

Plane Refracting Surfaces.



$$\frac{n_1}{s} = -\frac{n_2}{s'} \qquad s' = -\frac{n_2}{n_1} s.$$

Thin lenses. (biconvex)



$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

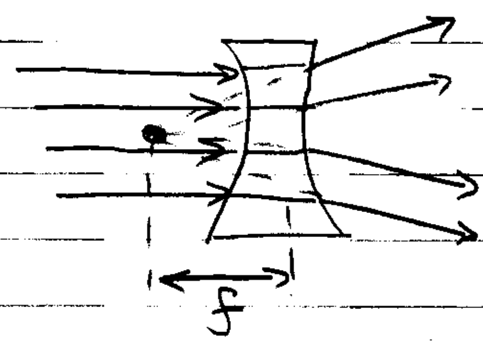
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Lensmakers' equation.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$M = \frac{h'}{h} = -\frac{s'}{s}.$$

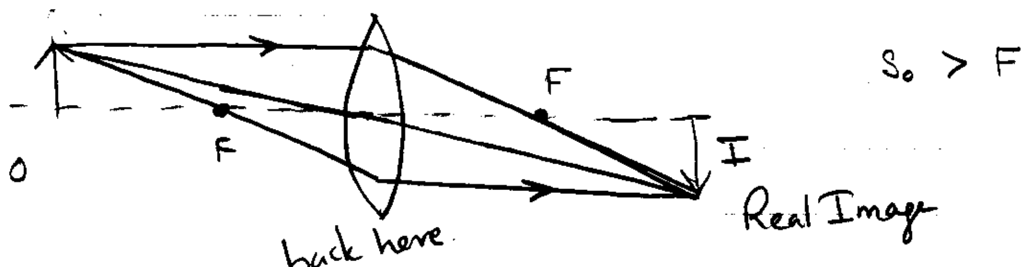
biconcave. f is negative.

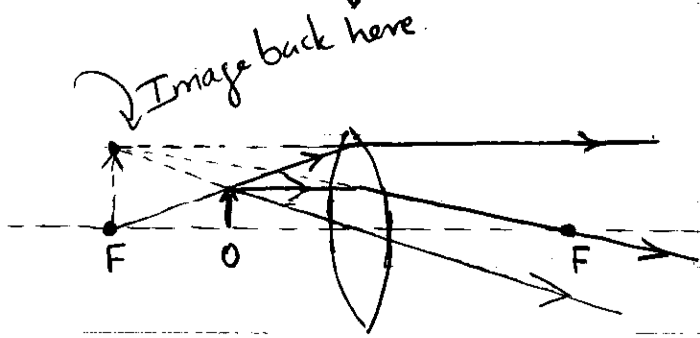


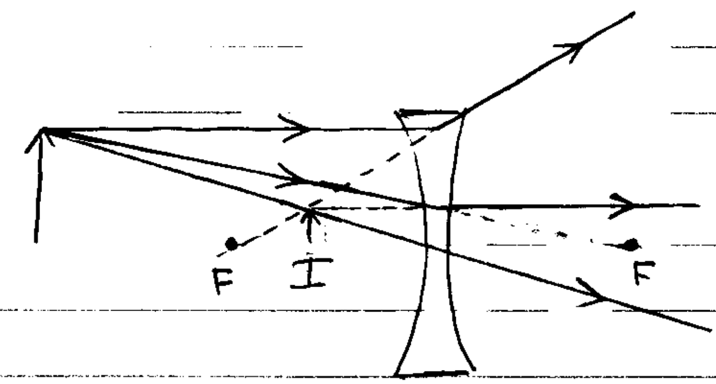
Sign Convention.

s + object in front of lens
s − object in back
s' + image in front
s' − image in back
R_1 and R_2 + if center of curvature is in back of lens
R_1 and R_2 − if center . . . in front of lens.

Ray diagrams for Thin Lenses







1. First ray parallel to optic axis. After being refracted this ray passes through one of the focal points.
2. Second ray - drawn to center of the lens. This ray continues in a straight line
3. Third ray drawn through F, and emerges from the lens parallel to optic axis.

Combination of Thin Lenses.

Two thin lenses used to form an image

① Image of first lens is calculated as if the second lens was not there. Light approaches the second lens as if it came from the image.

② Image of first lens treated as Object of second lens.

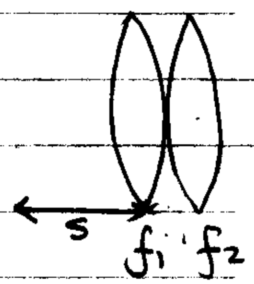
③ Image of second lens is the final image of the system

* if Image of first lens is to the right of the second lens, treat it as a virtual object for the second lens, i.e. s is negative.

Overall magnification is the product of the magnifications of the separate lenses.

i.e. $M = M_1 M_2 \ldots$

Consider



For l_1

$$\frac{1}{s} + \frac{1}{s_1'} = \frac{1}{f_1} \quad (a)$$

(Notice s_1' must be to the right of l_2).
⇒ s_1' is object to l_2 and is negative.

for l_2

$$-\frac{1}{s_1'} + \frac{1}{s'} = \frac{1}{f_2} \quad (b)$$

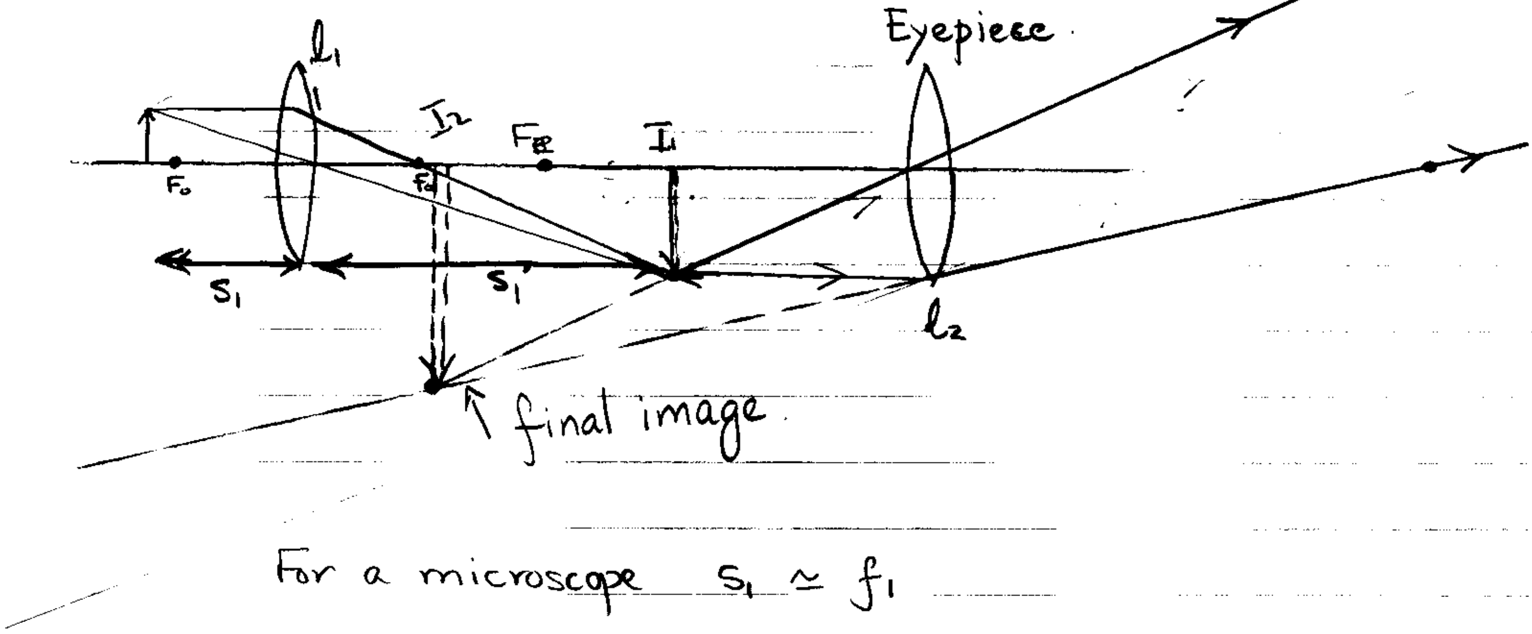
s' if final image a + b ⇒ $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_1} + \frac{1}{f_2}$.

↑ object locat. ↑ final image

⇒ Two thin lenses in contact act as one composite lens with focal length

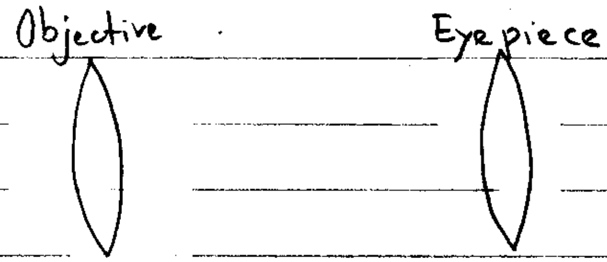
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Microscopes



For a microscope $s_1 \simeq f_1$

Telescope.



$s_1 \gg f_1$