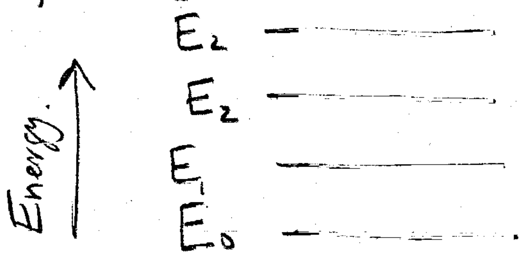
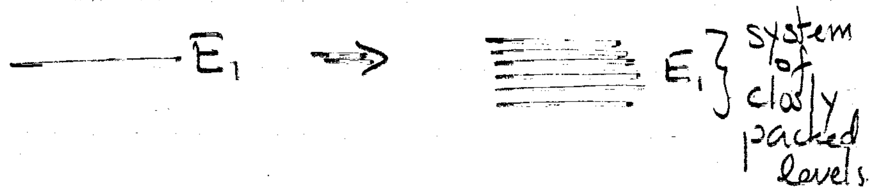
Radiation and Atomic Systems

Energy Levels.

All systems can be represented by a system of energy levels:



Each level can be more complex eg.



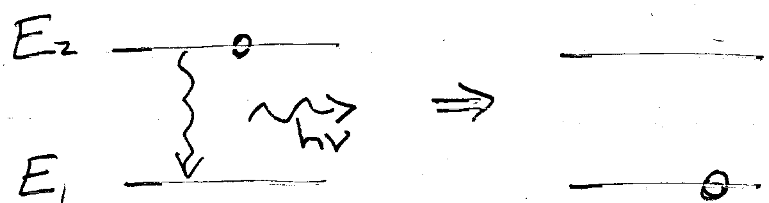
These discrete energy levels are called Eigenstates. Each eigenstate can be written as.

$$\psi_i(\vec{r},t) = u_i(\vec{r})\, e^{-iE_i t/\hbar}$$

where $|u_i(\vec{r})|^2 dx\,dy\,dz$ is the probability of finding an electron once in state i, in the volume element $dx, dy\, dz$, which is centered at $\vec{r}$., E_i is the energy of state i, $\hbar = \frac{h}{2\pi}$

h - Plancks constant $= 6.626 \times 10^{-34}$ Joule-second.

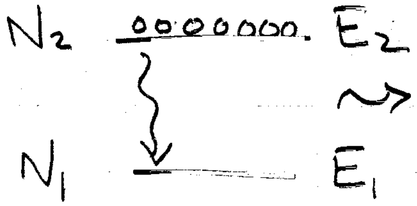
Spontaneous Emission.



$$h\nu = E_2 - E_1 \Big\}.\ \text{photon energy.}$$

Photon given off by an atom relaxing from state 2 to state 1.

Spontaneous lifetime.



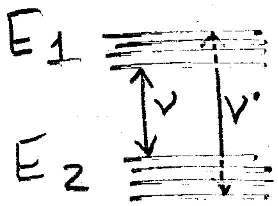
$$\frac{dN_2}{dt} = A_{21} N_2 \equiv \frac{N_2}{\tau^{21}_{spont}}$$

$$\tau^{21}_{spont} = A_{21}^{-1}$$

spontaneous lifetime   spontaneous transition rate

Spontaneous emission only occurs from higher energy states to lower energy states.

Lineshape function. - Homogeneous and Inhomogeneous broadening.



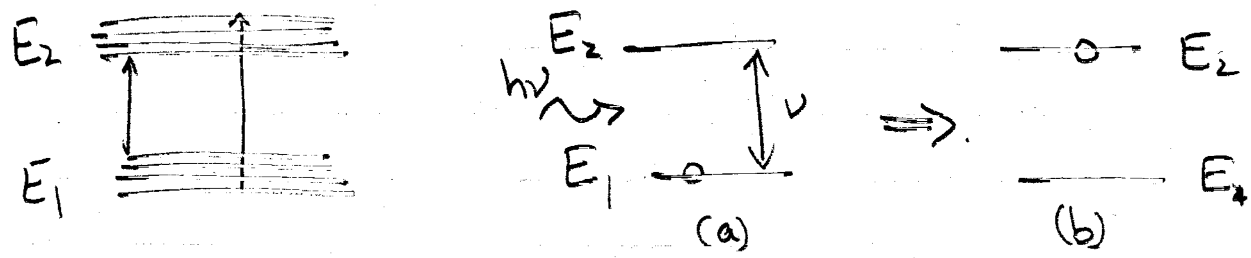
distribution of possible frequencies of emission.

$g(\nu)$ - lineshape function.

$$\int_{-\infty}^{\infty} g(\nu)\, d\nu = 1.$$

$g(\nu)\, d\nu$ is the probability that a given spontaneous emission from level 2 to level 1 will result in a photon with energy between ν and $\nu + d\nu$.

Additionally, $g(\nu)$ describes the absorption process. probability



Homogenous Broadening.

$$e(t) = E_0 e^{-t/\tau} \cos \omega_0 t = \frac{E_0}{2}\left[e^{i(\omega_0 + i\sigma/2)t} + e^{-i(\omega_0 - i\frac{\sigma}{2})t} \right]$$

$\frac{\sigma}{2} = \frac{1}{\tau}$ is the field decay rate.

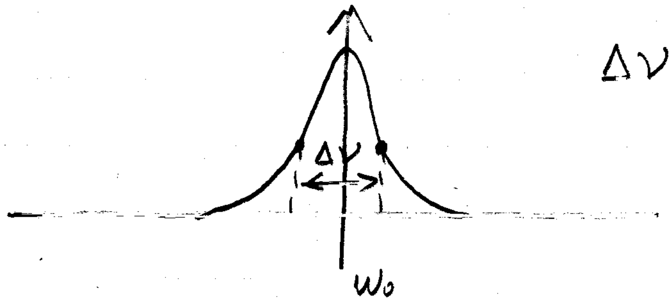
Fourier transform of $e(t)$

$$\int_0^\infty e(t)\, e^{-i\omega t}\, dt$$

$$= E_0 \left[\frac{i}{(\omega_0 - \omega + i\frac{\sigma}{2})} - \frac{i}{(\omega_0 + \omega - i\frac{\sigma}{2})} \right]$$

Spectral density $|E(\omega)|^2$

$$|E(\omega)|^2 \propto \frac{1}{(\omega - \omega_0)^2 + \left(\frac{\sigma}{2}\right)^2}$$



$$\Delta\nu = \frac{\sigma}{2\pi} = \frac{1}{\pi\tau}$$

In general $$\Delta\nu = \frac{1}{\pi}\left(\underset{\substack{\uparrow\\ \text{upper}\\ \text{state}}}{\tau_u^{-1}} + \underset{\substack{\uparrow\\ \text{lower}\\ \text{state}}}{\frac{1}{\tau_l}} + \underset{\substack{\uparrow\\ \text{upper}\\ \text{state}\\ \text{elastic}\\ \text{collisions}}}{\frac{1}{\tau_{cu}}} + \underset{\substack{\text{lower}\\ \text{state}\\ \text{elastic}\\ \text{collisions}}}{\frac{1}{\tau_{cl}}}\right)$$

$\therefore$ $$g(\nu) = \frac{\Delta\nu}{2\pi\left[(\nu-\nu_0)^2 + \left(\frac{\Delta\nu}{2}\right)^2\right]}.$$ ← homogenous broadening.

all atoms indistinguishable.

<u>Homogenous broadening.</u>

1. spontaneous lifetime broadening.
2. Collisions of atoms with photons.
3. Pressure broadening in a gas.

<u>Inhomogenous broadening.</u>

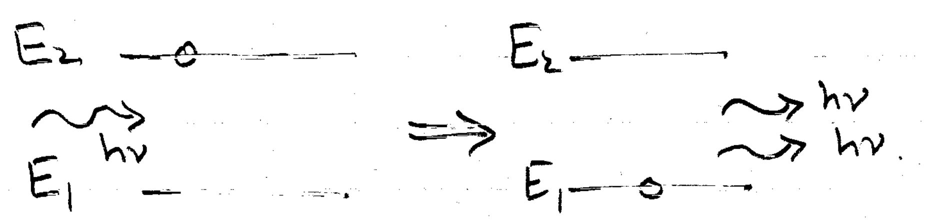
atoms are distinguishable.

$$\Delta\nu_D = 2\nu_0\sqrt{\frac{2kT}{Mc^2}\ln 2}$$

k: Boltzmann constant
T: temperature
c: speed of light
M: atomic mass.

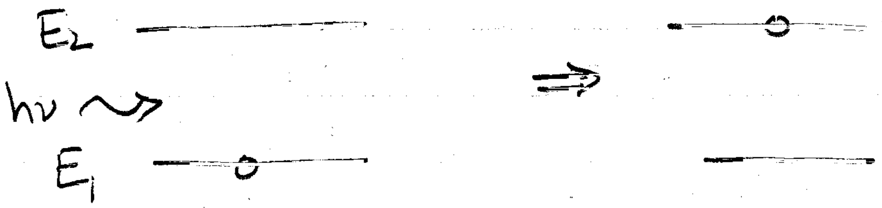
$$g(\nu) = \frac{2(\ln 2)^{1/2}}{\pi^{1/2}\Delta\nu_D}\cdot e^{-\left[4\ln 2\frac{(\nu-\nu_0)^2}{\Delta\nu_D^2}\right]}$$

Induced transitions.



Stimulated Emission.

(W'_{21}) induced $= B_{21}\,\rho(\nu)$. $\rho(\nu)$: energy density per unit frequency of incident light



Absorption.

(W'_{12}) induced $= B_{12}\,\rho(\nu)$.

Total downward transition $W'_{21} = B_{21}\rho(\nu) + A_{21}$

upward transition $W'_{12} = B_{12}\,\rho(\nu)$.

$$\rho(\nu) = \frac{8\pi n^3 h\nu^3}{c^3}\,\frac{1}{e^{h\nu/kT}-1}.$$

Thermal equilibrium:

$$N_2 W'_{21} = N_1 W'_{12}$$

N_2 & N_1 are populations of level 2 & 1.

$$\Rightarrow \quad N_2\left[B_{12}\,\rho(\nu) + A_{21}\right] = N_1 B_{12}\,\rho(\nu).$$

in thermal equilibrium $\quad \frac{N_2}{N_1} = e^{-h\nu/kT}$ substitute

Only statisfied if $\quad B_{12} = B_{21} \quad \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}.$

Einstein coefficients B_{12}, B_{21}, A_{21}.

$$W_i' = \frac{A_{21} c^3}{8\pi n^3 h\nu^3} \rho(\nu) = \frac{c^3}{8\pi n^3 h \nu^3 \tau_{spont}} \rho(\nu).$$

$\Rightarrow$ transition rate from $2 \to 1$ is identical to $1 \to 2$.

<u>Include lineshape</u>.

$$W_i' = \frac{c^3}{8\pi n^3 h \tau_{spont}} \int_{-\infty}^{\infty} \frac{\rho(\nu)\, g(\nu)}{\nu^3} d\nu$$

Realize if $\rho(\nu)$ is a δ function or $g(\nu)$ is a δ function.

we can use $\int_{-\infty}^{\infty} \delta(x-x_0) g(x)\, dx = g(x_0)$.

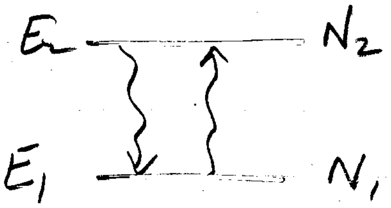
Let us assume a laser of frequency ν is incident.
then $\rho(\nu') = \rho_\nu\, \delta(\nu' - \nu)$.

$$\Rightarrow \quad W_i'(\nu) = \frac{c^3 \rho_\nu}{8\pi n^3 h \tau_{spont}\, \nu^3} g(\nu).$$

For a laser $I_\nu = \frac{c \rho_\nu}{n}$ (watts/m²)

$$\Rightarrow \quad W_i(\nu) = \frac{A_{21} c^2 I_\nu}{8\pi n^2 h \nu^3} g(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h \nu \tau_{spont}} g(\nu).$$

Absorption and Amplification.



Assume N_2 atoms in state E_2
N_1 atoms in state E_1

power

$$\frac{P}{\text{Volume}} = (N_2 - N_1)\, W_i\, h\nu$$

Radiation added coherently.

$$\therefore \quad \frac{dI_\nu}{dz} = (N_2 - N_1)\, \frac{c^2 g(\nu)}{8\pi n^2 \nu^2 \tau_{spont}}\, I_\nu .$$

$$\Rightarrow \quad I_\nu(z) = I_\nu(0)\, e^{\gamma(\nu) z}$$

$$\gamma(\nu) = (N_2 - N_1)\, \frac{c^2}{8\pi n^2 \cdot \nu^2 \tau_{spont}}\, g(\nu).$$

Gain: $N_2 > N_1 \Rightarrow$ exponential growth.
$N_1 > N_1 \Rightarrow$ attenuation.

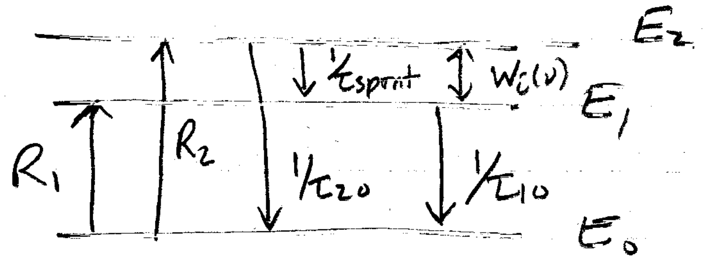
Thermal Equilibrium $\quad \frac{N_2}{N_1} = e^{-h\nu/kT}$ (system always absorbing.)

However, $N_2 > N_1 \Rightarrow$ population inverted.
temperature negative.

Read sections 5.4, 5.5.

<u>Gain Saturation in Homogeneous Media.</u>

Rate Equations.



$$\frac{1}{\tau_2} = \frac{1}{\tau_{spon}} + \frac{1}{\tau_{20}} \qquad W_i(\nu) = \frac{\lambda^2 g(\nu)}{8\pi n^2 h\nu \tau_{spont}} I_\nu$$

R_1, R_2 are the pumping rates ($m^{-3} s^{-1}$).

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - (N_2 - N_1) W_i(\nu).$$

$$\frac{dN_1}{dt} = R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{spont}} + (N_2 - N_1) W_i(\nu).$$

Steady state $\quad \dfrac{dN_2}{dt} = \dfrac{dN_1}{dt} = 0.$

$$\therefore \quad N_2 - N_1 = \Delta N = \frac{R_2\tau_2 - (R_1 + \delta R_2)\tau_1}{1 + [\tau_2 + (1-\delta)\tau_1]\, W_i(\nu)}.$$

$$\delta = \frac{\tau_2}{\tau_{spont}} \qquad \text{if } W_i(\nu) = 0 \qquad \Delta N^\circ = R_2\tau_2 - (R_1 + \delta R_2)\tau_1$$

$$N_2 - N_1 = \frac{\Delta N^\circ}{1 + \phi\,\tau_{spont}\,W_i(\nu)}$$

where $\phi = \delta\left[1 + (1-\delta)\frac{\tau_1}{\tau_2}\right]$

For an efficient laser system $\tau_2 \simeq \tau_{spont}$, $\delta \simeq 1$, $\tau_1 << \tau_2$, $\phi \simeq 1$.

$$N_2 - N_1 = \frac{\Delta N^\circ}{1 + \frac{I_\nu}{I_s(\nu)}}$$

$I_s(\nu)$ – saturation intensity.

$$I_s(\nu) = \frac{8\pi n^2 h\nu}{\phi\,\lambda^2 g(\nu)} = \frac{8\pi n^2 h\nu}{(\tau_2/\tau_{spont})\,\lambda^2 g(\nu)} = \frac{8\pi n^2 h\nu\,\Delta\nu}{(\tau_2/\tau_{spont})\,\lambda^2}$$

$$\gamma(\nu) = \frac{1}{1 + I_\nu/I_\nu(s)}\left(\frac{\Delta N^\circ \lambda^2}{8\pi n^2 \tau_{spont}}\right) g(\nu)$$

$$= \frac{\gamma_0(\nu)}{1 + \frac{I_\nu}{I_s(\nu)}}.$$

Inhomogeneous system

$$g(\nu)\,d\nu = \left[\int_{-\infty}^{\infty} p(\xi)\, g^{\xi}(\nu)\, d\xi \right] d\nu .$$

more complicated, similar equations.

Bottom line:

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{\sqrt{1 + \frac{I_\nu}{I_s}}}$$

← gain saturation sets in more slowly.

$$I_s(\nu) = \frac{4\pi^2 n^2 h\nu\, \Delta\nu}{\phi\, \lambda^2}$$