

Figure 6.3-9 (a) Variation of the refractive index $n(\theta)$ of the extraordinary wave with θ (the angle between the direction of propagation and the optic axis). (b) The \mathbf{E} and \mathbf{D} vectors for the ordinary wave (*o* wave) and the extraordinary wave (*e* wave). The circle with a dot at the center signifies that the direction of the vector is out of the plane of the paper, toward the reader.

Special Case: Uniaxial Crystals

In uniaxial crystals ($n_1 = n_2 = n_o$ and $n_3 = n_e$) the index ellipsoid is an ellipsoid of revolution. For a wave traveling at an angle θ with the optic axis the index ellipse has half-lengths n_o and $n(\theta)$, where

$$\frac{1}{n^2(\theta)} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2},$$

(6.3-15)

Refractive Index of the Extraordinary Wave

so that the normal modes have refractive indices $n_a = n_o$ and $n_b = n(\theta)$. The first mode, called the **ordinary wave**, has a refractive index n_o regardless of θ . The second mode, called the **extraordinary wave**, has a refractive index $n(\theta)$ varying from n_o when $\theta = 0^\circ$, to n_e when $\theta = 90^\circ$, in accordance with the ellipse shown in Fig. 6.3-9(a). The vector \mathbf{D} of the ordinary wave is normal to the plane defined by the optic axis and the direction of wave propagation \mathbf{k} , and the vectors \mathbf{D} and \mathbf{E} are parallel. The extraordinary wave, on the other hand, has a vector \mathbf{D} in the k - z plane, which is normal to \mathbf{k} , and \mathbf{E} is not parallel to \mathbf{D} . These vectors are illustrated in Fig. 6.3-9(b).

D. Rays, Wavefronts, and Energy Transport

The nature of waves in anisotropic media is best explained by examining the \mathbf{k} surface $\omega = \omega(k_1, k_2, k_3)$ obtained by equating the determinant of the matrix in (6.3-11) to zero as illustrated in Fig. 6.3-8. The \mathbf{k} surface describes the variation of the phase velocity $c = \omega/k$ with the direction $\hat{\mathbf{u}}$. The distance from the origin to the \mathbf{k} surface in the direction of $\hat{\mathbf{u}}$ is therefore inversely proportional to the phase velocity.

The group velocity may also be determined from the \mathbf{k} surface. In analogy with the group velocity $v = d\omega/dk$, which describes the velocity with which light pulses (wavepackets) travel (see Sec. 5.6), the group velocity for *rays* (localized beams, or spatial wavepackets) is the vector $\mathbf{v} = \nabla_{\mathbf{k}}\omega(\mathbf{k})$, the gradient of ω with respect to \mathbf{k} . Since the \mathbf{k} surface is the surface $\omega(k_1, k_2, k_3) = \text{constant}$, \mathbf{v} must be normal to the \mathbf{k} surface. Thus rays travel along directions normal to the \mathbf{k} surface.

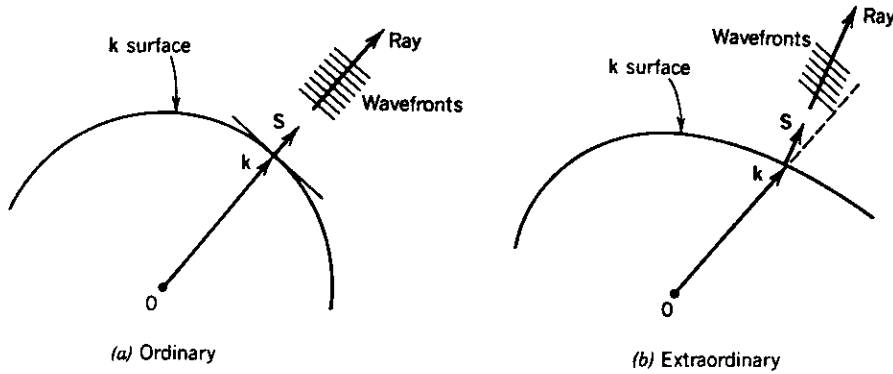


Figure 6.3-10 Rays and wavefronts for (a) spherical k surface, and (b) nonspherical k surface.

The Poynting vector $\mathbf{S} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^*$ is also normal to the k surface. This can be shown by assuming a fixed ω and two vectors \mathbf{k} and $\mathbf{k} + \Delta\mathbf{k}$ lying on the k surface. By taking the differential of (6.3-9) and (6.3-8) and using certain vector identities, it can be shown that $\Delta\mathbf{k} \cdot \mathbf{S} = 0$, so that \mathbf{S} is normal to the k surface. Consequently, \mathbf{S} is also parallel to the group velocity vector \mathbf{v} . The wavefronts are perpendicular to the wavevector \mathbf{k} (since the phase of the wave is $\mathbf{k} \cdot \mathbf{r}$). The wavefront normals are therefore parallel to the wavevector \mathbf{k} .

If the k surface is a sphere, as in isotropic media, for example, the vectors \mathbf{v} , \mathbf{S} , and \mathbf{k} are all parallel, indicating that rays are parallel to the wavefront normal \mathbf{k} and energy flows in the same direction, as illustrated in Fig. 6.3-10(a). On the other hand, if the k surface is not normal to the wavevector \mathbf{k} , as illustrated in Fig. 6.3-10(b), the rays and the direction of energy transport are not orthogonal to the wavefronts. Rays then have the "extraordinary" property of traveling at an oblique angle with their wavefronts [Fig. 6.3-10(b)].

Special Case: Uniaxial Crystals

In uniaxial crystals ($n_1 = n_2 = n_o$ and $n_3 = n_e$), the equation of the k surface $\omega = \omega(k_1, k_2, k_3)$ simplifies to

$$(k^2 - n_o^2 k_o^2) \left(\frac{k_1^2 + k_2^2}{n_e^2} + \frac{k_3^2}{n_o^2} - k_o^2 \right) = 0, \quad (6.3-16)$$

which has two solutions: a sphere,

$$k = n_o k_o, \quad (6.3-17)$$

and an ellipsoid of revolution,

$$\frac{k_1^2 + k_2^2}{n_e^2} + \frac{k_3^2}{n_o^2} = k_o^2. \quad (6.3-18)$$

Because of symmetry about the z axis (optic axis), there is no loss of generality in assuming that the vector \mathbf{k} lies in the y - z plane. Its direction is then characterized by the angle θ with the optic axis. It is therefore convenient to draw the k -surfaces only in the y - z plane—a circle and an ellipse, as shown in Fig. 6.3-11.

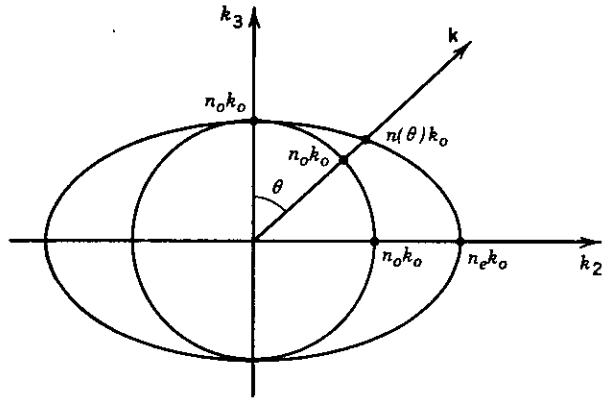


Figure 6.3-11 Intersection of the k surface with the y - z plane for a uniaxial crystal.

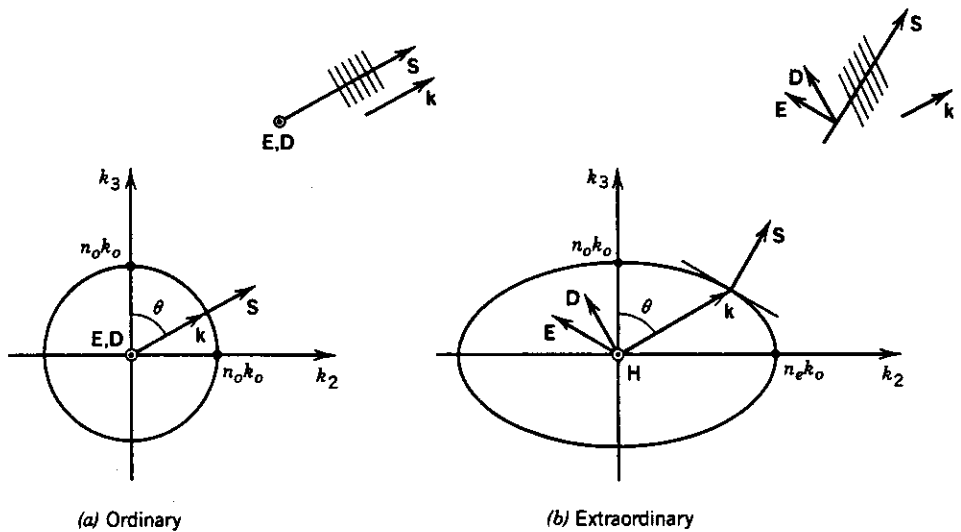


Figure 6.3-12 The normal modes for a plane wave traveling in a direction \mathbf{k} at an angle θ with the optic axis z of a uniaxial crystal are: (a) An ordinary wave of refractive index n_o polarized in a direction normal to the k - z plane. (b) An extraordinary wave of refractive index $n(\theta)$ [given by (6.3-15)] polarized in the k - z plane along a direction tangential to the ellipse (the k surface) at the point of its intersection with \mathbf{k} . This wave is "extraordinary" in the following ways: \mathbf{D} is not parallel to \mathbf{E} but both lie in the k - z plane; \mathbf{S} is not parallel to \mathbf{k} so that power does not flow along the direction of \mathbf{k} ; rays are not normal to wavefronts and the wave travels "sideways."

Given the direction $\hat{\mathbf{u}}$ of the vector \mathbf{k} , the wavenumber k is determined by finding the intersection with the k surfaces. The two solutions define the two normal modes, the ordinary and extraordinary waves. The ordinary wave has a wavenumber $k = n_o k_o$ regardless of direction, whereas the extraordinary wave has a wavenumber $n(\theta)k_o$, where $n(\theta)$ is given by (6.3-15), confirming earlier results obtained from the index-ellipsoid geometrical construction. The directions of rays, wavefronts, energy flow, and field vectors \mathbf{E} and \mathbf{D} for the ordinary and extraordinary waves in a uniaxial crystal are illustrated in Fig. 6.3-12.

E. Double Refraction

Refraction of Plane Waves

We now examine the refraction of a plane wave at the boundary between an isotropic medium (say air, $n = 1$) and an anisotropic medium (a crystal). The key principle is that the wavefronts of the incident wave and the refracted wave must be matched at the boundary. Because the anisotropic medium supports two modes of distinctly different phase velocities, one expects that for each incident wave there are two refracted waves with two different directions and different polarizations. The effect is called **double refraction** or **birefringence**.

The phase-matching condition requires that

$$k_o \sin \theta_1 = k \sin \theta, \quad (6.3-19)$$

where θ_1 and θ are the angles of incidence and refraction. In an anisotropic medium, however, the wave number $k = n(\theta)k_o$ is itself a function of θ , so that

$$\sin \theta_1 = n(\theta) \sin \theta, \quad (6.3-20)$$

a modified Snell's law. To solve (6.3-19), we draw the intersection of the k surface with the plane of incidence and search for an angle θ for which (6.3-19) is satisfied. Two solutions, corresponding to the two normal modes, are expected. The polarization state of the incident light governs the distribution of energy among the two refracted waves.

Take, for example, a uniaxial crystal and a plane of incidence parallel to the optic axis. The k surfaces intersect the plane of incidence in a circle and an ellipse (Fig. 6.3-13). The two refracted waves that satisfy the phase-matching condition are:

- An ordinary wave of orthogonal polarization (TE) at an angle $\theta = \theta_o$ for which

$$\sin \theta_1 = n_o \sin \theta_o;$$

- An extraordinary wave of parallel polarization (TM) at an angle $\theta = \theta_e$, for which

$$\sin \theta_1 = n(\theta_e) \sin \theta_e,$$

where $n(\theta)$ is given by (6.3-15).

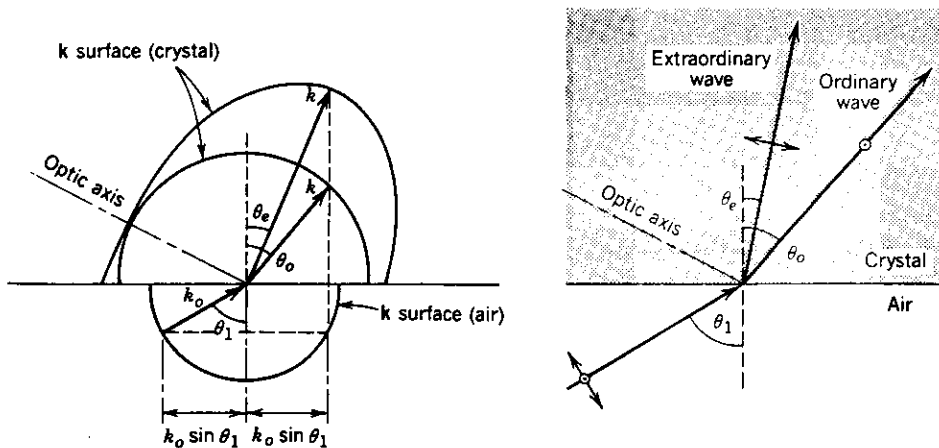


Figure 6.3-13 Determination of the angles of refraction by matching projections of the k vectors in air and in a uniaxial crystal.

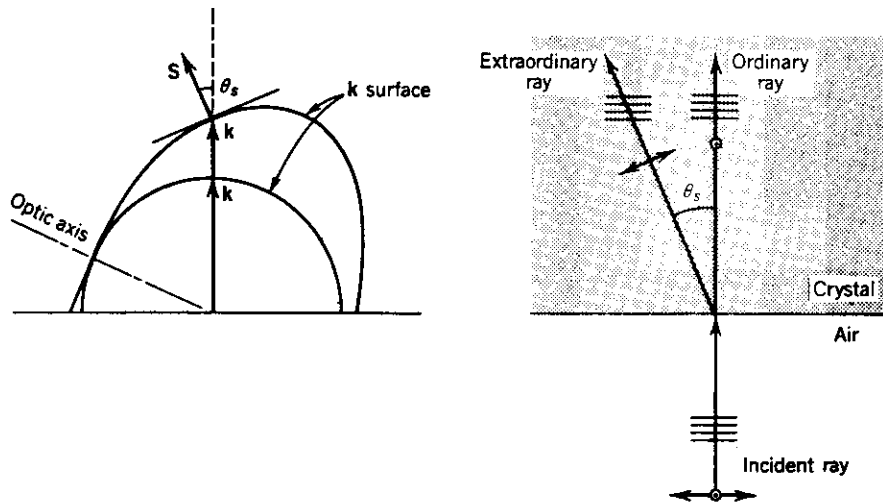


Figure 6.3-14 Double refraction at normal incidence.

If the incident wave carries the two polarizations, the two refracted waves will emerge.

Refraction of Rays

The previous analysis dealt with the refraction of plane waves. The refraction of rays is different since rays in an anisotropic medium do not necessarily travel in a direction normal to the wavefronts. In air, before entering the crystal, the wavefronts are normal to the rays. The refracted wave must have a wavevector satisfying the phase-matching condition, so that Snell's law (6.3-20) applies, with the angle of refraction θ determining the direction of \mathbf{k} . Since the direction of \mathbf{k} is not the direction of the ray, Snell's law is not applicable to rays.

An example that dramatizes the deviation from Snell's law is that of normal incidence at a uniaxial crystal whose optic axis is neither parallel nor perpendicular to the crystal boundary. The incident wave has a \mathbf{k} vector normal to the boundary. To ensure phase matching, the refracted waves must also have wavevectors in the same direction. Intersections with the \mathbf{k} surface yield two points corresponding to two waves. The ordinary ray is parallel to \mathbf{k} . But the extraordinary ray points in the direction of the normal to the \mathbf{k} surface, at an angle θ_s with the normal to the crystal boundary, as illustrated in Fig. 6.3-14. Thus normal incidence creates oblique refraction. Note, however, that the principle of phase matching is still maintained; wavefronts of both

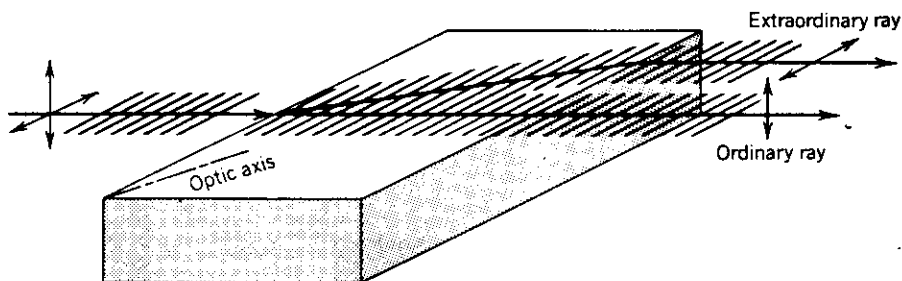


Figure 6.3-15 Double refraction through an anisotropic plate. The plate serves as a polarizing beamsplitter.

refracted rays are parallel to the crystal boundary and to the wavefront of the incident ray.

When light rays are transmitted through a plate of anisotropic material as described above, the two rays refracted at the first surface refract at the second surface, creating two laterally separated rays with orthogonal polarizations, as illustrated in Fig. 6.3-15.

6.4 OPTICAL ACTIVITY AND FARADAY EFFECT

A. Optical Activity

Certain materials act naturally as polarization rotators, a property known as optical activity. Their normal modes are circularly polarized, instead of linearly polarized waves; the waves with right- and left-circular polarizations travel at different phase velocities. Optical activity is found in materials in which the molecules have an inherently helical character. Examples are quartz, selenium, tellurium, and tellurium oxide (TeO_2). Many organic materials exhibit optical activity. The rotatory power and the sense of rotation are also sensitive to the chemical structure and concentration of solutions (this effect has been used, for example, to measure sugar content in solutions).

It will be shown subsequently that an optically active medium with right- and left-circular-polarization phase velocities c_o/n_+ and c_o/n_- acts as a polarization rotator with an angle of rotation $\pi(n_- - n_+)d/\lambda_o$ proportional to the distance d . The rotatory power (angle per unit length) of the optically active medium is therefore

$$\rho = \frac{\pi(n_- - n_+)}{\lambda_o} \quad (6.4-1)$$

Rotatory Power

The direction of rotation of the polarization plane is in the same sense as that of the circularly polarized component of the greater phase velocity (smaller refractive index). If $n_+ < n_-$, ρ is positive and the rotation is in the same direction as the electric field vector of the right circularly polarized wave [clockwise when viewed from the direction toward which the wave is approaching, as illustrated in Fig. 6.4-1(a)].

The optically active medium is a spatially dispersive medium since the relation between $\mathbf{D}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})$ is not local. $\mathbf{D}(\mathbf{r})$ at position \mathbf{r} is determined not only by $\mathbf{E}(\mathbf{r})$, but also by $\mathbf{E}(\mathbf{r}')$ at points \mathbf{r}' in the immediate vicinity of \mathbf{r} [since it is dependent on the derivatives in $\nabla \times \mathbf{E}(\mathbf{r})$]. Spatial dispersiveness is analogous to temporal dispersiveness, which is caused by the noninstantaneous response of the medium (see Sec. 5.2B).

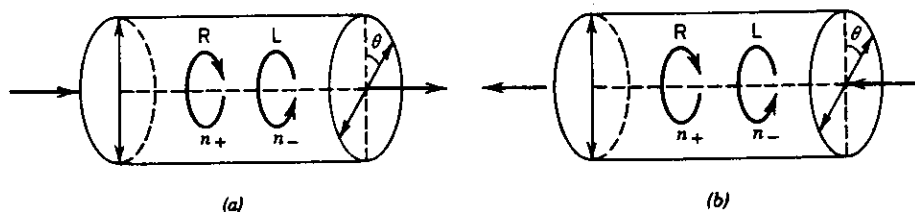


Figure 6.4-1 (a) Rotation of the plane of polarization in an optically active medium is a result of the difference in the velocities of the two circular polarizations. In this illustration, the right circularly polarized wave (R) is faster than the left circularly polarized wave (L), i.e., $n_+ < n_-$, so that ρ is positive. (b) If the wave in (a) is reflected after traversing the medium, the plane of polarization rotates in the opposite direction and the wave retraces itself.

Equation (6.4-1) may be obtained by decomposing the linearly polarized wave into a sum of right and left circularly polarized waves of equal amplitudes (see Exercise 6.1-1),

$$\begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} = \frac{1}{2} e^{-j\theta} \begin{bmatrix} 1 \\ j \end{bmatrix} + \frac{1}{2} e^{j\theta} \begin{bmatrix} 1 \\ -j \end{bmatrix},$$

where θ is the initial angle of the polarization plane. After a distance d of propagation in the medium, phase shifts $\varphi_+ = 2\pi n_+ d/\lambda_o$ and $\varphi_- = 2\pi n_- d/\lambda_o$, respectively, are encountered by the right and left circularly polarized waves, so that the new Jones vector is

$$\frac{1}{2} e^{-j\theta} e^{-j\varphi_+} \begin{bmatrix} 1 \\ j \end{bmatrix} + \frac{1}{2} e^{j\theta} e^{-j\varphi_-} \begin{bmatrix} 1 \\ -j \end{bmatrix} = e^{-j\varphi_o} \begin{bmatrix} \cos\left(\theta - \frac{\varphi}{2}\right) \\ \sin\left(\theta - \frac{\varphi}{2}\right) \end{bmatrix},$$

where $\varphi_o = \frac{1}{2}(\varphi_+ + \varphi_-)$ and $\varphi = \varphi_- - \varphi_+ = 2\pi(n_- - n_+)d/\lambda_o$. This Jones vector represents a linearly polarized wave with the plane of polarization rotated by an angle $\varphi/2 = \pi(n_- - n_+)d/\lambda_o$, as indicated above.

Medium Equations

We now show that a dielectric medium characterized by the medium equation

$$\mathbf{D} = \epsilon \mathbf{E} + \epsilon_o \xi j\omega \mathbf{B} = \epsilon \mathbf{E} - \epsilon_o \xi \nabla \times \mathbf{E}, \quad (6.4-2)$$

where ξ is a constant, is optically active. This medium relation arises in molecular structures with a helical character. In these structures, a time-varying magnetic flux density \mathbf{B} induces a circulating current that sets up an electric dipole moment (and hence polarization) proportional to $j\omega \mathbf{B} = -\nabla \times \mathbf{E}$, which is responsible for the last term in (6.4-2).

The optically active medium is a spatially dispersive medium since the relation between $\mathbf{D}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})$ is not local. $\mathbf{D}(\mathbf{r})$ at position \mathbf{r} is determined not only by $\mathbf{E}(\mathbf{r})$, but also by $\mathbf{E}(\mathbf{r}')$ at points \mathbf{r}' in the immediate vicinity of \mathbf{r} [since it is dependent on the derivatives in $\nabla \times \mathbf{E}(\mathbf{r})$]. Spatial dispersiveness is analogous to temporal dispersiveness, which is caused by the noninstantaneous response of the medium (see Sec. 5.2B).

We proceed to show that the two normal modes of a medium satisfying (6.4-2) are circularly polarized waves and we determine the velocities c_o/n_+ and c_o/n_- in terms of the constant ξ .

Normal Modes of the Optically Active Medium

Consider the propagation of a plane wave $\mathbf{E}(\mathbf{r}) = \mathbf{E} \exp(-j\mathbf{k} \cdot \mathbf{r})$ in a medium satisfying (6.4-2). Setting $\mathbf{D}(\mathbf{r}) = \mathbf{D} \exp(-j\mathbf{k} \cdot \mathbf{r})$, (6.4-2) yields

$$\mathbf{D} = \epsilon \mathbf{E} + j\epsilon_o \mathbf{G} \times \mathbf{E}, \quad (6.4-3)$$

where

$$\mathbf{G} = \xi \mathbf{k} \quad (6.4-4)$$

is known as the **gyration vector**. Clearly, the vector \mathbf{D} is not parallel to \mathbf{E} since the vector $\mathbf{G} \times \mathbf{E}$ in (6.4-3) is perpendicular to \mathbf{E} . The relation between \mathbf{D} and \mathbf{E} is therefore dependent on the wavevector \mathbf{k} , which is not surprising since the medium is

spatially dispersive. (This is analogous to the dependence of the dielectric properties of a temporally dispersive medium on ω .)

For simplicity, we assume that ϵ has uniaxial symmetry (with indices n_o and n_e), use the principal axes of the tensor ϵ as a coordinate system, and consider only waves propagating along the optic axis. The first term in (6.4-3) then corresponds to propagation of an ordinary wave of refractive index n_o .

To prove that the normal modes are circularly polarized, consider the two circularly polarized waves of electric-field vectors $\mathbf{E} = (E_0, \pm jE_0, 0)$ and wavevector $\mathbf{k} = (0, 0, k)$. The + and - signs correspond to right and left circularly polarized cases, respectively. Substituting in (6.4-3), we obtain $\mathbf{D} = (D_0, \pm jD_0, 0)$, where $D_0 = \epsilon_o(n_o^2 \pm G)E_0$. It follows that $\mathbf{D} = \epsilon_o n_{\pm}^2 \mathbf{E}$, where

$$n_{\pm} = (n_o^2 \pm G)^{1/2}, \quad (6.4-5)$$

so that for either of the two circularly polarized waves the vector \mathbf{D} is parallel to the vector \mathbf{E} . Equation (6.3-10) is satisfied if the wavenumber $k = n_{\pm} k_0$. Thus the right and left circularly polarized waves propagate, without change of their state of polarization, with refractive indices n_+ and n_- , respectively. They are the normal modes for this medium.

EXERCISE 6.4-1

Rotatory Power of an Optically Active Medium. Show that if $G \ll n_o$, the rotatory power of an optically active medium (rotation of the polarization plane per unit length) is approximately given by

$$\rho \approx -\frac{\pi G}{\lambda_o n_o}. \quad (6.4-6)$$

The rotatory power is strongly dependent on the wavelength. Since G is proportional to k , as indicated by (6.4-4), it is inversely proportional to the wavelength λ_o . Thus the rotatory power in (6.4-6) is inversely proportional to λ_o^2 . In addition, the refractive index n_o is itself wavelength dependent. The rotatory power ρ of quartz is ≈ 31 deg/mm at $\lambda_o = 500$ nm and ≈ 22 deg/mm at 600 nm; for silver thiogallate (AgGaS_2) ρ is ≈ 700 deg/mm at 490 nm and ≈ 500 deg/mm at 500 nm.

B. Faraday Effect

Certain materials act as polarization rotators when placed in a static magnetic field, a property known as the Faraday effect. The angle of rotation is proportional to the distance, and the rotatory power ρ (angle per unit length) is proportional to the component B of the magnetic flux density in the direction of wave propagation,

$$\rho = VB, \quad (6.4-7)$$

where V is known as the Verdet constant.

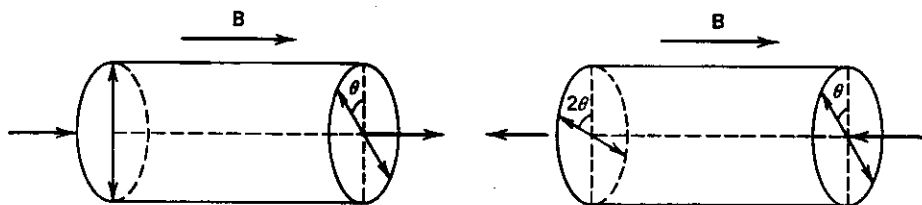


Figure 6.4-2 Polarization rotation in a medium exhibiting the Faraday effect. The sense of rotation is invariant to the direction of travel of the wave.

The sense of rotation is governed by the direction of the magnetic field: for $V > 0$, the rotation is in the direction of a right-handed screw pointing in the direction of the magnetic field. In contradistinction to optical activity, the sense of rotation does not reverse with the reversal of the direction of propagation of the wave (Fig. 6.4-2). When a wave travels through a Faraday rotator, reflects back onto itself, and travels once more through the rotator in the opposite direction, it undergoes twice the rotation.

The medium equation for materials exhibiting the Faraday effect is

$$\mathbf{D} = \epsilon \mathbf{E} + j\epsilon_0 \gamma \mathbf{B} \times \mathbf{E}, \quad (6.4-8)$$

where \mathbf{B} is the magnetic flux density and γ is a constant of the medium that is called the **magnetogyration coefficient**. This relation originates from the interaction of the static magnetic field \mathbf{B} with the motion of electrons in the molecules under the influence of the optical electric field \mathbf{E} .

To establish an analogy between the Faraday effect and optical activity (6.4-8) is written as

$$\mathbf{D} = \epsilon \mathbf{E} + j\epsilon_0 \mathbf{G} \times \mathbf{E}, \quad (6.4-9)$$

where

$$\mathbf{G} = \gamma \mathbf{B}. \quad (6.4-10)$$

Equation (6.4-9) is identical to (6.4-3) with the vector $\mathbf{G} = \gamma \mathbf{B}$ in Faraday rotators playing the role of the gyration vector $\mathbf{G} = \xi \mathbf{k}$ in optically active media. Note that in the Faraday effect \mathbf{G} is independent of \mathbf{k} , so that reversal of the direction of propagation does not reverse the sense of rotation of the polarization plane. This property can be used to make optical isolators, as explained in Sec. 6.6.

With this analogy, and using (6.4-6), we conclude that the rotatory power of the Faraday medium is $\rho \approx -\pi G / \lambda_o n_o = -\pi \gamma B / \lambda_o n_o$, from which the Verdet constant (the rotatory power per unit magnetic flux density) is

$$V \approx -\frac{\pi \gamma}{\lambda_o n_o}. \quad (6.4-11)$$

Clearly, the Verdet constant is a function of the wavelength λ_o .

Materials that exhibit the Faraday effect include glasses, yttrium-iron-garnet (YIG), terbium-gallium-garnet (TGG), and terbium-aluminum-garnet (TbAlG). The Verdet constant V of TbAlG is $V = -1.16 \text{ min/cm-Oe}$ at $\lambda_0 = 500 \text{ nm}$.

6.5 OPTICS OF LIQUID CRYSTALS

Liquid Crystals

The liquid-crystal state is a state of matter in which the elongated (typically cigar-shaped) molecules have orientational order (like crystals) but lack positional order (like liquids). There are three types (phases) of liquid crystals, as illustrated in Fig. 6.5-1:

- In **nematic** liquid crystals the molecules tend to be parallel but their positions are random.
- In **smectic** liquid crystals the molecules are parallel, but their centers are stacked in parallel layers within which they have random positions, so that they have positional order in only one dimension.
- The **cholesteric** phase is a distorted form of the nematic phase in which the orientation undergoes helical rotation about an axis.

Liquid crystallinity is a *fluid* state of matter. The molecules change orientation when subjected to a force. For example, when a thin layer of liquid crystal is placed between two parallel glass plates the molecular orientation is changed if the plates are rubbed; the molecules orient themselves along the direction of rubbing.

Twisted nematic liquid crystals are nematic liquid crystals on which a twist, similar to the twist that exists naturally in the cholesteric phase, is imposed by external forces (for example, by placing a thin layer of the liquid crystal material between two glass plates polished in perpendicular directions as shown in Fig. 6.5-2). Because twisted nematic liquid crystals have enjoyed the greatest number of applications in photonics (in liquid-crystal displays, for example), this section is devoted to their optical properties. The electro-optic properties of twisted nematic liquid crystals, and their use as optical modulators and switches, are described in Chap. 18.

Optical Properties of Twisted Nematic Liquid Crystals

The twisted nematic liquid crystal is an optically *inhomogeneous anisotropic medium* that acts locally as a uniaxial crystal, with the optic axis parallel to the molecular

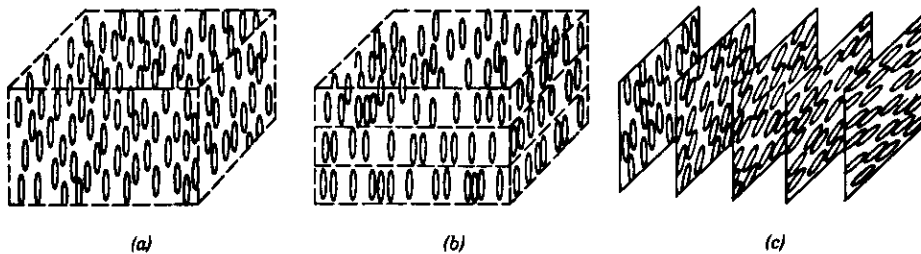


Figure 6.5-1 Molecular organizations of different types of liquid crystals: (a) nematic; (b) smectic; (c) cholesteric.

Substituting from (6.5-5) and (6.1-15)

$$\mathbf{T}_r \mathbf{R}(\Delta\theta) = \begin{bmatrix} \exp\left(-j\beta \frac{\Delta z}{2}\right) & 0 \\ 0 & \exp\left(j\beta \frac{\Delta z}{2}\right) \end{bmatrix} \begin{bmatrix} \cos \alpha \Delta z & \sin \alpha \Delta z \\ -\sin \alpha \Delta z & \cos \alpha \Delta z \end{bmatrix}. \quad (6.5-8)$$

Using (6.5-7) and (6.5-8), the Jones matrix \mathbf{T} of the device can, in principle, be determined in terms of the parameters α , β , and $d = N\Delta z$.

When $\alpha \ll \beta$, we can assume that the incremental rotation matrix $\mathbf{R}(\Delta\theta)$ is approximately an identity matrix and obtain

$$\begin{aligned} \mathbf{T} &\approx \mathbf{R}(-\theta_N) [\mathbf{T}_r]^N \mathbf{R}(\theta_1) = \mathbf{R}(-\alpha N \Delta z) \begin{bmatrix} \exp\left(-j\beta \frac{\Delta z}{2}\right) & 0 \\ 0 & \exp\left(j\beta \frac{\Delta z}{2}\right) \end{bmatrix}^N \\ &= \mathbf{R}(-\alpha N \Delta z) \begin{bmatrix} \exp\left(-j\beta N \frac{\Delta z}{2}\right) & 0 \\ 0 & \exp\left(j\beta N \frac{\Delta z}{2}\right) \end{bmatrix}. \end{aligned}$$

In the limit as $N \rightarrow \infty$, $\Delta z \rightarrow 0$, and $N\Delta z \rightarrow d$,

$$\mathbf{T} = \mathbf{R}(-\alpha d) \begin{bmatrix} \exp\left(-j\beta \frac{d}{2}\right) & 0 \\ 0 & \exp\left(j\beta \frac{d}{2}\right) \end{bmatrix}. \quad (6.5-9)$$

This Jones matrix represents a wave retarder of retardation βd with the slow axis along the x direction, followed by a polarization rotator with rotation angle αd . If the original wave is linearly polarized along the x direction the wave retarder provides only a phase shift; the device then simply rotates the polarization by an angle αd equal to the twist angle.

6.6 POLARIZATION DEVICES

This section is a brief description of a number of devices that are used to modify the state of polarization of light. The basic principles of most of these devices have been discussed earlier in this chapter.

A. Polarizers

A polarizer is a device that transmits the component of the electric field in the direction of its transmission axis and blocks the orthogonal component. This preferential treatment of the two components of the electric field is achieved by selective absorption, selective reflection from an isotropic medium, or selective reflection/refraction at the boundary of an anisotropic medium.

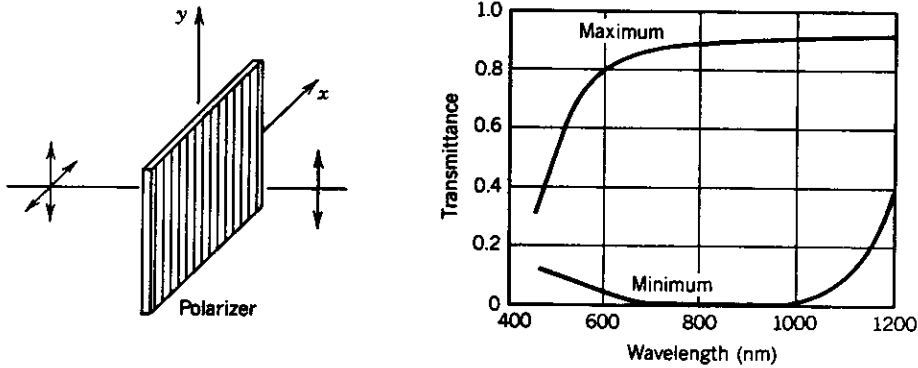


Figure 6.6-1 Power transmittances of a typical dichroic polarizer with the polarization plane of the light aligned for maximum and minimum transmittance.

Polarization by Selective Absorption (Dichroism)

The absorption of light by certain anisotropic materials, called **dichroic materials**, depends on the direction of the electric field (Fig. 6.6-1). These materials have anisotropic molecular structures whose response is sensitive to the direction of the applied field. The most common dichroic material is the Polaroid H-sheet (basically a sheet of polyvinyl alcohol heated and stretched in a certain direction then impregnated with iodine atoms).

Polarization by Selective Reflection

The reflection of light from the boundary between two dielectric isotropic materials is polarization dependent (see Sec. 6.2). At the Brewster angle of incidence, light of TM polarization is not reflected (i.e., is totally refracted). At this angle, only the TE component of the incident light is reflected, so that the reflector serves as a polarizer (Fig. 6.6-2).

Polarization by Selective Refraction in Anisotropic Media (Polarizing Beamsplitters)

When light refracts at the surface of an anisotropic crystal the two polarizations refract at different angles and are spatially separated (see Sec. 6.3E and Fig. 6.3-15). This is an excellent way of obtaining polarized light from unpolarized light. The device usually

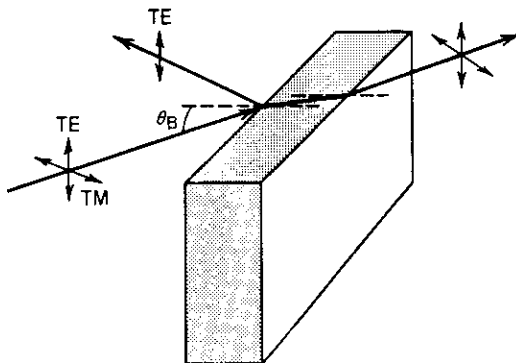


Figure 6.6-2 Brewster-angle polarizer.

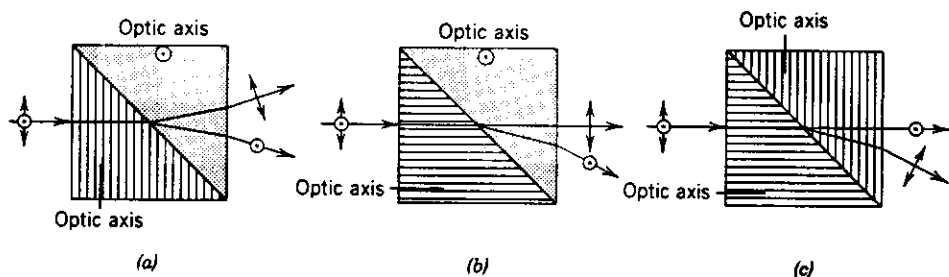


Figure 6.6-3 Polarizing prisms: (a) Wollaston prism; (b) Rochon prism; (c) Sénarmont prism. The directions and polarizations of the exiting waves differ in the three cases.

takes the form of two cemented prisms made of anisotropic (uniaxial) crystals in different orientations, as illustrated by the examples in Fig. 6.6-3. These prisms serve as polarizing beamsplitters.

B. Wave Retarders

The wave retarder is characterized by its retardation Γ and its fast and slow axes (see Sec. 6.1B). The normal modes are linearly polarized waves polarized in the directions of the axes, and the velocities are different. Upon transmission through the retarder, a relative phase shift Γ between these modes ensues.

Wave retarders are often made of anisotropic materials. As explained in Sec. 6.3B, when light travels along a principal axis of a crystal (say the z axis), the normal modes are linearly polarized waves pointing along the two other principal axes (x and y axes). The two modes travel with the principal refractive indices n_1 and n_2 . If $n_1 < n_2$, the x axis is the fast axis. If the plate has a thickness d , the phase retardation is $\Gamma = (n_2 - n_1)k_0 d = 2\pi(n_2 - n_1)d/\lambda_0$. The retardation is directly proportional to the thickness d and inversely proportional to the wavelength λ_0 (note, however, that $n_2 - n_1$ itself is wavelength dependent).

The refractive indices of mica, for example, are 1.599 and 1.594 at $\lambda_0 = 633$ nm, so that $\Gamma/d \approx 15.8\pi$ rad/mm. A $63.3\text{-}\mu\text{m}$ thin sheet is a half-wave retarder ($\Gamma \approx \pi$).

Control of Light Intensity by Use of a Wave Retarder and Two Polarizers

The power (or intensity) transmittance of a system constructed from a wave retarder of retardation Γ placed between two crossed polarizers, at 45° with respect to the retarder's axes, as shown in Fig. 6.6-4, is

$$\mathcal{T} = \sin^2 \frac{\Gamma}{2}. \quad (6.6-1)$$

This may be obtained by use of Jones matrices or by examining the polarization ellipse of the retarded light as a function of Γ and determining the component in the direction of the output polarizer, as illustrated in Fig. 6.6-4. If $\Gamma = 0$, no light is transmitted since the polarizers are orthogonal. If $\Gamma = \pi$, all the light is transmitted since the retarder rotates the polarization 90° , making it match the transmission axis of the second polarizer.

The intensity of the transmitted light can be controlled by altering the retardation Γ (for example, by changing the indices n_1 and n_2). This is the basic principle underlying the electro-optic modulators discussed in Chap. 18.

Furthermore, since Γ depends on d , slight variations in the thickness of a sample can be monitored by examining the pattern of the transmitted light. Also since Γ is

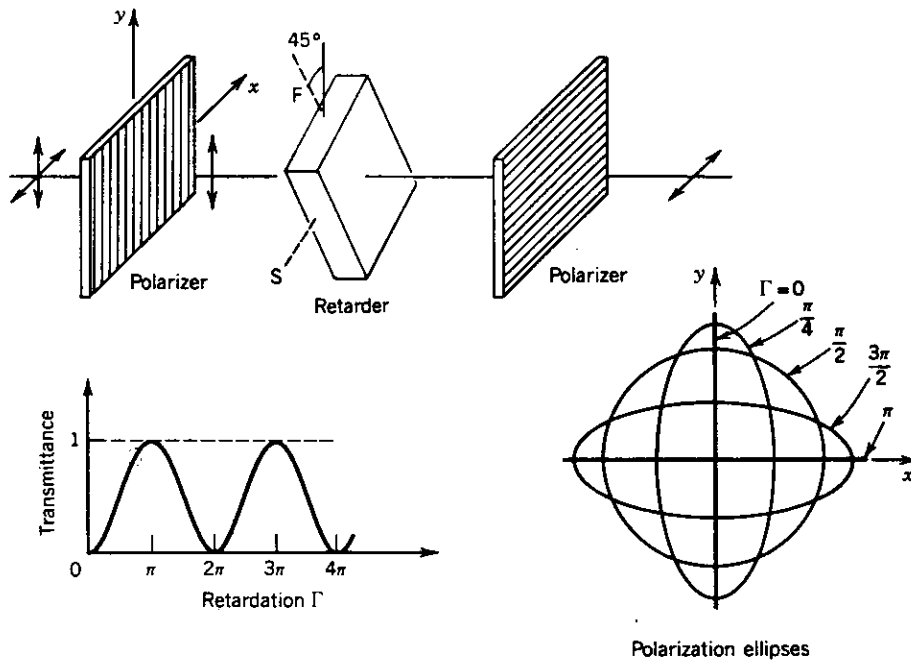


Figure 6.6-4 Controlling light intensity by use of a wave retarder with variable retardation Γ between two crossed polarizers.

wavelength dependent, the transmittance of the system is frequency sensitive. The system therefore serves as a filter, but the selectivity is not very sharp. Other configurations using wave retarders and polarizers can be used to construct narrowband transmission filters.

C. Polarization Rotators

A polarization rotator rotates the plane of polarization of linearly polarized light by a fixed angle, maintaining its linearly polarized nature. Optically active media and materials exhibiting the Faraday effect act as polarization rotators, as shown in Sec. 6.4. The twisted nematic liquid crystal also acts as a polarization rotator under certain conditions, as shown in Sec. 6.5.

If a polarization rotator is placed between two polarizers, the amount of transmitted light depends on the rotation angle. The intensity of light can be controlled (modulated) if the angle of rotation is controlled by some external means (e.g., by varying the magnetic flux density applied to a Faraday rotator, or by changing the molecular orientation of a liquid crystal by means of an applied electric field). Electro-optic modulation of light and liquid-crystal display devices are discussed in Chap. 18.

Optical Isolators

An optical isolator is a device that transmits light in only one direction, thus acting as a "one-way valve." Optical isolators are useful in preventing reflected light from returning back to the source. This type of feedback can have deleterious effects on the operation of certain light sources (semiconductor lasers, for example).

A system made of a polarizing beamsplitter followed by a quarter-wave retarder acts as an isolator. Light traveling in the forward direction is polarized by the cube, then circularly polarized by the retarder. Upon reflection from a mirror beyond the retarder,

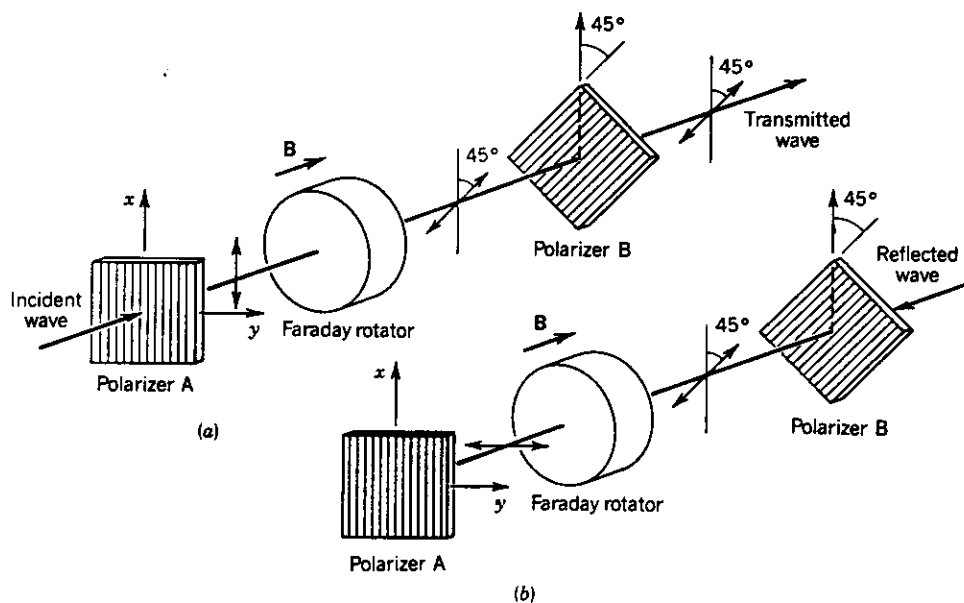


Figure 6.6-5 An optical isolator using a Faraday rotator transmits light in one direction, as in (a), and blocks light in the opposite direction, as in (b).

the sense of rotation is reversed (left to right, or vice versa), so that upon transmission back through the retarder it becomes polarized in the orthogonal direction and is therefore blocked by the polarizing cube (see Problem 6.1-6). Although this type of isolator can offer attenuation of the backward wave up to 30 dB (0.1%), it operates only over a narrow wavelength range.

A Faraday rotator placed between two polarizers making a 45° angle with each other can also be used as an optical isolator. The magnetic flux density applied to the rotator is adjusted so that the polarization is rotated by 45° in the direction of a right-handed screw pointing in the z direction [Fig. 6.6-5(a)]. Light traveling from left to right crosses polarizer A, rotates 45° , and is transmitted through polarizer B. However, light traveling in the opposite direction [Fig. 6.6-5(b)], although it crosses polarizer B, rotates an additional 45° and is blocked by polarizer A. A Faraday rotator cannot be replaced by an optically active or liquid-crystal polarization rotator since, in those devices, the sense of rotation is such that the polarization of the reflected wave retraces that of the incident wave and is therefore transmitted back through the polarizers to the source. Faraday-rotator isolators made of yttrium-iron-garnet (YIG) or terbium-gallium-garnet (TGG), for example, can offer an attenuation of the backward wave up to 90 dB, over a relatively wide wavelength range.

READING LIST

General

See also the list of general books on optics in Chapter 1.

D. S. Kliger, J. W. Lewis, and C. E. Randall, *Polarized Light in Optics and Spectroscopy*, Academic Press, Boston, 1990.

J. F. Nye, *Physical Properties of Crystals: Their Representation by Tensors and Matrices*, Oxford University Press, New York, 1967, 2nd ed. 1984.