

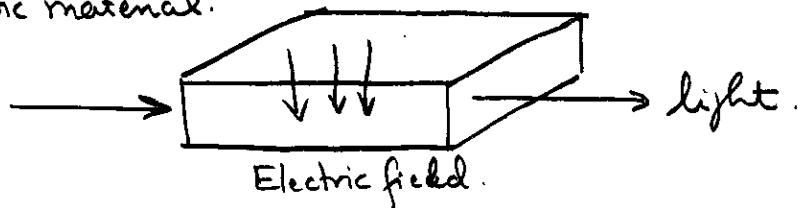
Read information in handouts - pages 218-234.

- You have the background to understand these.
- we can discuss them some time.

Electro-optics.

Electro-optic Effect.

Electro-optic material.



E-O effect is the change in the refractive index resulting from the application of an DC or low frequency electric field.

- A field applied to an anisotropic electro-optic material modifies its refractive index and thereby its effects on polarized light.

- Refractive index changes that are proportional to \vec{E}_{applied} - linear electro-optic effect or the Pockels effect.

- Refractive index changes $\propto (\vec{E}_{\text{applied}})^2 \Rightarrow$ quadratic electro-optic effect or the Kerr Effect.

Suppose $\Delta n \sim 10^{-5}$ & we travel 10^5 wavelengths.

1 wavelength $\sim 1 \mu\text{m}$

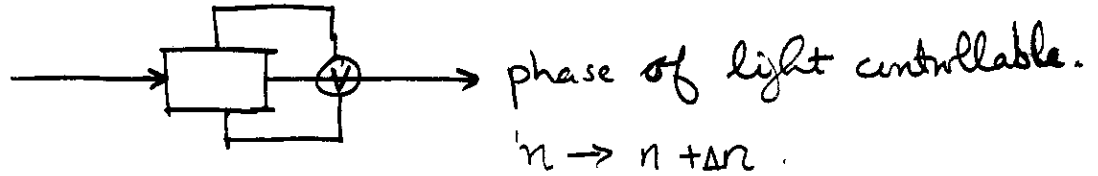
\Rightarrow

$k \cdot n \cdot d \rightarrow k(n + \Delta n)d$.

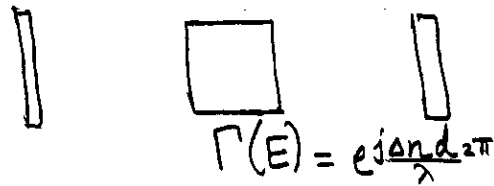
$$k_0 \Delta n d = \frac{2\pi \Delta n d}{\lambda} = 2\pi \Rightarrow \text{additional } 2\pi \text{ phase shift.}$$

Why interesting?

- ① Lens made of a material whose $n = n(E_{\text{applied}}) \Rightarrow$ controllable focal length
- ② Prism whose beam bending is controllable can be used for a scanning device.
- ③ Optical phase modulator.



- ④ Controllable wave retarders from anisotropic crystals.
- ⑤ Polarization rotation switch.



Pockels Effect.

$$n(E) = n + a_1 E + \frac{1}{2} a_2 E^2 + \dots$$

$$n = n(E=0), \quad a_1 = \left. \frac{dn}{dE} \right|_{E=0}, \quad a_2 = \left. \left(\frac{d^2 n}{dE^2} \right) \right|_{E=0}.$$

We write

$$r = \frac{-2a_1}{n^3} \quad s = \frac{-a_2}{n^3}.$$

$$\Rightarrow n(E) = n - \frac{1}{2} r n^3 E - \frac{1}{2} s n^3 E^2 + \dots$$

typically very small terms

Recall $\eta = \frac{\epsilon_0}{\epsilon} = \frac{1}{n^2}$

$$\begin{aligned} \Delta \eta &= \left(\frac{d\eta}{dn} \right) \Delta n = \left(-2/n^3 \right) \left(-\frac{1}{2} r n^3 E - \frac{1}{2} s n^3 E^2 \right) \\ &= r E + s E^2. \end{aligned}$$

$$n(E) = n + \underbrace{rE} + \underbrace{sE^2} : \text{impermeability.}$$

$$n = n(0)$$

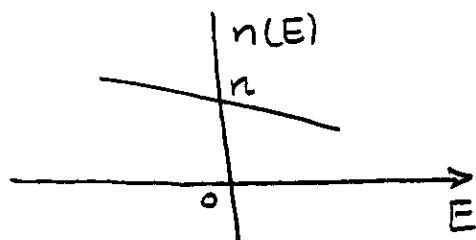
explains definitions of r & s . (n has simple form).

Pockels Effect.

$$sE^2 \ll rE.$$

$$\Rightarrow n(E) = n - \frac{1}{2} r n^3 E$$

r - Pockels coefficient or
linear electro-optic coefficient.



Pockels medium.
(Pockels cell)

$$10^{-12} < r < 10^{-10} \text{ m/V} \quad (1 \rightarrow 100 \text{ pm/V}).$$

Example $E = 10^6 \text{ V/m}$ $10 \text{ kV across } 1 \text{ cm}$.

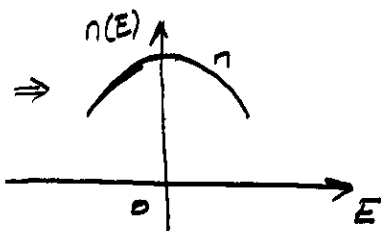
$$\frac{1}{2} r n^3 E \sim 10^{-6} \text{ to } 10^{-4}$$

Pockels cells: $\text{NH}_4\text{H}_2\text{PO}_4$ (ADP), KH_2PO_4 (KDP), LiNbO_3 , LiTaO_3
 CdTe .

Kerr Effect.

centrosymmetric materials: liquids, gases, certain crystals

$n(E)$ must be an even symmetric function



$$\Rightarrow \left. \frac{dn}{dE} \right|_{E=0} = 0 \Rightarrow r = 0$$

$$n(E) = n - \frac{1}{2} s n^3 E^2$$

Kerr medium (or a Kerr cell).

s - Kerr coefficient or quadratic
electro-optic coefficient.

$$10^{-18} < s < 10^{-14} \text{ m}^2/\text{V}^2 \text{ in crystals.}$$

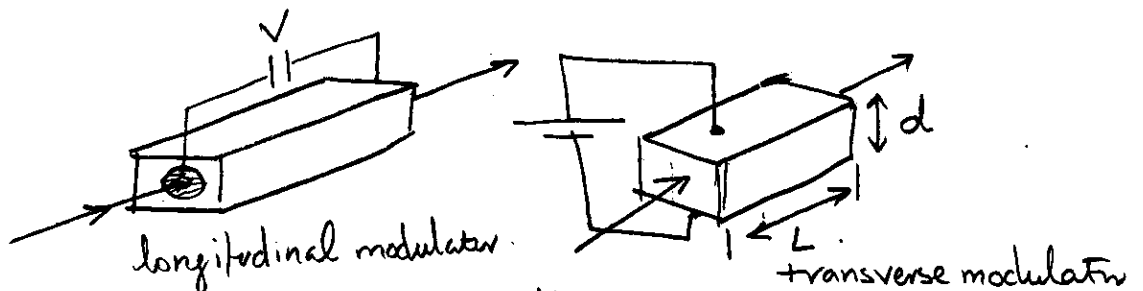
$$10^{-22} < s < 10^{-19} \text{ m}^2/\text{V}^2 \text{ in liquids.}$$

$$E = 10^6 \text{ V/m}$$

$$\frac{1}{2} s n^3 E^2 \sim 10^{-6} \text{ to } 10^{-2} \text{ in crystals}$$

$$10^{-10} \text{ to } 10^{-7} \text{ in liquids.}$$

Phase modulators.



$$\phi = n(E)k_0 L$$

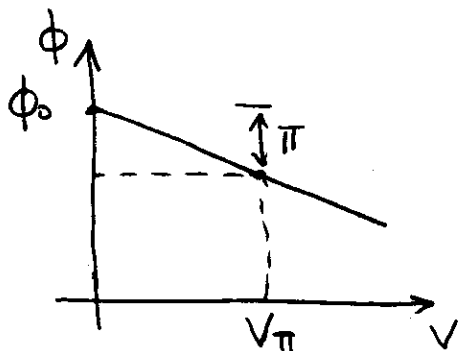
$$= \frac{2\pi n(E)L}{\lambda_0}$$

(plane wave e^{jkz} phase)
 λ_0 - free space wavelength.

$$\phi = \phi_0 - \frac{\pi r n^3 E L}{\lambda_0}$$

$$\phi_0 = \frac{2\pi n L}{\lambda_0}$$

$$E = \frac{V}{d}$$



$$\phi = \phi_0 - \pi \frac{V}{V_\pi}$$

$$V_\pi = \frac{d \lambda_0}{L r n^3}$$

Half-wave Voltage.

V_π - half-wave voltage - phase changes by π

\Rightarrow We can modulate the phase with application of the voltage.

V_π - important characteristic of the modulator.

- depends on material properties (r & n).
- depends on λ_0
- depends on aspect ratio $\frac{d}{L}$.

If V applied along length $\Rightarrow d = L$.

r - depends on direction of propagation.
 - because crystal is anisotropic.

$V_\pi \sim 1 \rightarrow 10^3$ V for longitudinal modulators.
 100 's V for transverse mods.

Modulation speeds \rightarrow MHz \rightarrow GHz, easily.

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Dynamic Wave retarders.

anisotropic medium. - Two normal modes $\sim \frac{c_0}{n_1}, \frac{c_0}{n_2}$
see n_1 and n_2

Application of electric field E modifies two refractive indices.

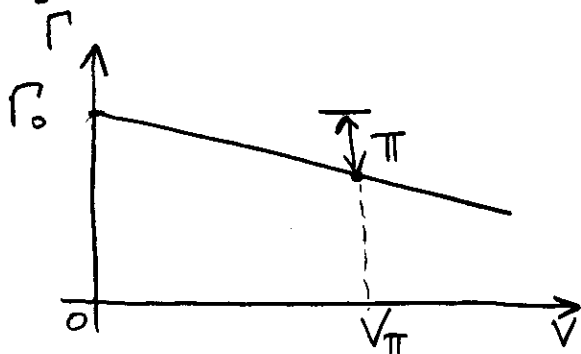
$$n_1(E) \approx n_1 - \frac{1}{2} r_1 n_1^3 E$$

$$n_2(E) \approx n_2 - \frac{1}{2} r_2 n_2^3 E$$

$r_1, r_2 \sim$ Pockels coefficients. After distance L
phase retardation:

$$\begin{aligned} \Gamma &= k_0 (n_1(E) - n_2(E)) L \\ &= k_0 (n_1 - n_2) L - \frac{1}{2} k_0 (r_1 n_1^3 - r_2 n_2^3) E L. \end{aligned}$$

E - applying voltage V between two surfaces of the medium.



$$\Gamma = \Gamma_0 - \pi \frac{V}{V_\pi} \quad \text{Phase retardation.}$$

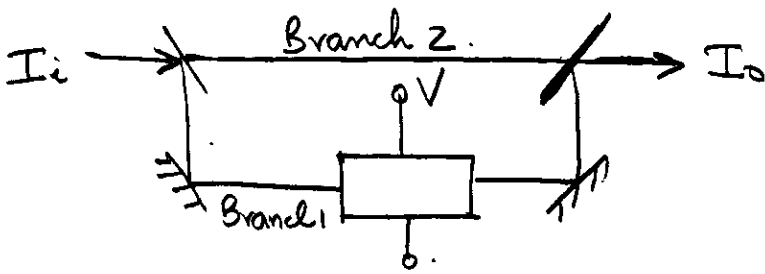
$\Gamma_0 = k_0 (n_1 - n_2) L$ - phase retardation with $E=0$.

$$V_\pi = \frac{d}{L} \cdot \frac{\lambda_0}{r_1 n_1^3 - r_2 n_2^3}.$$

\Rightarrow electrical controllable dynamic wave retarder.

Intensity Modulators: Use of a phase modulator in an Interferometer.

Mach-Zehnder interferometer.



By adding fields at the output. Beamsplitters divide optical power equally \Rightarrow

$$I_o = \frac{1}{2}I_i + \frac{1}{2}I_i \cos \phi = I_i \cos^2 \frac{\phi}{2}$$

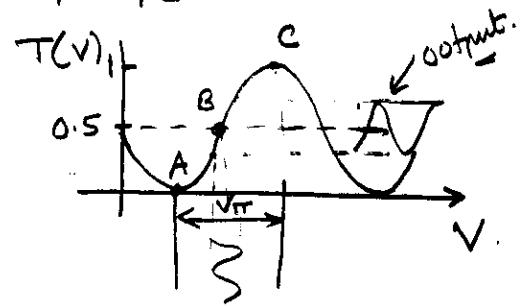
$\phi = \phi_1 - \phi_2$ - difference between phase shifts encountered by light.

$$T = \frac{I_o}{I_i} = \cos^2 \left(\frac{\phi}{2} \right)$$

Add modulator: in branch 1 $\Rightarrow \phi_1 = \phi_0 - \frac{\pi V}{V_\pi}$

$$\Rightarrow \phi = \phi_1 - \phi_2 = \phi_0 - \frac{\pi V}{V_\pi} \quad \phi_0 = \phi_0 - \phi_2$$

$$\Rightarrow T(V) = \cos^2 \left(\frac{\phi_0}{2} - \frac{\pi V}{2V_\pi} \right)$$

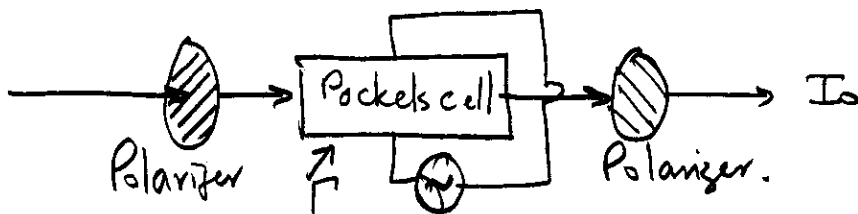


$$\phi_0 = \frac{\pi}{2}, T = 0.5 \quad \text{or} \quad \phi_0 = n2\pi \quad T(0) = 1$$

$$T(V_\pi) = 0$$

modulator switches the light on/off as V $0/V_\pi$.

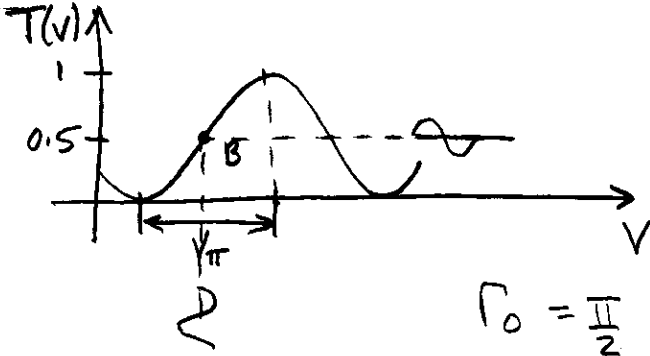
Intensity Modulators: Use of a retarder between crossed polarizer.



$$T = \sin^2\left(\frac{\Gamma}{2}\right)$$

if retarder is a Pockels cell, Γ linearly dependent on applied voltage V .

$$\Rightarrow \Gamma = \Gamma_0 - \frac{\pi V}{V_\pi} \Rightarrow T = \sin^2\left(\frac{\Gamma_0}{2} - \frac{\pi}{2} \frac{V}{V_\pi}\right)$$



Linear operation: cell is biased to point B.

$$\Gamma_0 = \frac{\pi}{2} \quad V \ll V_\pi$$

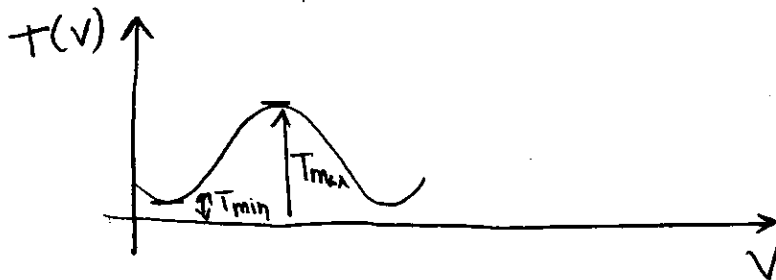
$$T(V) = \sin^2\left(\frac{\pi}{4} - \frac{\pi}{2} \frac{V}{V_\pi}\right) = T(0) + \frac{dT}{dV}\bigg|_{V=0} V = \frac{1}{2} - \frac{\pi}{2} \frac{V}{V_\pi}$$

Eaylor series expansion.

$\frac{\pi}{2V_\pi}$ - sensitivity of modulator.

Γ_0 - adjusted optically (adding phase retarder, a compensator) or electrically by adding a constant bias voltage to V .
i.e., $V = V_0 + V_{mod}$.

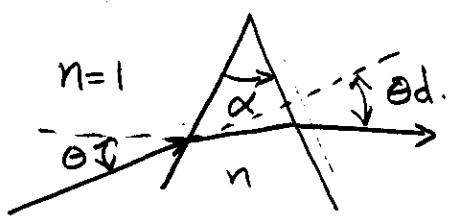
Ratio between maximum and minimum transmittances is called the extinction ratio.



real devices.

Ratios $\frac{T_{max}}{T_{min}} > 1000:1$ are possible.

Prism as Scanner.



$$\theta_d = \theta - \alpha + \sin^{-1} \left((n^2 - \sin^2 \theta)^{1/2} \sin \alpha - \sin \theta \cos \alpha \right)$$

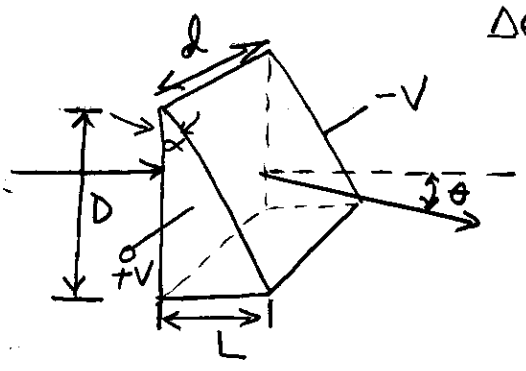
determined by applying Snell's law twice.

if α, θ small. $\Rightarrow \theta_d \approx (n-1)\alpha$.

\Rightarrow if we use Pockel effect.

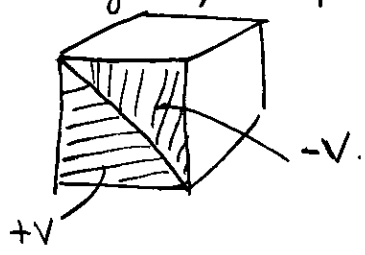
$$\theta_d = (n-1)\alpha$$

$$\Delta \theta_d = \alpha \Delta n = -\frac{1}{2} \alpha r n^3 E = -\frac{1}{2} \alpha r n^3 \frac{V}{d}$$



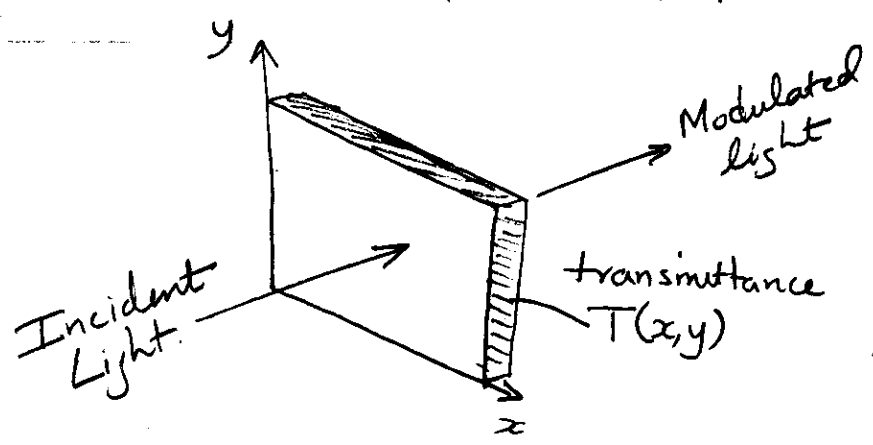
Incident light is scanned.

More convenient to use triangularly shaped electrodes on a cubic crystal.



Spatial Light Modulators.

- modulates intensity of light at different positions by prescribed factors.

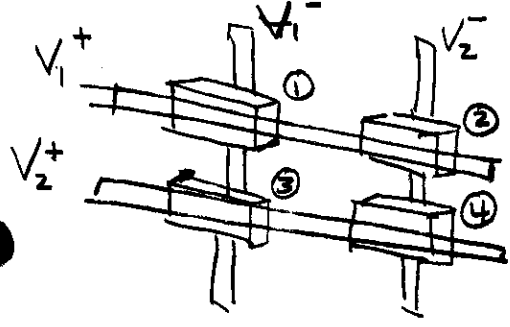


$$I_{out}(x,y) = I_i(x,y) T(x,y)$$

If $I_i(x,y)$ is uniform in intensity $\Rightarrow I_{out} \propto T(x,y)$.

\therefore we read the image $T(x,y)$

$T(x,y)$ is controllable.



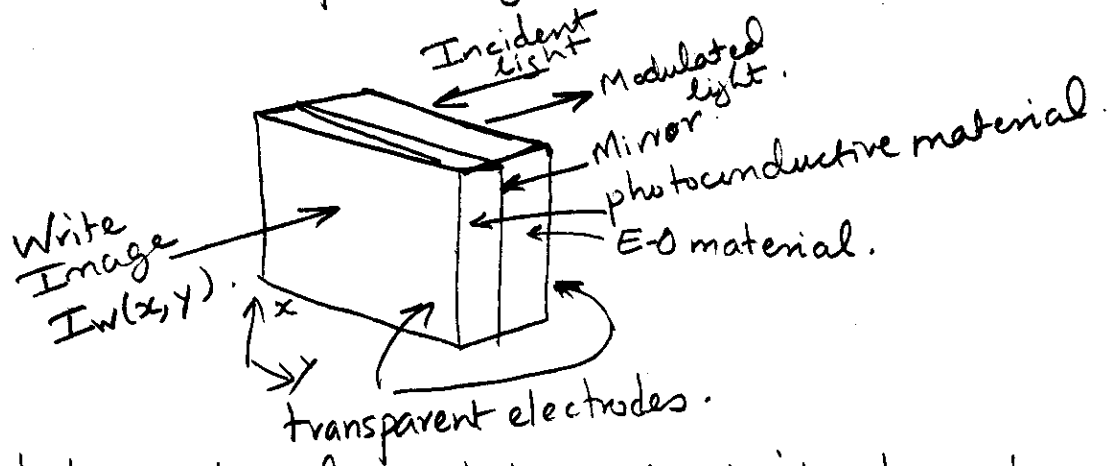
$$\left. \begin{aligned}
 V_0 &= V_1^+ - V_1^- \\
 V_2 &= V_1^+ - V_2^- \\
 V_3 &= V_2^+ - V_1^- \\
 V_4 &= V_2^+ - V_2^-
 \end{aligned} \right\}$$

can modify voltage across each device.
 \Rightarrow controllable transmission

\Rightarrow each e-o modulator acts as a pixel as each gets smaller. $T(x_i, y_i) \rightarrow T(x, y)$.
 $i = 1, 2, 3, \dots$

(Used in Liquid-crystal spatial light modulators used for display).

Optically addressed E-O spatial light modulator



Photoconductive material: photo conductivity depends on light intensity. $G(x, y)$.

$I_w(x, y)$ - image written in spatial intensity.

$$G(x, y) \propto I_w(x, y) \quad E(x, y) \propto \frac{1}{G(x, y)} \propto \frac{1}{I_w(x, y)}$$

$$T(x, y) \propto E(x, y) \propto \frac{1}{I_w(x, y)}$$

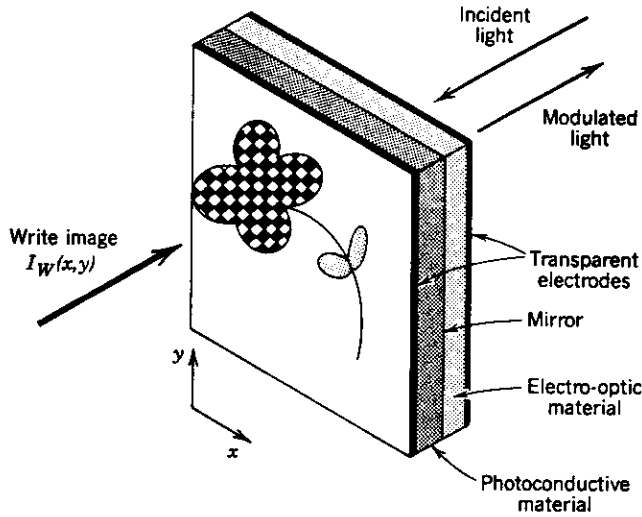


Figure 18.1-14 The electro-optic spatial light modulator uses a photoconductive material to create a spatial distribution of electric field which is used to control an electro-optic material.

The Pockels Readout Optical Modulator

An ingenious implementation of this principle is the Pockels readout optical modulator (PROM). The device uses a crystal of bismuth silicon oxide, $\text{Bi}_{12}\text{SiO}_{20}$ (BSO), which has an unusual combination of optical and electrical properties: (1) it exhibits the electro-optic (Pockels) effect; (2) it is photoconductive for blue light, but not for red light; and (3) it is a good insulator in the dark. The PROM (Fig. 18.1-15) is made of a thin wafer of BSO sandwiched between two transparent electrodes. The light that is to be modulated (read light) is transmitted through a polarizer, enters the BSO layer, and is reflected by a dichroic reflector, whereupon it crosses a second polarizer. The reflector reflects red light but is transparent to blue light. The PROM is operated as

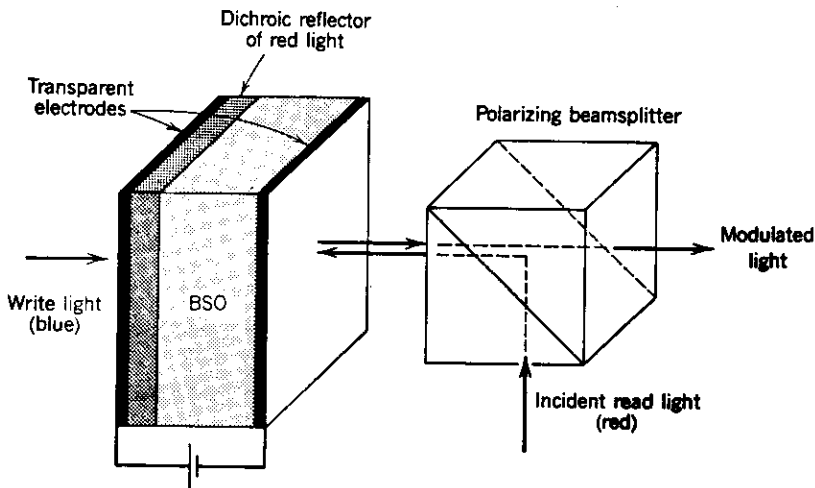


Figure 18.1-15 The Pockels readout optical modulator (PROM).